

MYSTERY PICTURE

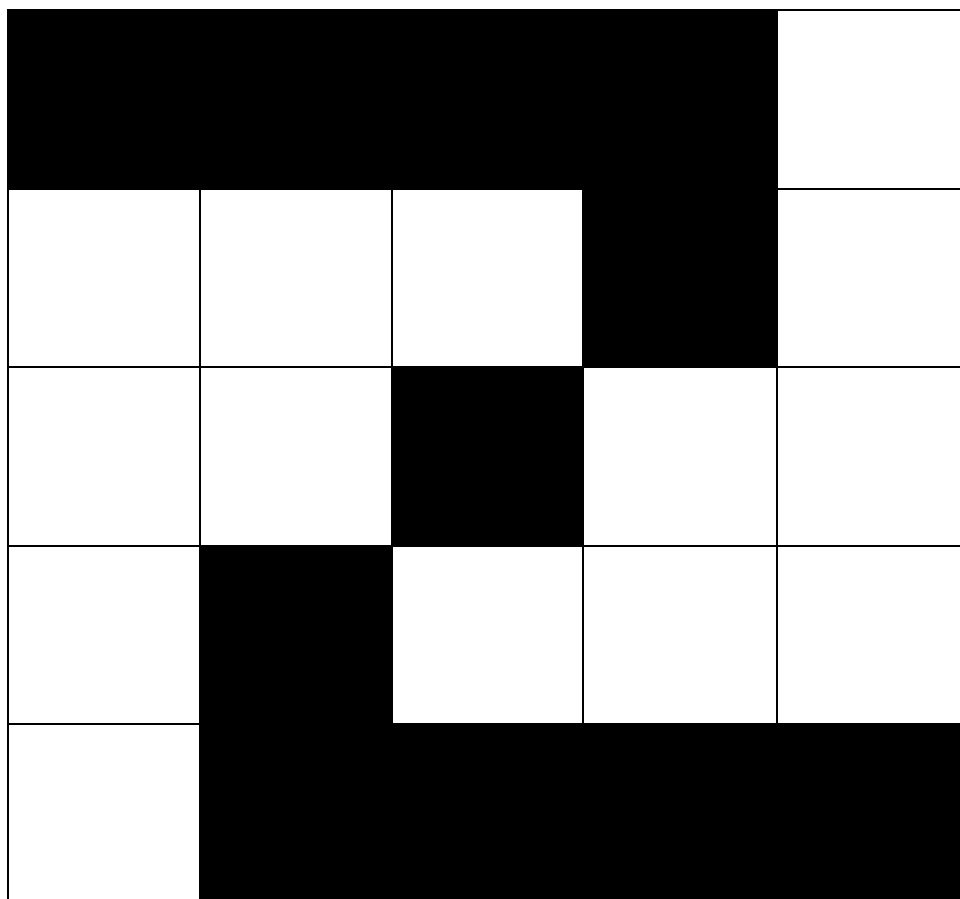


CONSTRUCTING PERPENDICULAR LINES

Perpendicular Through a Point: Do the Marks Prove the Line?

25 strangers' attempts · on the line and off it · a wrong square breaks the picture

FREE



25 squares · 1 student sheet · answer key · 4 pages

Geometry · Grades 8–10 · chooseyourlesson.com

1. **Hand out page 3**, one per student, and a compass each. Nothing is measured and no protractor is needed. That is the whole preparation.
2. **Say this once:** *“A line that looks perpendicular proves nothing. It must run through P. It must also run through a second point, where two arcs of one opening cross. Those two arcs are drawn from the two points where the first arc from P cut k.”*
3. **Fill a square with slanted lines if its marks prove the line is the perpendicular to k through P. Leave it white if they do not.** A picture appears. Eleven of the 25 are filled.

Student sheet	page 3, one per student. The compass-opening box is printed on it.
Answers	page 4 — the finished picture and the reason for every square.

Each square is somebody else's finished attempt, and the student judges it. Both kinds of task are mixed. P is on k in fourteen squares and off it in eleven. Each of the fourteen white squares shows one mistake. With P on k, the second pair of arcs has the same opening as the first arc, so the two arcs meet only at P and the line is put in by eye (three). The second pair of arcs has two different openings, because the compass was changed (three). The first arc is drawn as two arcs with two different openings (two). There are no arcs, and the line is drawn by eye (two). The line k is bisected instead, so the new line misses P (two). The second pair is so small that it never meets (one). The first arc never reaches k (one).

Seven white squares look exactly right. A line drawn by eye can look better than a real construction.

The two cases have different rules. With P on k, the second pair of arcs must be larger than the first arc. With P off k, the second pair may have the same opening as the first arc, and then the pair crosses at P itself. A student who only checks that the line runs through P and one crossing gets four squares wrong. Those are exactly the squares where an opening was not kept the same. A student who only checks that the line runs through P and two crossings gets four wrong. Those are exactly the squares with P off k and the pair equal to the first arc, where the line has only P and one crossing on it. This difference between the two cases is a good thing to discuss when the picture is done.

Guessing does not work. Every simple guessing rule, such as filling every square whose line runs through P, gets at least seven squares wrong. The picture then has holes that a student can see from their own desk.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture

Do the marks prove the line?

Name _____

Period _____

Date _____

Each square is somebody's finished attempt at the line through P perpendicular to the line k . Every arc was left on. The one dot is P . Do the marks prove the other line is the perpendicular to k through P ? If yes, fill the square with slanted lines. If no, leave it white. A line that only looks perpendicular is white. Use your compass to compare two arcs. You do not measure anything. When you finish, look at the picture. If it does not look right, check a square again.

How the construction works. When P is on line k : draw an arc from P that cuts k twice. From the two cuts, draw two arcs of the same size, bigger than the first arc, so that they cross. The line goes through P and that crossing. When P is not on line k : draw an arc from P that cuts k twice. From the two cuts, draw two arcs of the same size that cross on the other side of k . The line goes through P and that crossing.

Last lesson: you bisected an angle. You drew an arc that cuts both sides of the angle. From those two points, you drew two arcs of the same size that cross.

1 	2 	3 	4 	5
6 	7 	8 	9 	10
11 	12 	13 	14 	15
16 	17 	18 	19 	20
21 	22 	23 	24 	25

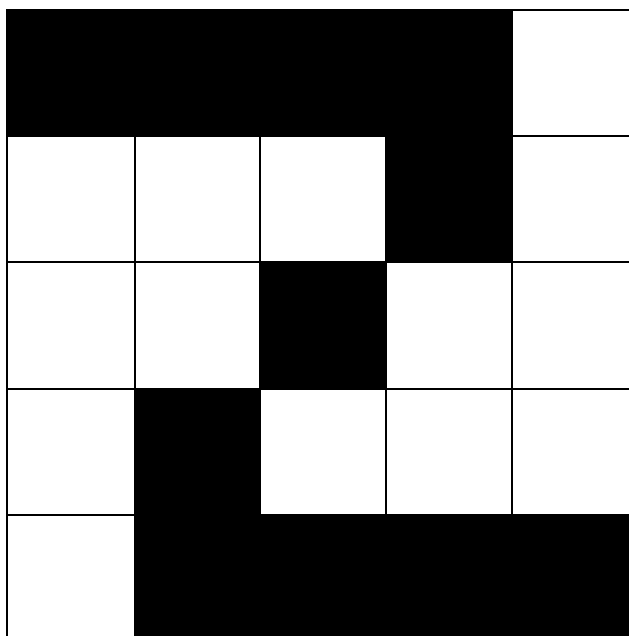


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Mystery Picture — answers

For the teacher

The finished sheet is **the letter Z: a bar across the top, a stroke falling from its right end down to the left, and a bar across the bottom**. The table reads left to right, the way the sheet is read; the squares are numbered on the student's sheet.



1–5	FILL P on k, second pair larger than the first arc	FILL P off k, second pair same as the first arc	FILL P off k, second pair at another opening	FILL P on k, second pair larger than the first arc	white P on k, second pair same as the first arc
6–10	white second pair has two openings	white first arc drawn with two openings	white no arcs: drawn by eye	FILL P off k, second pair same as the first arc	white bisects k, misses P
11–15	white second pair never meets	white first arc does not reach k	FILL P off k, second pair at another opening	white P on k, second pair same as the first arc	white second pair has two openings
16–20	white first arc drawn with two openings	FILL P on k, second pair larger than the first arc	white no arcs: drawn by eye	white bisects k, misses P	white P on k, second pair same as the first arc
21–25	white second pair has two openings	FILL P off k, second pair same as the first arc	FILL P off k, second pair at another opening	FILL P on k, second pair larger than the first arc	FILL P off k, second pair same as the first arc

No square depends on sharp eyes. Two arcs either have the same opening or differ by at least 4 mm and at least 30% of the smaller one. So a compass set on one arc clearly misses the other. A crossing of arcs is either on a line or at least 2.5 mm away from it. A line either runs through P or passes at least 2.5 mm from it.