

MYSTERY PICTURE

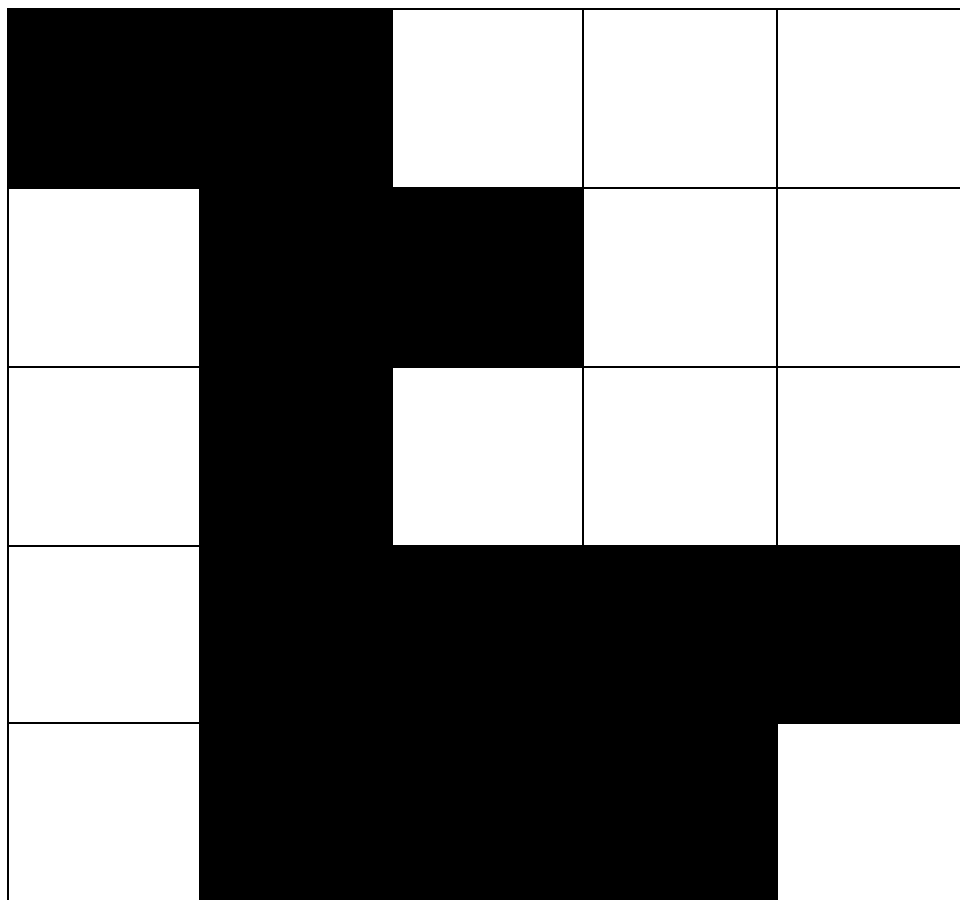


THE ANGLE BISECTOR THEOREM

The Angle Bisector & the Two Distances

25 squares · sitting · 25–35 minutes

FREE



1. **Hand out page 3.** That is the whole preparation.
2. **Say this once:** “A stroke is only a distance if it has the little square on it. If there is no square, there is no distance. Then you cannot say anything about it.”
3. **Fill the square with slanted pencil lines if the marks force the dot to be the same distance from both sides. Leave it white if they do not.** A picture appears: a boot: a shaft down the left, with the foot turning right at the bottom, and twelve of the 25 squares are filled.

Student sheet page 3, one per student. It carries the three marks as pictures.
Answers page 4 — the finished picture and the reason for every square.

A ruler is no use on this sheet, and that is deliberate. Every one of the 25 drawings is drawn with its two strokes within 1.1 mm of each other — the widest gap on the sheet, and no class ruler separates that. A student who measures gets the same answer on all twenty-five and a picture that is a solid block. The marks are the only evidence there is, and the student sheet says so.

The three kinds of white square are three different reasons. On five the angle is marked halved and only one stroke carries a right-angle mark — so the other is just a stroke, not the distance to that side, and the theorem has nothing to say. On two neither stroke is marked square. On six both strokes *are* distances and nothing says the angle is halved. All three are white, because the question is *do the marks force it*.

The one they will argue about. On those six squares the two strokes are drawn the same length and a student will say “but you can *see* they are equal”. They can see it; nothing on the figure says it. That is the whole lesson in one square.

The angle opens differently on every square — 13 openings between 40° and 106° , and 25 different turns of the figure between them, with no side ever horizontal. A class that has only seen this drawn with one side along the bottom of the page measures straight down and straight across instead of perpendicular to each side.

Slanted lines and not “color it in”, and the picture is the marking. Twelve squares of pencil shading comes out pale grey, which is the one thing that does not survive a school copier; hatching survives and leaves the drawing readable underneath. One square wrong and the arrow has a bite out of it, and the student sees that from their own desk before you do — so you collect nothing to know who has it.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture

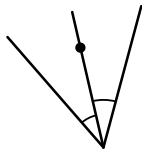
The marks decide, not the drawing. Do not measure.

Name _____

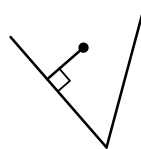
Period _____

Date _____

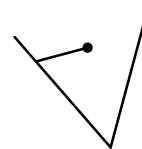
Do the marks force the dot to be the *same distance* from both sides of the angle? Yes: fill it with slanted lines. No: leave it white. Every drawing has its two strokes within 1.1 mm of each other, so a ruler tells you nothing. Only the marks do. **A picture appears. If it does not look right, a square is wrong.**



two arcs
the ray halves the angle



a square at the foot
this stroke is the distance to that side



no square
just a stroke, not a distance

1 	2 	3 	4 	5
6 	7 	8 	9 	10
11 	12 	13 	14 	15
16 	17 	18 	19 	20
21 	22 	23 	24 	25

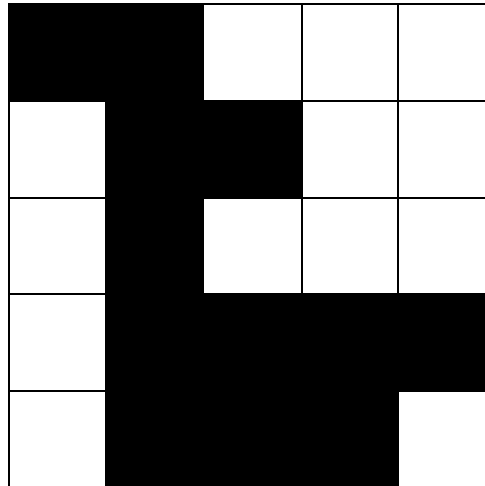


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Mystery Picture — answers

For the teacher

The picture is a boot: a shaft down the left, with the foot turning right at the bottom, and if what is on a student's sheet is not that, a square is wrong. The table reads left to right, the way the sheet is read; the squares are numbered on the student's sheet.



1–5	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 1 square <i>white — one stroke is not a distance</i>	2 squares · no arcs <i>white — nothing halves the angle</i>	arcs · no squares <i>white — neither is a distance</i>
6–10	2 squares · no arcs <i>white — nothing halves the angle</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 1 square <i>white — one stroke is not a distance</i>	2 squares · no arcs <i>white — nothing halves the angle</i>
11–15	arcs · 1 square <i>white — one stroke is not a distance</i>	arcs · 2 squares <i>FILL — must be</i>	2 squares · no arcs <i>white — nothing halves the angle</i>	arcs · no squares <i>white — neither is a distance</i>	2 squares · no arcs <i>white — nothing halves the angle</i>
16–20	arcs · 1 square <i>white — one stroke is not a distance</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>
21–25	2 squares · no arcs <i>white — nothing halves the angle</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 2 squares <i>FILL — must be</i>	arcs · 1 square <i>white — one stroke is not a distance</i>

The rule, in one line: the two distances are forced equal when the angle is marked halved through the dot *and both* strokes carry a right-angle mark. Take either square away and that stroke stops being a distance, and the theorem is about distances.

Every white square is worth a sentence, and they are not all the same sentence. Five have one stroke marked square and one not; two have neither marked; six have both marked and no arcs, so both strokes really are distances and nothing whatever says they are equal. The last of those three is the one a class fights about, because the drawing shows them equal — and the drawing is not a reason. On this sheet it never is: all 25 figures are drawn that way on purpose.