

CUT & GLUE SORT

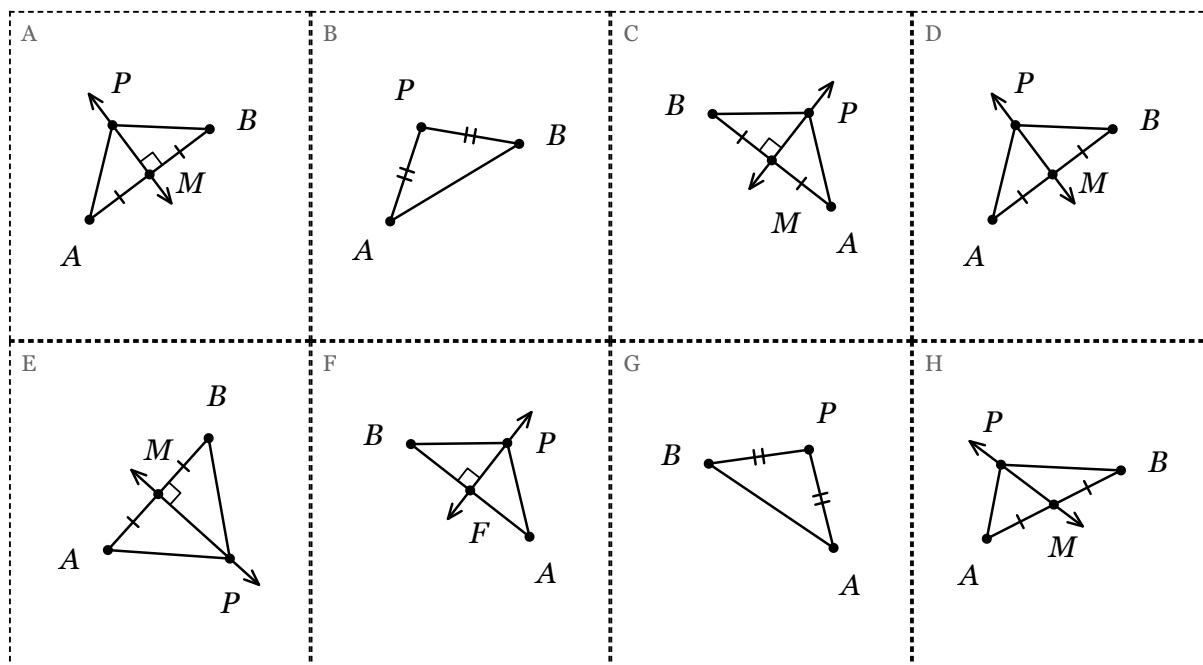


THE PERPENDICULAR BISECTOR THEOREM

The Perpendicular Bisector — which half of the theorem?

So $PA = PB$ · so P is on the line · neither
25 minutes, including the cutting

FREE



1. **Hand out both sheets, one of each per student.** Each student needs scissors and glue. You cut nothing yourself; the students cut out the pieces.
2. **Say this once:** *“A perpendicular bisector needs two things. It goes through the midpoint, and it is perpendicular. A figure can show one without the other. Then the theorem tells you nothing.”*
3. **Cut out the twelve pieces and glue each into its row. Every row gets exactly 4.**

Both sheets pages 3 and 4, one of each per student.

Answers page 5.

The top two rows are the two halves of one theorem. Telling them apart is the main work. A piece in the top row gives the student the *line*: two ticks show the midpoint, a square shows the right angle, and P is on the line. What follows is $PA = PB$. A piece in the middle row gives only the *two equal distances*. What follows is that P is on the line. A class that has only seen the first half will put the middle-row pieces in the top row.

The bottom row is the heart of the lesson. Each piece in it shows one of the two conditions but not the other. Two go through the ticked midpoint with no right angle. Two have a right angle where the line crosses AB, but nothing marks the midpoint. Many students think a line through the midpoint must be perpendicular. The bottom row shows that it need not be.

Two pairs of pieces are twins. Go over them at the end. The twins are A and D, and C and F. Each pair is one drawing printed twice, and one mark was taken off the bottom-row copy. On that copy the two distances really are equal on the paper, but the marks do not say so. Hold a pair up side by side. Together they show the whole lesson.

Marking takes ten seconds a sheet. Every row holds 4 pieces. A row with more or fewer is wrong before you read a single piece. The letters give nothing away, because they are shuffled. On the bottom-row pieces that are not twins, the two distances differ by 6 to 7 millimeters. So a ruler sorts some of that row, but not all of it.

Plan for twenty-five minutes. Cutting out twelve pieces takes five to eight minutes, before anyone thinks about a midpoint.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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What does each figure let you say?

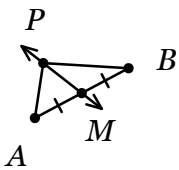
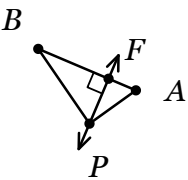
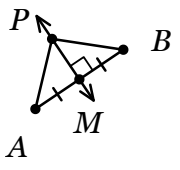
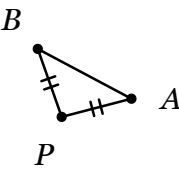
The perpendicular bisector

Name _____

Period _____

Date _____

Cut out the twelve pieces and glue each into the row it belongs in. **The marks are all you are told.** Two pieces are drawn so the two distances look equal, and nothing on them says so. Every row gets 4.

 Through the midpoint and nothing else $PA = PB$ does not follow	 Perpendicular and nothing else $PA = PB$ does not follow	 Both marks: the perpendicular bisector so $PA = PB$	 The two distances marked equal so P is on the line
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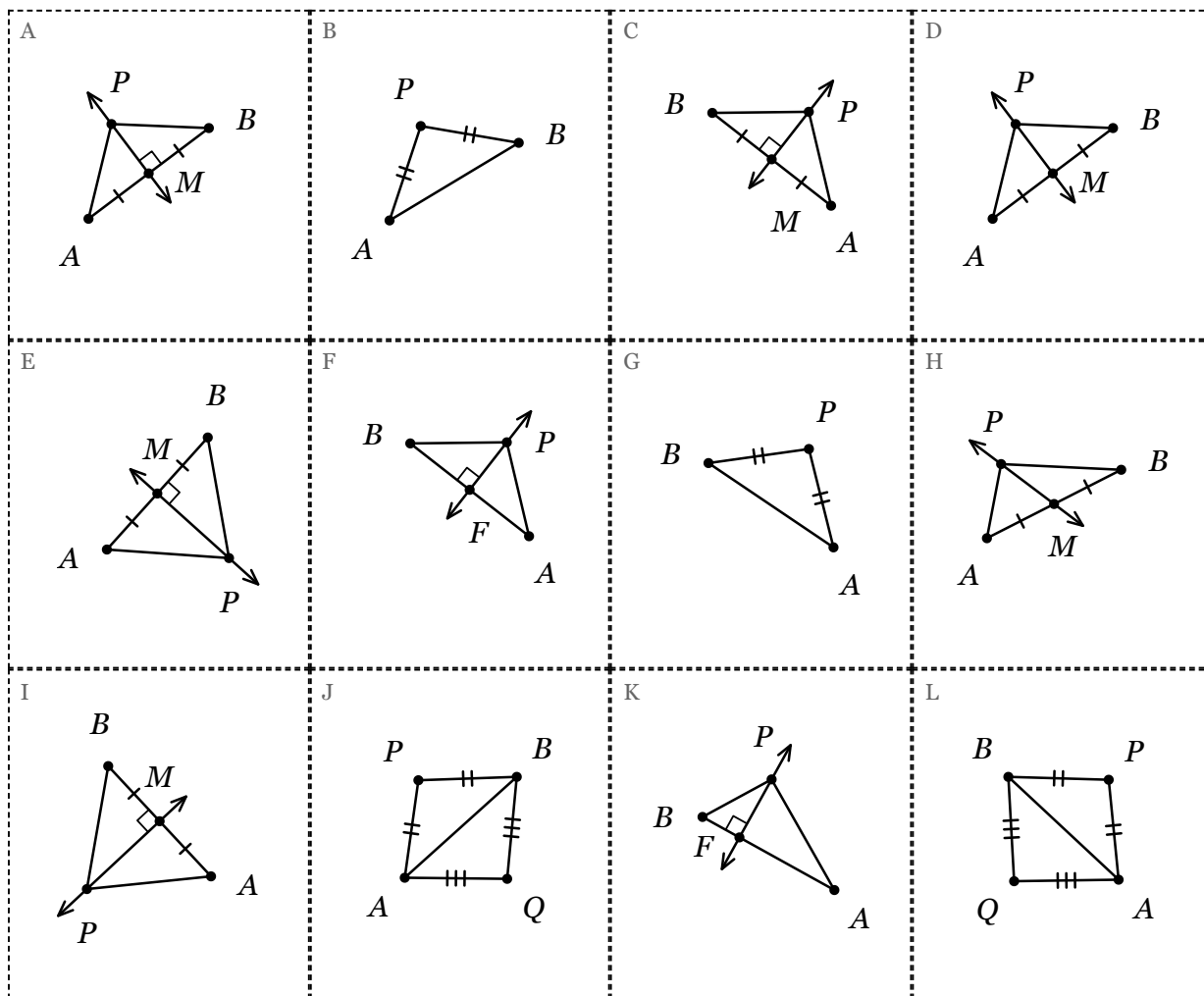
So $PA = PB$: the line is given

So P is on the perpendicular bisector of AB : the two distances are given

Neither follows: one condition is missing: one condition marked, the other not

The twelve pieces

Cut along the dotted lines



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Four pieces go in each row. A row with more or fewer pieces is wrong before you read any piece.

So $PA = PB$ **A:** The ticks mark the midpoint of AB , and the square marks a right angle. So the line is the perpendicular bisector. P is on it, so $PA = PB$.
C: The ticks mark the midpoint of AB , and the square marks a right angle. So the line is the perpendicular bisector. P is on it, so $PA = PB$.
E: The ticks mark the midpoint of AB , and the square marks a right angle. So the line is the perpendicular bisector. P is on it, even far out along the line, so $PA = PB$.
I: The ticks mark the midpoint of AB , and the square marks a right angle. So the line is the perpendicular bisector. P is on it, even far out along the line, so $PA = PB$.

So P is on the perpendicular bisector of AB **B:** The ticks show $PA = PB$, and no line is drawn. A point the same distance from A and B is on the perpendicular bisector of AB , so P is on it.
G: The ticks show $PA = PB$, and no line is drawn. A point the same distance from A and B is on the perpendicular bisector of AB , so P is on it.
J: The ticks show that P and Q are each the same distance from A as from B . A point the same distance from A and B is on the perpendicular bisector of AB , so P is on it.
L: The ticks show that P and Q are each the same distance from A as from B . A point the same distance from A and B is on the perpendicular bisector of AB , so P is on it.

Neither follows: one condition is missing **D:** The line goes through the ticked midpoint of AB , but no right angle is marked. So it may not be the perpendicular bisector, and nothing follows. It is piece A with the right angle mark taken off.
F: The square marks a right angle, but nothing marks the midpoint of AB . So the line may not be the perpendicular bisector, and nothing follows. It is piece C with the midpoint ticks taken off.
H: The line goes through the ticked midpoint of AB , but no right angle is marked. So it may not be the perpendicular bisector, and nothing follows.
K: The square marks a right angle, but nothing marks the midpoint of AB . So the line may not be the perpendicular bisector, and nothing follows.

Why the top two rows are different. The theorem works in both directions: $PA = PB$ **if and only if** P is on the perpendicular bisector of AB . One direction starts from the line and gives the two equal distances. The other starts from the two equal distances and gives the line. A piece goes in the top row when the figure gives the line. It goes in the middle row when the figure gives the equal distances.

Why the bottom row has no answer. A perpendicular bisector needs two conditions: it goes through the midpoint, and it is perpendicular. Every piece in that row shows one condition but not the other. So there is no perpendicular bisector on those figures, and neither direction of the theorem can be used. “Nothing follows” is the correct answer.

Do not use triangle congruence or the isosceles triangle theorem to explain these answers. The class has not learned them yet at this point. Both directions of this theorem are proved with reflections, which the class has already met.