

MYSTERY PICTURE



INDIRECT PROOF

Indirect Proof: Is the Assumption Correct?

25 squares · a statement and an assumption in each · fill or leave white

FREE

¹ To prove: $AB > CD$. Assume temporarily that $AB < CD$.	² To prove: $\angle B$ is not obtuse. Assume temporarily that $\angle B$ is obtuse.	³ To prove: Exactly one of the points A, B, C lies on line n. Assume temporarily that none of A, B, C lies on line n, or at least two of them do.	⁴ To prove: $\angle 1$ and $\angle 2$ are supplementary. Assume temporarily that $m\angle 1 + m\angle 2 \neq 180^\circ$.	⁵ To prove: $m\angle P > 40^\circ$. Assume temporarily that $m\angle P < 40^\circ$.
⁶ To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that $\angle B$ is a right angle.	⁷ To prove: All three angles of $\triangle ABC$ are acute. Assume temporarily that no angle of $\triangle ABC$ is acute.	⁸ To prove: $m\angle Q \leq 60^\circ$. Assume temporarily that $m\angle Q \geq 60^\circ$.	⁹ To prove: $\angle B$ is not obtuse. Assume temporarily that $\angle B$ is acute.	¹⁰ To prove: AB is not perpendicular to CD. Assume temporarily that AB is parallel to CD.
¹¹ To prove: M is not the midpoint of AB. Assume temporarily that M is the midpoint of AB.	¹² To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that $m\angle A \neq 90^\circ$.	¹³ To prove: At most one of the points A, B, C lies on line n. Assume temporarily that exactly two of A, B, C lie on line n.	¹⁴ To prove: $\angle K$ is not acute. Assume temporarily that $\angle K$ is not obtuse.	¹⁵ To prove: $\angle 1$ and $\angle 2$ are supplementary. Assume temporarily that $\angle 1$ and $\angle 2$ are complementary.
¹⁶ To prove: All three angles of $\triangle ABC$ are acute. Assume temporarily that at least one angle of $\triangle ABC$ is not acute.	¹⁷ To prove: M is not the midpoint of AB. Assume temporarily that $AM \neq MB$.	¹⁸ To prove: Exactly one of the points A, B, C lies on line n. Assume temporarily that none of A, B, C lies on line n.	¹⁹ To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that if $m\angle A \neq 90^\circ$, then $\angle B$ is a right angle.	²⁰ To prove: $\triangle ABC$ has at most one obtuse angle. Assume temporarily that $\triangle ABC$ has no obtuse angle.
²¹ To prove: $\angle K$ is not acute. Assume temporarily that $\angle K$ is obtuse.	²² To prove: $AB > CD$. Assume temporarily that $AB \leq CD$.	²³ To prove: $m\angle P \neq 40^\circ$. Assume temporarily that $m\angle P = 40^\circ$.	²⁴ To prove: $\triangle ABC$ has at most one obtuse angle. Assume temporarily that $\triangle ABC$ has at least two obtuse angles.	²⁵ To prove: AB is not perpendicular to CD. Assume temporarily that AB is not perpendicular to CD.

Mystery Picture — how to run it

25–35 minutes

1. **Hand out page 3, one per student.** Students need a pencil. Nothing else is needed.
2. **Say this once:** *“A correct assumption says exactly that the statement is false. Use the two questions on the sheet to test it.”*
3. **Check the picture on page 4.** A correct sheet shows the letter C, with 9 squares filled.

Student sheet page 3, one per student.

Answers page 4: the picture, and why each white square is wrong.

Every square is the first step of an indirect proof. The student reads a statement and a temporary assumption and decides if the assumption is correct. The student's sheet gives two questions to test it. A correct assumption can never be true together with the statement, and it can never be false together with it.

Each of the 16 white squares shows a common error. In 5 of them the assumption is the everyday opposite, or it keeps the equal case: “acute” for “not obtuse”, or $<$ for $>$. In 4, words such as “all” or “at most one” are denied the wrong way, for example “none” for “all”. In 2, the assumption denies the if part or is the inverse. In 3, it changes the condition instead of denying it. In 2, it says the same as the statement. Students often say that an indirect proof assumes what it wants to prove. It does not. A correct assumption says exactly that the statement is false. It is made only to show that it leads to something impossible.

In an if-then statement, the if part is given. So the assumption denies the then part. The student's sheet says this in one line. 3 squares share one if-then statement, and only 1 of them is correct.

Guessing does not draw the picture. Every simple guessing rule, such as filling every square whose assumption drops a “not”, gets at least 6 of the 25 squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture

Is the assumption correct?

Name _____

Period _____

Date _____

Each square has a statement to prove and a temporary assumption. A correct assumption says exactly that the statement is false. We make it only to show that it leads to something impossible. Is the assumption in the square correct? If yes, fill the square with slanted lines. If no, leave it white. When you finish, a picture appears. If part of it looks wrong, check that square again.

Test the assumption with two questions.

1. Could the statement and the assumption both be true?
2. Could they both be false?

If either answer is yes, the assumption is wrong. If both answers are no, it is correct.
For an "If ..., then ..." statement, the if part is given.
Test the assumption against the then part.

acute: less than 90° · **right:** exactly 90° · **obtuse:** more than 90° and less than 180°

$\leq$ means less than or equal to · $\geq$ means greater than or equal to · $\neq$ means not equal to

supplementary: the two measures add to 180° · **complementary:** they add to 90° · **M is the midpoint of AB:** M is on AB and $AM = MB$

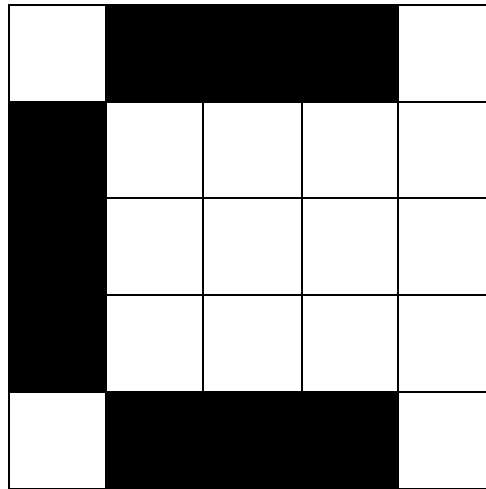
1 To prove: $AB > CD$. Assume temporarily that $AB < CD$.	2 To prove: $\angle B$ is not obtuse. Assume temporarily that $\angle B$ is obtuse.	3 To prove: Exactly one of the points A, B, C lies on line n. Assume temporarily that none of A, B, C lies on line n, or at least two of them do.	4 To prove: $\angle 1$ and $\angle 2$ are supplementary. Assume temporarily that $m\angle 1 + m\angle 2 \neq 180^\circ$.	5 To prove: $m\angle P > 40^\circ$. Assume temporarily that $m\angle P < 40^\circ$.
6 To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that $\angle B$ is a right angle.	7 To prove: All three angles of $\triangle ABC$ are acute. Assume temporarily that no angle of $\triangle ABC$ is acute.	8 To prove: $m\angle Q \leq 60^\circ$. Assume temporarily that $m\angle Q \geq 60^\circ$.	9 To prove: $\angle B$ is not obtuse. Assume temporarily that $\angle B$ is acute.	10 To prove: AB is not perpendicular to CD. Assume temporarily that AB is parallel to CD.
11 To prove: M is not the midpoint of AB. Assume temporarily that M is the midpoint of AB.	12 To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that $m\angle A \neq 90^\circ$.	13 To prove: At most one of the points A, B, C lies on line n. Assume temporarily that exactly two of A, B, C lie on line n.	14 To prove: $\angle K$ is not acute. Assume temporarily that $\angle K$ is not obtuse.	15 To prove: $\angle 1$ and $\angle 2$ are supplementary. Assume temporarily that $\angle 1$ and $\angle 2$ are complementary.
16 To prove: All three angles of $\triangle ABC$ are acute. Assume temporarily that at least one angle of $\triangle ABC$ is not acute.	17 To prove: M is not the midpoint of AB. Assume temporarily that $AM \neq MB$.	18 To prove: Exactly one of the points A, B, C lies on line n. Assume temporarily that none of A, B, C lies on line n.	19 To prove: If $m\angle A = 90^\circ$, then $\angle B$ is not a right angle. Assume temporarily that if $m\angle A \neq 90^\circ$, then $\angle B$ is a right angle.	20 To prove: $\triangle ABC$ has at most one obtuse angle. Assume temporarily that $\triangle ABC$ has no obtuse angle.
21 To prove: $\angle K$ is not acute. Assume temporarily that $\angle K$ is obtuse.	22 To prove: $AB > CD$. Assume temporarily that $AB \leq CD$.	23 To prove: $m\angle P \neq 40^\circ$. Assume temporarily that $m\angle P = 40^\circ$.	24 To prove: $\triangle ABC$ has at most one obtuse angle. Assume temporarily that $\triangle ABC$ has at least two obtuse angles.	25 To prove: AB is not perpendicular to CD. Assume temporarily that AB is not perpendicular to CD.



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Mystery Picture — answers

For the teacher



The letter C: 9 squares filled.

1. White. It is the everyday opposite. It leaves out $AB = CD$. Both are false when $AB = 10$, $CD = 10$.

2. Correct: fill.

3. Correct: fill.

4. Correct: fill.

5. White. It is the everyday opposite. It leaves out $m\angle P = 40^\circ$. Both are false when $m\angle P = 40^\circ$.

6. Correct: fill.

7. White. The complement of “all” is “at least one is not”, not “none”. Both are false when $m\angle A = 160^\circ$, $m\angle B = 10^\circ$, $m\angle C = 10^\circ$.

8. White. It keeps the case $m\angle Q = 60^\circ$. Both are true when $m\angle Q = 60^\circ$.

9. White. It is the everyday opposite. It leaves out the right angle. Both are true when $m\angle B = 10^\circ$.

10. White. It uses an unrelated condition. Parallel lines are not perpendicular either. Both are true when $AB \parallel CD$.

11. Correct: fill.

12. White. It denies the if part, which is given. Both are false when $m\angle A = 90^\circ$, $m\angle B = 90^\circ$.

13. White. The complement of “at most one” is “at least two”, not “exactly two”. Both are false when A is on n, B is on n, C is on n.

14. White. It keeps the “not” and changes the word. Both are true when $m\angle K = 90^\circ$.

15. White. It uses an unrelated condition. “Complementary” is not “not supplementary”. Both are false when $m\angle 1 = 10^\circ$, $m\angle 2 = 10^\circ$.

16. Correct: fill.

17. White. It says the same as the statement. This assumes what was to be proved. Both are true when $AM = 1$, $MB = 10$.

18. White. It leaves out “two or more”. Both are false when A is on n, B is on n, C is off n.

19. White. It turns the statement into another if-then sentence (the inverse). Both are true when $m\angle A = 90^\circ$, $m\angle B = 10^\circ$.

20. White. The complement of “at most one” is “at least two”. Both are true when $m\angle A = 90^\circ$, $m\angle B = 80^\circ$, $m\angle C = 10^\circ$.

21. White. It is the everyday opposite. It leaves out the right angle. Both are true when $m\angle K = 100^\circ$.

22. Correct: fill.

23. Correct: fill.

24. Correct: fill.

25. White. It says the same as the statement. This assumes what was to be proved. Both are true when AB and CD meet, the angle between AB and CD = 10° .