

ANSWER MAZE



INDUCTIVE REASONING, CONJECTURE AND COUNTEREXAMPLE

Counterexamples: Check 1 First, and One Case Is Enough

One page · 10–15 minutes · nothing to prepare

FREE

1

Claim: If a number x is greater than 1, then $x \times x$ is greater than $x + 1$. For $x = 0$: $x \times x = 0$, and $x + 1 = 1$. For $x = 1.5$: $x \times x = 2.25$, and $x + 1 = 2.5$. Which value of x is a counterexample?

0 → go to box 4

1.5 → go to box 5

Hint: Check 1 comes first. Is x greater than 1?

2

Claim: If n is a whole number, then $n \times n + n + 11$ is prime. It works for every n from 0 to 9. For $n = 10$, $n \times n + n + 11 = 121$, and $121 = 11 \times 11$. Is the claim true?

yes → go to box 6

no → you are out

Hint: Look at the case $n = 10$. Does it pass both checks?

3

Claim: If a whole number is odd and greater than 1, then it is prime. Which number is a counterexample to the claim?

10 → go to box 4

9 → go to box 7

Hint: Check 1 comes first. Is the number odd, and is it greater than 1?

4

Claim: If a shape is a triangle, then exactly one of its sides is the longest. Which shape is a counterexample: a triangle with sides 5, 5 and 5, or a four-sided shape with sides 6, 4, 6 and 4?

the triangle → go to box 7

the four-sided shape → go to box 3

Hint: Check 1 comes first. Is the shape a triangle?

Answer Maze — how to run it

10–15 minutes

1. **Hand out page 3.** Nothing else is needed.
2. **Say this once:** *“Check 1 first: does the case fit the if part? One case that passes both checks shows that a claim is false.”*
3. **Start at box 3.** Six boxes and you are out. **Write every box number in the row at the bottom as you go.**

Student sheet	page 3, one per student. The two checks are printed on it.
Answers	page 4.

The route is 3 → 7 → 1 → 5 → 8 → 2 → out. Boxes 4 and 6 are not on it. They look like every other box, and both of their answers lead back onto the route.

Each box off the route catches one mistake. Three boxes send a wrong answer to box 4. A student there skipped check 1 and chose a case the claim is not about. For example, 10 is not prime, but 10 is not odd, so the claim says nothing about it. Three boxes send a wrong answer to box 6. A student there trusted the cases that agree with the claim. They say things like “6 is a special case”, “it works for ten numbers, so it is true” or “one case is not enough”.

What the row at the bottom tells you. A clean run is **six numbers**. A longer row shows a mistake. The number just before 4 or 6 is the box the student got wrong.



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Answer Maze

Start at box 3. Answer, then go to the box your answer names.

Name _____

Period _____

Date _____

A counterexample passes two checks. Check 1: the case fits the *if* part of the claim. **Check 2:** for this case, the *then* part is false. Both checks: it is a counterexample. If check 1 fails, the claim is not about this case.

One case can show that a claim is false. No number of cases can show that it is true.

1

Claim: If a number x is greater than 1, then $x \times x$ is greater than $x + 1$. For $x = 0$: $x \times x = 0$, and $x + 1 = 1$. For $x = 1.5$: $x \times x = 2.25$, and $x + 1 = 2.5$. Which value of x is a counterexample?

0 → go to box 4

1.5 → go to box 5

Hint: Check 1 comes first. Is x greater than 1?

2

Claim: If n is a whole number, then $n \times n + n + 11$ is prime. It works for every n from 0 to 9. For $n = 10$, $n \times n + n + 11 = 121$, and $121 = 11 \times 11$. Is the claim true?

yes → go to box 6

no → you are out

Hint: Look at the case $n = 10$. Does it pass both checks?

3

Claim: If a whole number is odd and greater than 1, then it is prime. Which number is a counterexample to the claim?

10 → go to box 4

9 → go to box 7

Hint: Check 1 comes first. Is the number odd, and is it greater than 1?

4

Claim: If a shape is a triangle, then exactly one of its sides is the longest. Which shape is a counterexample: a triangle with sides 5, 5 and 5, or a four-sided shape with sides 6, 4, 6 and 4?

the triangle → go to box 7

the four-sided shape → go to box 3

Hint: Check 1 comes first. Is the shape a triangle?

5

Claim: If a whole number is a multiple of 4 and a multiple of 6, then it is a multiple of 24. Kim finds one case. 12 is a multiple of 4 and a multiple of 6, but 12 is not a multiple of 24. Is one case enough to show that the claim is false?

yes → go to box 8

no → go to box 6

Hint: Look at the case 12. Does it pass both checks?

6

Lee tries 50 whole numbers on a claim. Lee cannot find a counterexample. Does that show that the claim has no counterexample?

yes → go to box 7

no → go to box 1

Hint: Look at what Lee checks, and at what Lee does not check.

7

Priya's claim: If a whole number is a multiple of 3, then it is odd. It works for 3, 9, 15, 21 and 27. It does not work for 6, because 6 is even. Is Priya's claim true?

no → go to box 1

yes → go to box 6

Hint: Look at the case 6. Does it pass both checks?

8

Claim: If a whole number ends in 6, then it is a multiple of 6. Which number is a counterexample to the claim?

16 → go to box 2

20 → go to box 4

Hint: Check 1 comes first. Does the number end in 6?

Write every box number here as you go. This helps you find your way back if you lose your place. **A clean run is six numbers.**

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Answer Maze — answers

For the teacher

The route, in order. Boxes 4 and 6 are not on the route. They are at the bottom of the table. A student reaches them only with a wrong answer.

3 → 7 → 1 → 5 → 8 → 2 → out

	Box	Answer	Wrong answer, and the box it goes to	Right answer goes to
1	Box 3	9	10 → box 4 (skipped check 1)	box 7
2	Box 7	no	yes → box 6 (trusted the cases that agree)	box 1
3	Box 1	1.5	0 → box 4 (skipped check 1)	box 5
4	Box 5	yes	no → box 6 (trusted the cases that agree)	box 8
5	Box 8	16	20 → box 4 (skipped check 1)	box 2
6	Box 2	no	yes → box 6 (trusted the cases that agree)	out
	Box 4	the triangle	the four-sided shape → box 3 (skipped check 1)	box 7
	Box 6	no	yes → box 7 (trusted the cases that agree)	box 1

Why each right answer is right. Box 3: 9 is odd and greater than 1, and $9 = 3 \times 3$ is not prime. 10 is not prime either, but 10 is even, so the claim is not about it. Box 1: 1.5 is greater than 1, and 2.25 is not greater than 2.5. 0 is not greater than 1. Box 8: 16 ends in 6 and is not a multiple of 6. 20 does not end in 6. Box 4: the triangle 5, 5, 5 has three longest sides. The four-sided shape is not a triangle. Boxes 7, 5 and 2: one case that passes both checks shows that a claim is false, however many cases agree with it. Box 6: not finding a counterexample does not show that there is none.