



INDIRECT PROOF

Indirect Proof: Assume, Collide, Close

8 boxes · the three steps of an indirect proof · no dead ends

FREE

5

To prove: $m\angle P > 40^\circ$. What is the temporary assumption?

- Assume temporarily that $m\angle P < 40^\circ$. **Go to box 3.**
- Assume temporarily that $m\angle P \leq 40^\circ$. **Go to box 2.**

Hint: Check an angle of exactly 40° .

2

To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. What is the temporary assumption?

- Assume temporarily that $\angle B$ is a right angle. **Go to box 7.**
- Assume temporarily that $m\angle A \neq 90^\circ$. **Go to box 3.**

Hint: Find the then part of the statement.

7

To prove: All three angles of $\triangle ABC$ are acute. What is the temporary assumption?

- Assume temporarily that no angle of $\triangle ABC$ is acute. **Go to box 8.**
- Assume temporarily that at least one angle of $\triangle ABC$ is not acute. **Go to box 4.**

Hint: Ask the second question: could both sentences be false?

4

$\angle 1$ and $\angle 2$ are complementary. $m\angle 1 = 89^\circ$. Can both sentences be true?

- Yes. They can both be true. **Go to box 1.**
- No. The meaning of “complementary” forbids it. **Go to box 8.**

Hint: Find $m\angle 2$ first.

1. **Hand out page 3.** Students start at box 5. Nothing else is needed.
2. **Say this once:** *“An indirect proof has three steps. First, assume for now that the statement is false. Next, find two sentences that cannot both be true. Last, say what is proved.”*
3. **Check the row of box numbers at the foot.** A clean run is 5, 2, 7, 4, 1, 6, then Finish.

Student sheet page 3, one per student.

Answers page 4: the route, and the right answer in every box.

The boxes follow the three steps of the method. In 5 boxes the student chooses the temporary assumption. In 2 boxes the student decides whether two sentences can both be true. In 1 box the student chooses the last sentence of the proof. Every wrong answer is a common error. It sends the student to a box off the route (3 and 8) or back one step. Every box has a task, so nobody is stuck.

“They can both be true” is right in one box and wrong in another. In box 4 the angles are 89° and 1° . That looks strange, but the angles are complementary, so both sentences can be true. Many students call this a contradiction. In box 1 the definition of midpoint forbids $AM = 5$ and $MB = 7$, so they cannot both be true.

Box 6 asks for the last sentence of the proof. The wrong answer, “Therefore the opposite is true”, does not say which statement is meant. The right ending says two things: the assumption is false, and the statement is true.

Guessing does not reach Finish. Every simple guessing rule, such as always taking the first answer, goes wrong within the first 3 boxes of the route. The hints say what to look at, never the answer.



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Name _____

Period _____

Date _____

Start at box 5. Read the task. Choose one of the two answers. Then go to the box it names.

Test the assumption with two questions.

1. Could the statement and the assumption both be true?
2. Could they both be false?

If either answer is yes, the assumption is wrong. If both answers are no, it is correct.

For an "If ..., then ..." statement, the if part is given. Test the assumption against the then part.

acute: less than 90° · **right:** exactly 90° · **obtuse:** more than 90° and less than 180°

$\leq$ means less than or equal to · $\geq$ means greater than or equal to · $\neq$ means not equal to

supplementary: the two measures add to 180° · **complementary:** they add to 90° · **M is the midpoint of AB:** M is on AB and $AM = MB$

1

M is the midpoint of AB. $AM = 5$ and $MB = 7$. Can both sentences be true?

- Yes. They can both be true. **Go to box 4.**
- No. The meaning of "midpoint" forbids it. **Go to box 6.**

Hint: Compare AM and MB.

2

To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. What is the temporary assumption?

- Assume temporarily that $\angle B$ is a right angle. **Go to box 7.**
- Assume temporarily that $m\angle A \neq 90^\circ$. **Go to box 3.**

Hint: Find the then part of the statement.

3

To prove: If $m\angle 1 = 90^\circ$, then $\angle 1$ is not acute. What is the temporary assumption?

- Assume temporarily that if $m\angle 1 \neq 90^\circ$, then $\angle 1$ is acute. **Go to box 5.**
- Assume temporarily that $\angle 1$ is acute. **Go to box 2.**

Hint: Find the then part of the statement.

4

$\angle 1$ and $\angle 2$ are complementary. $m\angle 1 = 89^\circ$. Can both sentences be true?

- Yes. They can both be true. **Go to box 1.**
- No. The meaning of "complementary" forbids it. **Go to box 8.**

Hint: Find $m\angle 2$ first.

5

To prove: $m\angle P > 40^\circ$. What is the temporary assumption?

- Assume temporarily that $m\angle P < 40^\circ$. **Go to box 3.**
- Assume temporarily that $m\angle P \leq 40^\circ$. **Go to box 2.**

Hint: Check an angle of exactly 40° .

6

To prove: $\angle 2$ is not a right angle. Given: $m\angle 2 = 70^\circ$. Assume temporarily that $\angle 2$ is a right angle. Then $m\angle 2 = 90^\circ$. But the given says $m\angle 2 = 70^\circ$. What is the last sentence?

- So the assumption is false, and $\angle 2$ is not a right angle. **Go to Finish.**
- Therefore the opposite is true. **Go to box 1.**

Hint: A proof ends with the statement it proves.

7

To prove: All three angles of $\triangle ABC$ are acute. What is the temporary assumption?

- Assume temporarily that no angle of $\triangle ABC$ is acute. **Go to box 8.**
- Assume temporarily that at least one angle of $\triangle ABC$ is not acute. **Go to box 4.**

Hint: Ask the second question: could both sentences be false?

8

To prove: Exactly one of the points A, B, C lies on line n. What is the temporary assumption?

- Assume temporarily that none of A, B, C lies on line n, or at least two of them do. **Go to box 4.**
- Assume temporarily that none of A, B, C lies on line n. **Go to box 7.**

Hint: Ask the second question: could both sentences be false?

Write each box number here, in order, as you go. Start with 5.

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Maze — answers

For the teacher

The route: $5 \rightarrow 2 \rightarrow 7 \rightarrow 4 \rightarrow 1 \rightarrow 6 \rightarrow$ Finish. Boxes 3 and 8 are off the route.

Box	Place on the route	Right answer	Right answer goes to
1	step 5	No. The meaning of “midpoint” forbids it. <i>The other answer is wrong. The definition of midpoint forbids $AM = 5$ and $MB = 7$, so this is a real contradiction.</i>	6
2	step 2	Assume temporarily that $\angle B$ is a right angle. <i>The other answer is wrong. It denies the if part, but the if part is given.</i>	7
3	off the route	Assume temporarily that $\angle 1$ is acute. <i>The other answer is wrong. It is the inverse, another if-then sentence, and not an assumption.</i>	2
4	step 4	Yes. They can both be true. <i>The other answer is wrong. The numbers look strange ($m\angle 2 = 1^\circ$), but nothing forbids them.</i>	1
5	step 1	Assume temporarily that $m\angle P \leq 40^\circ$. <i>The other answer is wrong. It is the everyday opposite, and it leaves out $m\angle P = 40^\circ$.</i>	2
6	step 6	So the assumption is false, and $\angle 2$ is not a right angle. <i>The other answer is wrong. “The opposite” does not say which statement is meant.</i>	Finish
7	step 3	Assume temporarily that at least one angle of $\triangle ABC$ is not acute. <i>The other answer is wrong. “None” is not the complement of “all”.</i>	4
8	off the route	Assume temporarily that none of A, B, C lies on line n, or at least two of them do. <i>The other answer is wrong. “None” leaves out two or more points.</i>	4