

MYSTERY PICTURE

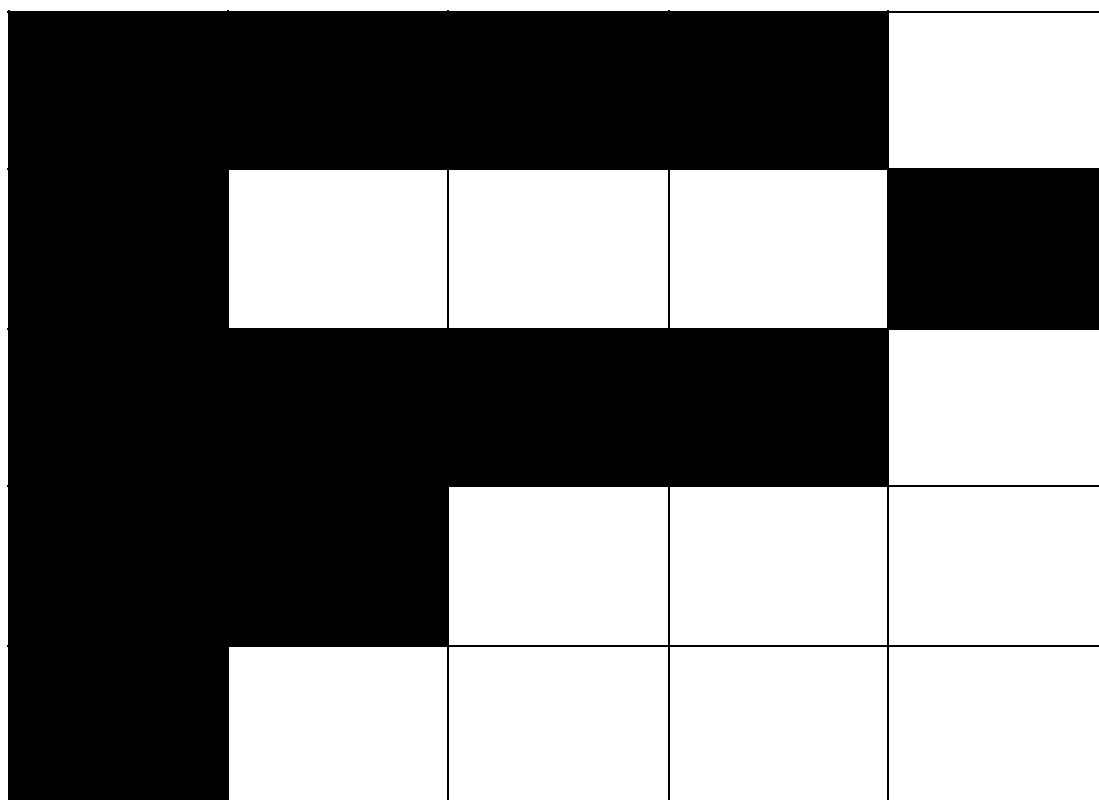


THE PERPENDICULAR BISECTOR THEOREM

The Perpendicular Bisector — is $PA = PB$?

25 squares · 25–35 minutes · wrong answers show in the picture

FREE



1. **Hand out page 3**, one per student. Nothing else is needed.
2. **Say this once:** *“A perpendicular bisector needs two things. It goes through the midpoint, and it is perpendicular. One of them on its own is not enough.”*
3. **Fill the square if the marks force $PA = PB$. Leave it white if they do not.** A picture appears. Thirteen of the 25 squares are filled.

Student sheet page 3, one per student. It shows the two conditions as pictures across the top.
Answers page 4. It shows the finished picture and the reason for every square.

There are two kinds of filled square. In eight, the line has both marks and P is on it, so $PA = PB$ follows. In the other five, there is no line at all, and ticks mark PA and PB equal, so the equality is given. Because of these five, a student cannot fill a square just because it has a right-angle mark. They have to read the ticks.

There are several kinds of white square. Two tick the two halves of AB but say nothing about the angle. Two mark the right angle but say nothing about the midpoint. Two show a line with no marks at all. The last six are twins, explained below. All twelve are white, because the marks do not force $PA = PB$ on any of them.

The six twins matter most. The pairs are 1 and 5, 3 and 8, 6 and 15, 11 and 19, 13 and 22, 16 and 25. Each pair is one drawing printed twice, and one mark was taken off the white copy. On that copy the two distances really *are* equal on the paper, but the marks do not say so. At the end, put a pair on the board side by side. Together they show the whole lesson.

A ruler does not solve this sheet. Every drawing is to scale. On most white squares the two lengths differ, by 4 to 7 mm. On the twins they do not. So a student who fills every square that looks equal gets 19 of the 25 right. The letter P then has six squares wrong, and the student can see that from their own desk. **Use slanted lines, not shading.** Thirteen squares of pencil shading come out pale gray, and pale gray does not survive a school copier.

Do not use triangle congruence or the isosceles triangle theorem to explain these answers. The class has not learned them yet at this point. Both directions of this theorem are proved with reflections, which the class has already met.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture

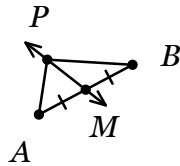
The marks decide, not the drawing

Name _____

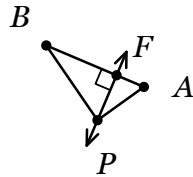
Period _____

Date _____

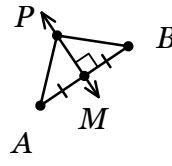
Do the marks force $PA = PB$? Yes: fill it with slanted lines. No: leave it white. A line is the perpendicular bisector of AB only with *both* marks. Do not measure: some are drawn to look equal and the marks still do not say so. **A picture appears. If it does not look right, a square is wrong.**



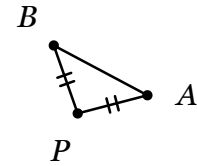
Through the midpoint
and nothing else
 $PA = PB$ does **not** follow



Perpendicular
and nothing else
 $PA = PB$ does **not** follow



Both marks:
the perpendicular bisector
so $PA = PB$



The two distances
marked equal
so P is on the line

1 	2 	3 	4 	5
6 	7 	8 	9 	10
11 	12 	13 	14 	15
16 	17 	18 	19 	20
21 	22 	23 	24 	25

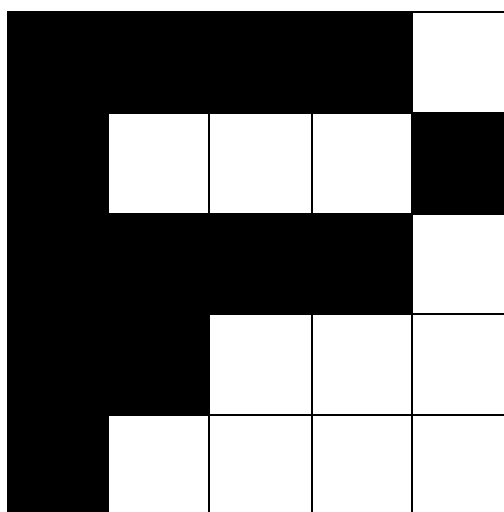


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Mystery Picture — answers

For the teacher

The picture is the letter P. The table reads left to right, like the numbered squares on the student's sheet.



1–5	both marks <i>FILL</i>	$PA = PB$ ticked <i>FILL</i>	both marks <i>FILL</i>	$PA = PB$ ticked <i>FILL</i>	midpoint only (twin of 1) <i>white</i>
6–10	both marks <i>FILL</i>	midpoint only <i>white</i>	right angle only (twin of 3) <i>white</i>	right angle only <i>white</i>	$PA = PB$ ticked <i>FILL</i>
11–15	both marks <i>FILL</i>	$PA = PB$ ticked <i>FILL</i>	both marks <i>FILL</i>	$PA = PB$ ticked <i>FILL</i>	midpoint only (twin of 6) <i>white</i>
16–20	both marks <i>FILL</i>	both marks <i>FILL</i>	no marks <i>white</i>	right angle only (twin of 11) <i>white</i>	midpoint only <i>white</i>
21–25	both marks <i>FILL</i>	right angle only (twin of 13) <i>white</i>	right angle only <i>white</i>	no marks <i>white</i>	midpoint only (twin of 16) <i>white</i>

Squares 1, 3, 6, 11, 13, 16, 17, 21: the ticks mark the midpoint of AB and the square marks a right angle. So the line is the perpendicular bisector, and P is on it, so $PA = PB$. Fill. **Squares 2, 4, 10, 12, 14:** there is no line, and the ticks mark PA and PB equal. So $PA = PB$ is given. Fill. **Squares 5, 7, 15, 20, 25:** the midpoint is marked, but no right angle is. The line may not be the perpendicular bisector, so nothing follows. White. **Squares 8, 9, 19, 22, 23:** a right angle is marked, but the midpoint is not. The line may not be the perpendicular bisector, so nothing follows. White. **Squares 18, 24:** the line has no marks at all, so nothing follows. White.

The twins are 1 and 5, 3 and 8, 6 and 15, 11 and 19, 13 and 22, 16 and 25. Each pair is one drawing printed twice, with one mark taken off the white copy. On the white copy the two distances really are equal on the paper, but the marks do not force it.