

CUT & GLUE SORT



INDIRECT PROOF

Indirect Proof: Can Both Be True?

12 pieces · 3 rows · a definition, the angle sum, or no contradiction

FREE

A In $\triangle DEF$, $m\angle D = 70^\circ$ and $m\angle E = 60^\circ$. $m\angle F = 60^\circ$.	B $m\angle 1 = 89.5^\circ$. $\angle 1$ is a right angle.	C In $\triangle DEF$, $\angle F$ is obtuse. $m\angle D = 1^\circ$ and $m\angle E = 1^\circ$.	D In $\triangle ABC$, $\angle B$ is obtuse. $m\angle B = 90^\circ$.
E Ray BD bisects $\angle ABC$ and $m\angle ABD = 40^\circ$. $\angle ABC$ is acute.	F In $\triangle ABC$, $\angle A$ is a right angle. $\angle B$ is a right angle.	G M is the midpoint of AB . $AM = 4$ and $MB = 6$.	H In $\triangle ABC$, $m\angle A = 89^\circ$. All three angles of $\triangle ABC$ are acute.

Cut & Glue Sort — how to run it

25 minutes, with the cutting

1. **Hand out pages 3 and 4, one of each per student.** Students need scissors and glue. Nothing else is needed.
2. **Say this once:** *“A contradiction needs something to point at: a definition or a fact. If there is nothing to point at, the two sentences can both be true.”*
3. **Check with page 5.** Every row gets 4 pieces.

Student sheets	pages 3 (the rows) and 4 (the pieces), one of each per student.
Answers	page 5.

An indirect proof ends when two statements cannot both be true. The student must be able to point at the reason. In this lesson, the reason is almost always a definition, or the fact that the three angles of a triangle add to 180° . Students accept this angle sum for now, and it is proved later. So row 2 calls it an accepted fact.

Row 3 holds pieces whose two sentences can both be true. They look strange (angles of 1° , an angle of 89° in an acute triangle, a midpoint at 5.4), but nothing forbids them. Many students stop at “that can't be right” and call this a contradiction. It is not. Strange numbers appear in the other rows too, such as an angle of 89.5° called right, or two angles of 1° in a triangle. So a strange number does not decide the row.

No shortcut sorts the sheet. Triangles appear in every row, and every simple guessing rule puts at most 8 of the 12 pieces in the right row.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort

Can both be true?

Name _____

Period _____

Date _____

Each piece has two sentences about one figure. Can they both be true? If not, find what says no: the meaning of a word, or the angle sum of a triangle. Glue the piece in the row that says it.

1. They cannot both be true. The meaning of a word forbids it.

A definition says no: acute, right, obtuse, midpoint, bisects.

2. They cannot both be true. The three angles of a triangle add to 180° .

An accepted fact says no. No single word does.

3. They can both be true. It may look strange, but nothing forbids it.

This is not a contradiction.

Cut & Glue Sort — the pieces

Cut along the dashed lines

acute: less than 90° · **right:** exactly 90° · **obtuse:** more than 90° and less than 180°

M is the midpoint of AB: M is on AB and $AM = MB$ · **ray BD bisects $\angle ABC$:** $m\angle ABD = m\angle DBC$

Cut out the 12 pieces. Read the two sentences on each one. Then glue the piece in a row on the other sheet.

A In $\triangle DEF$, $m\angle D = 70^\circ$ and $m\angle E = 60^\circ$. $m\angle F = 60^\circ$.	B $m\angle 1 = 89.5^\circ$. $\angle 1$ is a right angle.	C In $\triangle DEF$, $\angle F$ is obtuse. $m\angle D = 1^\circ$ and $m\angle E = 1^\circ$.	D In $\triangle ABC$, $\angle B$ is obtuse. $m\angle B = 90^\circ$.
E Ray BD bisects $\angle ABC$ and $m\angle ABD = 40^\circ$. $\angle ABC$ is acute.	F In $\triangle ABC$, $\angle A$ is a right angle. $\angle B$ is a right angle.	G M is the midpoint of AB. $AM = 4$ and $MB = 6$.	H In $\triangle ABC$, $m\angle A = 89^\circ$. All three angles of $\triangle ABC$ are acute.
I In $\triangle KLM$, $m\angle K = 1^\circ$ and $m\angle L = 1^\circ$. $\angle M$ is acute.	J In $\triangle PQR$, ray PS bisects $\angle P$. $m\angle QPS = 30^\circ$ and $m\angle SPR = 35^\circ$.	K M is the midpoint of AB. $AM = 5.4$ and $AB = 10.8$.	L In $\triangle PQR$, $\angle P$ is obtuse. $\angle Q$ is obtuse.



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Cut & Glue Sort — answers

For the teacher

Row	What the row says, and the reason for each piece	Pieces
1	They cannot both be true. The meaning of a word forbids it. B: the meaning of “right angle”. D: the meaning of “obtuse”. G: the meaning of “midpoint”. J: the meaning of “bisects”.	B, D, G, J
2	They cannot both be true. The three angles of a triangle add to 180°. A: the angle sum. F: the angle sum. I: the angle sum. L: the angle sum.	A, F, I, L
3	They can both be true. It may look strange, but nothing forbids it. C: both are true when $m\angle F = 178^\circ$, $m\angle D = 1^\circ$, $m\angle E = 1^\circ$. E: both are true when $m\angle ABD = 40^\circ$, $m\angle DBC = 40^\circ$, $m\angle ABC = 80^\circ$. H: both are true when $m\angle A = 89^\circ$, $m\angle B = 89^\circ$, $m\angle C = 2^\circ$. K: both are true when $AM = 5.4$, $MB = 5.4$, $AB = 10.8$.	C, E, H, K