

MYSTERY PICTURE



WHY SSA IS NOT A CONGRUENCE CRITERION

Do the Facts Prove It?

25 squares, no drawings · SAS or SSA? · a picture appears

FREE

1 △PQR and △XYZ $\angle Q \cong \angle Y$ $\angle P \cong \angle X$ $\angle R \cong \angle Z$	2 △ABC and △DEF $AB \cong DE$ $BC \cong EF$ $\angle A \cong \angle D$	3 △ABC and △DEF $AB \cong DE$ $AC \cong DF$ $\angle A \cong \angle D$	4 △GHK and △LMN $\angle G \cong \angle L$ $HK \cong MN$ $\angle H \cong \angle M$	5 △JKL and △RST $JL \cong RT$ $\angle K \cong \angle S$ $JK \cong RS$
6 △MNP and △UVW $\angle N \cong \angle V$ $\angle P \cong \angle W$ $\angle M \cong \angle U$	7 △DEF and △QRS $\angle F \cong \angle S$ $DE \cong QR$ $EF \cong RS$	8 △PQR and △KLM $\angle P \cong \angle K$ $PQ \cong KL$ $\angle Q \cong \angle L$	9 △XYZ and △ABC $XZ \cong AC$ $\angle Z \cong \angle C$ $ZY \cong CB$	10 △XYZ and △ABC $XZ \cong AC$ $\angle Z \cong \angle C$ $XY \cong AB$
11 △GHJ and △STU $\angle J \cong \angle U$ $\angle G \cong \angle S$ $\angle H \cong \angle T$	12 △KLM and △PQR $LM \cong QR$ $KM \cong PR$ $\angle L \cong \angle Q$	13 △RST and △DEF $\angle R \cong \angle D$ $\angle T \cong \angle F$ $ST \cong EF$	14 △UVW and △GHJ $UV \cong GH$ $VW \cong HJ$ $UW \cong GJ$	15 △BCD and △KLM $\angle C \cong \angle L$ $\angle D \cong \angle M$ $\angle B \cong \angle K$
16 △JKL and △XYZ $KL \cong YZ$ $\angle J \cong \angle X$ $\angle K \cong \angle Y$	17 △MNP and △RST $\angle N \cong \angle S$ $MN \cong RS$ $NP \cong ST$	18 △ABC and △PQR $\angle B \cong \angle Q$ $\angle C \cong \angle R$ $AB \cong PQ$	19 △DEF and △JKL $DF \cong JL$ $EF \cong KL$ $\angle F \cong \angle L$	20 △DEF and △JKL $DF \cong JL$ $DE \cong JK$ $\angle F \cong \angle L$
21 △GHK and △UVW $\angle K \cong \angle W$ $GH \cong UV$ $\angle H \cong \angle V$	22 △LMN and △BCD $\angle L \cong \angle B$ $\angle N \cong \angle D$ $MN \cong CD$	23 △PQR and △XYZ $\angle Q \cong \angle Y$ $PQ \cong XY$ $QR \cong YZ$	24 △PQR and △XYZ $\angle Q \cong \angle Y$ $PQ \cong XY$ $PR \cong XZ$	25 △STU and △EFG $\angle T \cong \angle F$ $\angle U \cong \angle G$ $\angle S \cong \angle E$

1. **Give each student page 3** and a pencil.
2. **Say this once:** *“Each square gives three facts about two triangles. Do the facts prove the triangles congruent? If yes, fill the square with slanted lines. If no, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet page 3, one per student.

Answers page 4, for you.

The top of the sheet shows all six ways to give three parts. SSS, SAS, ASA and AAS prove two triangles congruent. SSA and AAA do not. So the real question in each square is: which of the six is it? What decides it is where the angle is. In SAS the angle is between the two sides: the angle's letter is the letter the two side names share. In SSA it is not.

No square has a drawing. The students read the letters. The facts are printed in mixed order, so the order of the lines does not tell the answer.

Three common mistakes. Some students treat SSA like SAS and fill every square with a side in it. Some think a part that is not between the other two is never enough, and leave AAS white. AAS does prove it, because when two angles match, the third angles match too. Some think SSA triangles are never congruent. That is not what the lesson says: SSA does not *guarantee* congruence, because two different triangles can have those three parts.

Look at squares 2 and 3, 9 and 10, 19 and 20, 23 and 24. In each pair, only one side is different. It moves from beside the angle to across from it, and SAS becomes SSA. If a student's picture is wrong in one of these squares, ask: *“Which letter do the two sides share? Is the angle at that letter?”*

Guessing does not work. Every simple guessing rule, such as filling every square that has a side in it, gets at least six squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it.

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Mystery Picture — do the facts prove it?

SSS, SAS, ASA, AAS, SSA, AAA

Name _____

Period _____

Date _____

Each square names two triangles and gives three facts about them. **Do the three facts prove the triangles congruent? If yes, fill the square with slanted lines. If no, leave it white.** Use the six pictures below. First find the two sides, then check where the angle is. When you finish, a picture appears. If part of it is missing, or one square does not fit, check that square again.

SSS proves it	SAS proves it	ASA proves it	AAS proves it	SSA does NOT prove it	AAA does NOT prove it

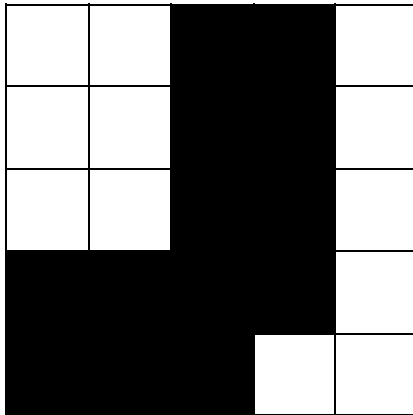
1 $\triangle PQR$ and $\triangle XYZ$ $\angle Q \cong \angle Y$ $\angle P \cong \angle X$ $\angle R \cong \angle Z$	2 $\triangle ABC$ and $\triangle DEF$ $AB \cong DE$ $BC \cong EF$ $\angle A \cong \angle D$	3 $\triangle ABC$ and $\triangle DEF$ $AB \cong DE$ $AC \cong DF$ $\angle A \cong \angle D$	4 $\triangle GHK$ and $\triangle LMN$ $\angle G \cong \angle L$ $HK \cong MN$ $\angle H \cong \angle M$	5 $\triangle JKL$ and $\triangle RST$ $JL \cong RT$ $\angle K \cong \angle S$ $JK \cong RS$
6 $\triangle MNP$ and $\triangle UVW$ $\angle N \cong \angle V$ $\angle P \cong \angle W$ $\angle M \cong \angle U$	7 $\triangle DEF$ and $\triangle QRS$ $\angle F \cong \angle S$ $DE \cong QR$ $EF \cong RS$	8 $\triangle PQR$ and $\triangle KLM$ $\angle P \cong \angle K$ $PQ \cong KL$ $\angle Q \cong \angle L$	9 $\triangle XYZ$ and $\triangle ABC$ $XZ \cong AC$ $\angle Z \cong \angle C$ $ZY \cong CB$	10 $\triangle XYZ$ and $\triangle ABC$ $XZ \cong AC$ $\angle Z \cong \angle C$ $XY \cong AB$
11 $\triangle GHJ$ and $\triangle STU$ $\angle J \cong \angle U$ $\angle G \cong \angle S$ $\angle H \cong \angle T$	12 $\triangle KLM$ and $\triangle PQR$ $LM \cong QR$ $KM \cong PR$ $\angle L \cong \angle Q$	13 $\triangle RST$ and $\triangle DEF$ $\angle R \cong \angle D$ $\angle T \cong \angle F$ $ST \cong EF$	14 $\triangle UVW$ and $\triangle GHJ$ $UV \cong GH$ $VW \cong HJ$ $UW \cong GJ$	15 $\triangle BCD$ and $\triangle KLM$ $\angle C \cong \angle L$ $\angle D \cong \angle M$ $\angle B \cong \angle K$
16 $\triangle JKL$ and $\triangle XYZ$ $KL \cong YZ$ $\angle J \cong \angle X$ $\angle K \cong \angle Y$	17 $\triangle MNP$ and $\triangle RST$ $\angle N \cong \angle S$ $MN \cong RS$ $NP \cong ST$	18 $\triangle ABC$ and $\triangle PQR$ $\angle B \cong \angle Q$ $\angle C \cong \angle R$ $AB \cong PQ$	19 $\triangle DEF$ and $\triangle JKL$ $DF \cong JL$ $EF \cong KL$ $\angle F \cong \angle L$	20 $\triangle DEF$ and $\triangle JKL$ $DF \cong JL$ $DE \cong JK$ $\angle F \cong \angle L$
21 $\triangle GHK$ and $\triangle UVW$ $\angle K \cong \angle W$ $GH \cong UV$ $\angle H \cong \angle V$	22 $\triangle LMN$ and $\triangle BCD$ $\angle L \cong \angle B$ $\angle N \cong \angle D$ $MN \cong CD$	23 $\triangle PQR$ and $\triangle XYZ$ $\angle Q \cong \angle Y$ $PQ \cong XY$ $QR \cong YZ$	24 $\triangle PQR$ and $\triangle XYZ$ $\angle Q \cong \angle Y$ $PQ \cong XY$ $PR \cong XZ$	25 $\triangle STU$ and $\triangle EFG$ $\angle T \cong \angle F$ $\angle U \cong \angle G$ $\angle S \cong \angle E$



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Mystery Picture — answers

For the teacher



The finished picture is a **boot**. Thirteen squares are filled.

A square is filled when its facts are SSS, SAS, ASA or AAS. It is white when they are SSA or AAA. For two sides and an angle, find the letter the two sides share. If the angle is at that letter, it is SAS. If not, it is SSA. AAS proves congruence because when two angles match, the third angles match too. AAA does not: the triangles have the same shape, but one can be bigger.

	Facts	Answer	Why
1	$\angle Q \cong \angle Y, \angle P \cong \angle X, \angle R \cong \angle Z$	white	There are three angles and no side, so it is AAA.
2	$AB \cong DE, BC \cong EF, \angle A \cong \angle D$	white	AB and BC meet at B, but the angle is at A. So it is SSA.
3	$AB \cong DE, AC \cong DF, \angle A \cong \angle D$	fill	AB and AC meet at A, and the angle is at A. So it is SAS.
4	$\angle G \cong \angle L, HK \cong MN, \angle H \cong \angle M$	fill	Side HK does not join the angles at G and H. So it is AAS.
5	$JL \cong RT, \angle K \cong \angle S, JK \cong RS$	white	JL and JK meet at J, but the angle is at K. So it is SSA.
6	$\angle N \cong \angle V, \angle P \cong \angle W, \angle M \cong \angle U$	white	There are three angles and no side, so it is AAA.
7	$\angle F \cong \angle S, DE \cong QR, EF \cong RS$	white	DE and EF meet at E, but the angle is at F. So it is SSA.
8	$\angle P \cong \angle K, PQ \cong KL, \angle Q \cong \angle L$	fill	Side PQ joins the two angles, at P and Q. So it is ASA.
9	$XZ \cong AC, \angle Z \cong \angle C, ZY \cong CB$	fill	XZ and ZY meet at Z, and the angle is at Z. So it is SAS.
10	$XZ \cong AC, \angle Z \cong \angle C, XY \cong AB$	white	XZ and XY meet at X, but the angle is at Z. So it is SSA.
11	$\angle J \cong \angle U, \angle G \cong \angle S, \angle H \cong \angle T$	white	There are three angles and no side, so it is AAA.
12	$LM \cong QR, KM \cong PR, \angle L \cong \angle Q$	white	LM and KM meet at M, but the angle is at L. So it is SSA.
13	$\angle R \cong \angle D, \angle T \cong \angle F, ST \cong EF$	fill	Side ST does not join the angles at R and T. So it is AAS.
14	$UV \cong GH, VW \cong HJ, UW \cong GJ$	fill	There are three pairs of sides, so it is SSS.
15	$\angle C \cong \angle L, \angle D \cong \angle M, \angle B \cong \angle K$	white	There are three angles and no side, so it is AAA.
16	$KL \cong YZ, \angle J \cong \angle X, \angle K \cong \angle Y$	fill	Side KL does not join the angles at J and K. So it is AAS.
17	$\angle N \cong \angle S, MN \cong RS, NP \cong ST$	fill	MN and NP meet at N, and the angle is at N. So it is SAS.
18	$\angle B \cong \angle Q, \angle C \cong \angle R, AB \cong PQ$	fill	Side AB does not join the angles at B and C. So it is AAS.
19	$DF \cong JL, EF \cong KL, \angle F \cong \angle L$	fill	DF and EF meet at F, and the angle is at F. So it is SAS.
20	$DF \cong JL, DE \cong JK, \angle F \cong \angle L$	white	DF and DE meet at D, but the angle is at F. So it is SSA.
21	$\angle K \cong \angle W, GH \cong UV, \angle H \cong \angle V$	fill	Side GH does not join the angles at K and H. So it is AAS.
22	$\angle L \cong \angle B, \angle N \cong \angle D, MN \cong CD$	fill	Side MN does not join the angles at L and N. So it is AAS.
23	$\angle Q \cong \angle Y, PQ \cong XY, QR \cong YZ$	fill	PQ and QR meet at Q, and the angle is at Q. So it is SAS.
24	$\angle Q \cong \angle Y, PQ \cong XY, PR \cong XZ$	white	PQ and PR meet at P, but the angle is at Q. So it is SSA.
25	$\angle T \cong \angle F, \angle U \cong \angle G, \angle S \cong \angle E$	white	There are three angles and no side, so it is AAA.