

MYSTERY PICTURE

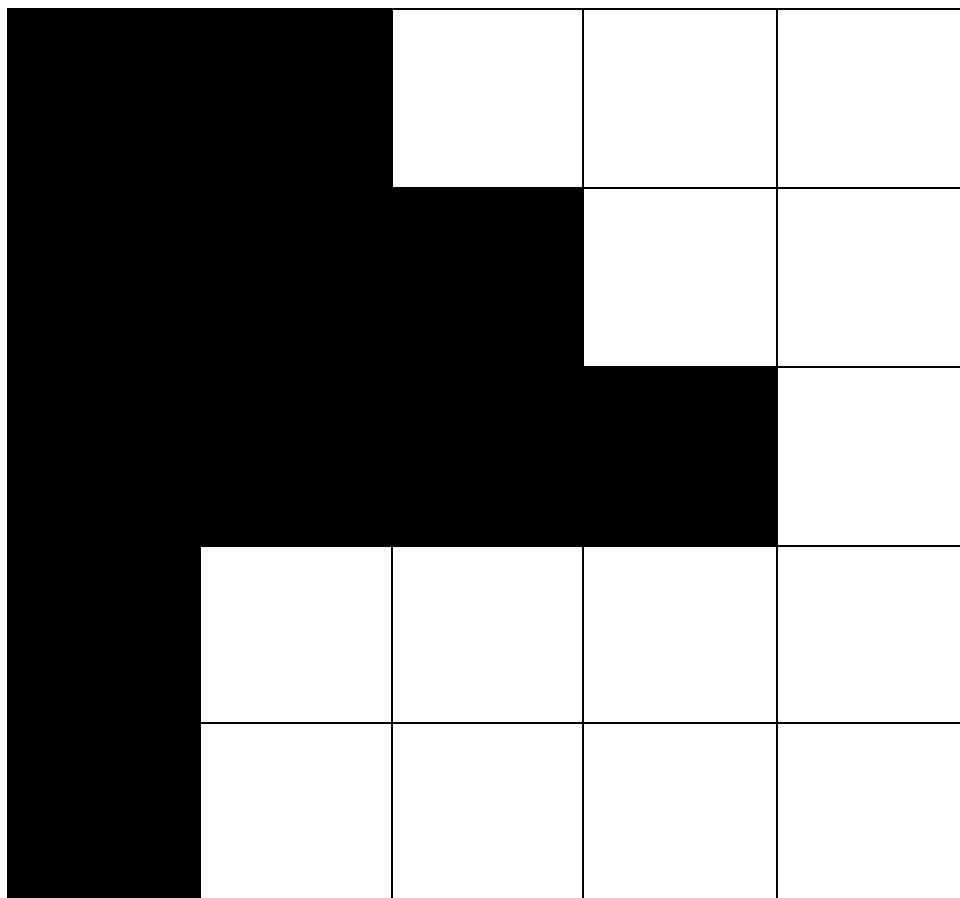


BICONDITIONALS AND DEFINITIONS

If and Only If: Is It True Both Ways?

25 sentences · check each one forwards and backwards · the picture checks the answers

FREE



Mystery Picture — how to run it

25–35 minutes

1. **Hand out page 3, one per student.** Nothing else is needed.
2. **Say this once:** *“An if and only if sentence makes two claims. Check it forwards. Then check it backwards. Fill the square only if both are true.”*
3. **A correct sheet shows a flag.** In a wrong square, look at which T or F the student circled.

Student sheet page 3, one per student. The example at the top shows the two directions.

Answers page 4: the finished picture, both truth values for every square, and a case that breaks each false direction.

Each square asks for two answers, not one. A common mistake is to write both directions and then judge the sentence by only one of them. So the student circles T or F for each direction. Of the 25 sentences, 11 are true both ways.

The false sentences fail in different ways. Six are true forwards and false backwards. Each of these is a true statement joined to a false converse, the most common mistake in this lesson. Another 6 are false forwards and true backwards, and 2 are false both ways. A student who checks only forwards, or only backwards, gets at least 6 squares wrong, and the flag looks wrong.

No square has a drawing. Some squares are about a midpoint or an angle bisector. To see that one of them is false, the student must picture a point that is not on the segment, or a ray outside the angle. The lesson uses the same cases. The answer key gives a case that breaks each false direction, in case a student asks.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture

Is it true both ways?

Name _____

Period _____

Date _____

An **if and only if** sentence makes two claims. Example: *A number is even if and only if it ends in 0, 2, 4, 6 or 8.*

Forwards: If a number is even, then it ends in 0, 2, 4, 6 or 8. **Backwards:** If a number ends in 0, 2, 4, 6 or 8, then it is even. Both are true here. A claim is false if even one case does not work.

In each square, circle T or F for each claim. If both are T, fill the square with slanted lines. If one is F, leave it white. When you finish, a picture appears. If part of it looks wrong, check that square again.

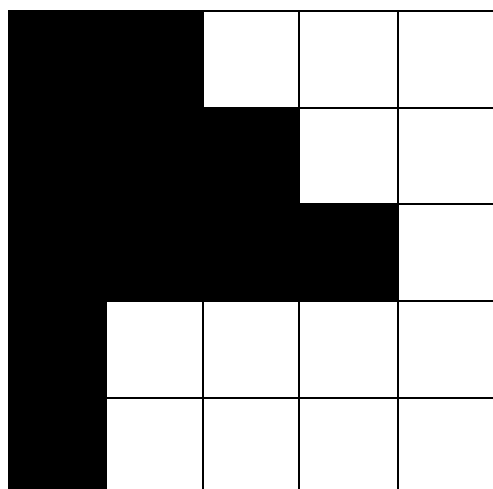
1 Two angles are supplementary if and only if their measures add to 180°. Forwards: T F Backwards: T F	2 M is the midpoint of segment AB if and only if M is on segment AB and $AM = MB$. Forwards: T F Backwards: T F	3 Two angles are supplementary if and only if they form a linear pair. Forwards: T F Backwards: T F	4 M is the midpoint of segment AB if and only if $AM = MB$. Forwards: T F Backwards: T F	5 An angle measures 90° or less if and only if it is acute. Forwards: T F Backwards: T F
6 An angle is acute if and only if it measures less than 90°. Forwards: T F Backwards: T F	7 Ray BD is the bisector of angle ABC if and only if D is in the interior of angle ABC and angles ABD and DBC are congruent. Forwards: T F Backwards: T F	8 An angle is a right angle if and only if it measures 90°. Forwards: T F Backwards: T F	9 Two angles are complementary if and only if both of them measure less than 90°. Forwards: T F Backwards: T F	10 Two lines intersect if and only if they are perpendicular. Forwards: T F Backwards: T F
11 Two angles form a linear pair if and only if they are adjacent and their other sides are opposite rays. Forwards: T F Backwards: T F	12 Two lines are perpendicular if and only if they meet to form a right angle. Forwards: T F Backwards: T F	13 An angle is obtuse if and only if it measures more than 90° and less than 180°. Forwards: T F Backwards: T F	14 Two angles are vertical angles if and only if their sides form two pairs of opposite rays. Forwards: T F Backwards: T F	15 Ray BD is the bisector of angle ABC if and only if angles ABD and DBC are congruent. Forwards: T F Backwards: T F
16 Two angles are complementary if and only if their measures add to 90°. Forwards: T F Backwards: T F	17 Two angles are congruent if and only if they are vertical angles. Forwards: T F Backwards: T F	18 Two angles are complementary if and only if they are adjacent. Forwards: T F Backwards: T F	19 An angle is obtuse if and only if it measures 90° or more. Forwards: T F Backwards: T F	20 Two angles are adjacent if and only if they form a linear pair. Forwards: T F Backwards: T F
21 An angle is a straight angle if and only if it measures 180°. Forwards: T F Backwards: T F	22 Two angles are supplementary if and only if at least one of them is not acute. Forwards: T F Backwards: T F	23 Two angles are congruent if and only if they are supplementary. Forwards: T F Backwards: T F	24 Two angles are supplementary if and only if each of them measures 90°. Forwards: T F Backwards: T F	25 Ray BD is the bisector of angle ABC if and only if D is in the interior of angle ABC. Forwards: T F Backwards: T F



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Mystery Picture — answers

For the teacher



The finished sheet is **a flag**.

T means true and F means false. Forwards means “if the first part, then the second part.” Backwards means “if the second part, then the first part.”

Square	Answer	Forwards, backwards	A case that breaks the false direction
1	FILL	T T	
2	FILL	T T	
3	white	F T	forwards: angles of 60° and 120° , at two different vertices
4	white	T F	backwards: M at (4, 3) with A at (0, 0) and B at (8, 0): $AM = 5$, $MB = 5$, not on segment AB
5	white	F T	forwards: an angle of 90°
6	FILL	T T	
7	FILL	T T	
8	FILL	T T	
9	white	T F	backwards: angles of 60° and 60° , at two different vertices
10	white	F T	forwards: two lines that meet at 60°
11	FILL	T T	
12	FILL	T T	
13	FILL	T T	
14	FILL	T T	
15	white	T F	backwards: angle ABC of 100° with D outside it: angle ABD = 130° , angle DBC = 130°
16	FILL	T T	
17	white	F T	forwards: angles of 60° and 60° , at two different vertices
18	white	F F	forwards: angles of 60° and 30° , at two different vertices; backwards: angles of 60° and 150° , adjacent angles
19	white	T F	backwards: an angle of 90°
20	white	F T	forwards: angles of 60° and 150° , adjacent angles
21	FILL	T T	
22	white	T F	backwards: angles of 60° and 90° , at two different vertices
23	white	F F	forwards: angles of 60° and 60° , at two different vertices; backwards: angles of 60° and 120° , at two different vertices
24	white	F T	forwards: angles of 60° and 120° , at two different vertices
25	white	T F	backwards: angle ABC of 100° with D inside it: angle ABD = 5° , angle DBC = 95°