

CUT & GLUE SORT

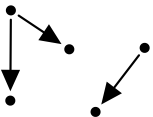


TRANSFORMATIONS AS FUNCTIONS

Is This Rule a Transformation?

A transformation · not one · cannot tell

FREE

A	B	C	D
 A rule was tried on these points. The arrows are every output it gave.	Every point goes to the point 2 cm to its right.	A rule sent P to P', Q to Q' and R to R'. No more is known about it.	Every point goes to a point 2 cm from it, in any direction.

1. **Hand out pages 3 and 4**, one of each per student, and scissors and glue.
2. **Say this once:** *“Check two things. Does every point of the plane get an output? Does each point get only one output? Testing a few points can show that a rule fails. It can never show that a rule works.”*
3. **Every row gets 4.** A row with more or fewer has a piece in the wrong place. (This is on your page only, not the student's.)

Student sheets pages 3 and 4, one of each per student.

Answers page 5.

The third row is the hard one: testing a few points cannot prove that a rule is a transformation. A transformation must give every point of the plane exactly one output. A record shows only a few points. The four pieces in *cannot tell* are records where no point has two outputs. The rule might still do something different to a point nobody tested. A record where one point has two outputs does prove something: that rule is not a transformation. It goes in the middle row.

A transformation does not have to move anything. “Every point goes to itself” is a transformation, although no point moves. “Every point goes to O” is one too, although every point lands on the same dot. Sending each point to its nearest point on line m sends many points to one output, and it is still a transformation.

A rule must give an output to every point, not only to the points of a figure. Many students think a rule acts only on the lettered corners of a triangle. A rule for the triangle's points only is not a transformation, because the rest of the plane gets no output. One small word also matters. “*The* point 2 cm to its right” gives one output. “A point 2 cm from it” gives a whole circle of outputs.

Guessing does not work. A student who sorts by a few key words, such as “every point” or “tried”, gets at most eight of the twelve pieces right.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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A transformation, not one, or cannot tell?

Rules on the points of the plane

Name _____

Period _____

Date _____

Cut out the twelve pieces on the next page. Each is a rule, or a record of what a rule did to a few points. **For each piece, decide: is it a transformation, is it not one, or can you not tell?** Glue each into its row. **A transformation is a rule that takes every point of the plane as an input and gives each input exactly one output.**



$$P \rightarrow P'$$

P is the preimage, the input. P' is its image, the output



$$T(P) = P'$$

the same statement. T names the rule

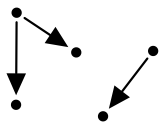
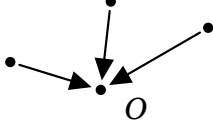
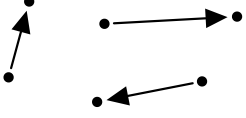
A TRANSFORMATION — *every point of the plane gets exactly one output*

NOT A TRANSFORMATION — *some point gets two outputs, or none*

CANNOT TELL — *a few points were tried. That can show a rule fails, but never that it works*

The twelve pieces

Cut along the dotted lines

A  A rule was tried on these points. The arrows are every output it gave.	B Every point goes to the point 2 cm to its right.	C A rule sent P to P', Q to Q' and R to R'. No more is known about it.	D Every point goes to a point 2 cm from it, in any direction.
E  Every point of the plane goes to O. Three of them are drawn.	F  A rule was tried on these points. The arrows are every output it gave.	G Each point of triangle ABC goes 2 cm to its right. The rule gives every other point no output.	H A rule was tried on five points. Each came out 2 cm to its right.
J Every point goes to itself. No point moves.	K  A rule was tried on these points. The arrows are every output it gave.	L A rule was tried on P and Q. P went to R. Q went to S and to T.	M Every point goes to the point of line m nearest to it.



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Cut & Glue Sort — answers

For the teacher

A TRANSFORMATION	B — Each point has one output: the point 2 cm to its right.	E — Each point has one output, O. Many points may share one output.	J — Each point has one output: itself. A transformation does not have to move anything.	M — Each point has one output. Two points may share the same output.
NOT A TRANSFORMATION	A — One point has two arrows going out of it, so it has two outputs.	D — “A point” 2 cm away can be any point on a circle. That is many outputs.	G — The points outside the triangle get no output.	L — Q has two outputs, S and T.
CANNOT TELL	C — Three points tell us nothing about a fourth point.	F — Each point has one arrow out. But only these few points were tested.	H — Only five points were tested, not every point.	K — Arrows may meet at one dot. But only these few points were tested.

A transformation gives every point of the plane exactly one output. A piece goes in the middle row only if some point gets more than one output, or no output. A piece goes in *cannot tell* if it shows only a few points and none of them breaks the rule.