

CUT & GLUE SORT

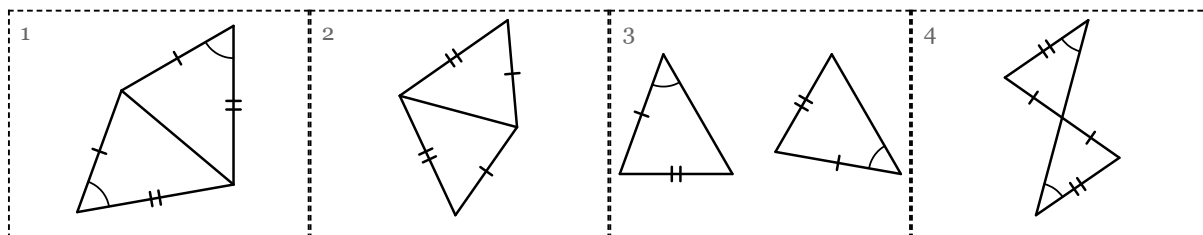


CHOOSING A TRIANGLE CONGRUENCE CRITERION

Marks Alone, a Free Pair, or Not Proved?

Choosing a congruence criterion · the pairs with no mark

FREE



Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. Nothing else is needed.
2. **Say this once:** *“First use only the marks. Then look for a side in both triangles, or two lines that cross. Those give a pair with no mark. Glue each piece in the first row that fits.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 shows SSS, SAS, ASA, AAS, HL and the two pairs with no mark as small drawings.
Answers	page 5.

What the rows test. Row 1 holds pieces whose marks already fit a criterion. Row 2 holds pieces that need a pair with no mark: a side that lies in both triangles (it is equal to itself), or the two facing angles where two segments cross (vertical angles are equal). Row 3 holds pieces where no criterion fits, even after those pairs are counted.

The last row is the most important. Many students believe that a pair of triangles on a worksheet is always congruent. In two pieces of row 3 the triangles share a side or cross, but the extra pair still leaves two sides and an angle that is not between them (SSA). In three pieces of row 3 the triangles are drawn the same, but the marks do not prove it. The drawing never decides. Only the marks and the pairs with no mark decide.

Rows 1 and 2. Some pieces in row 1 touch or cross, but their marks are already enough. In two pieces of row 2 there are three marks, but they make SSA until the free pair is added. So counting marks does not sort the pieces. Guessing does not work: every simple guessing rule places at most six of the twelve pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Marks alone, a free pair, or not proved?

Choosing a criterion

Name _____

Period _____

Date _____

Cut out the twelve pieces on the next page. Each piece shows two triangles. **First use only the marks. If they fit a criterion, glue the piece in row 1. If not, look for a free pair. If that makes a criterion fit, glue it in row 2. If not, glue it in row 3.**



SSS Three pairs of sides.



SAS The angle is between the two sides.



ASA The side is between the two angles.



AAS The side is not between the two angles.



HL A right angle, the long side, and one more side.



Free side A side in both triangles counts as a pair.



Free angles Where two lines cross, the two facing angles count as a pair.

Some pairs have no mark. We call them free pairs. A side that is in **both** triangles is equal to itself. Where two segments cross, the two **facing** angles are equal. Count these pairs too. Do not trust how the drawing looks. Only the marks and the free pairs count. Two sides and an angle that is not between them prove nothing. Three angles prove nothing.

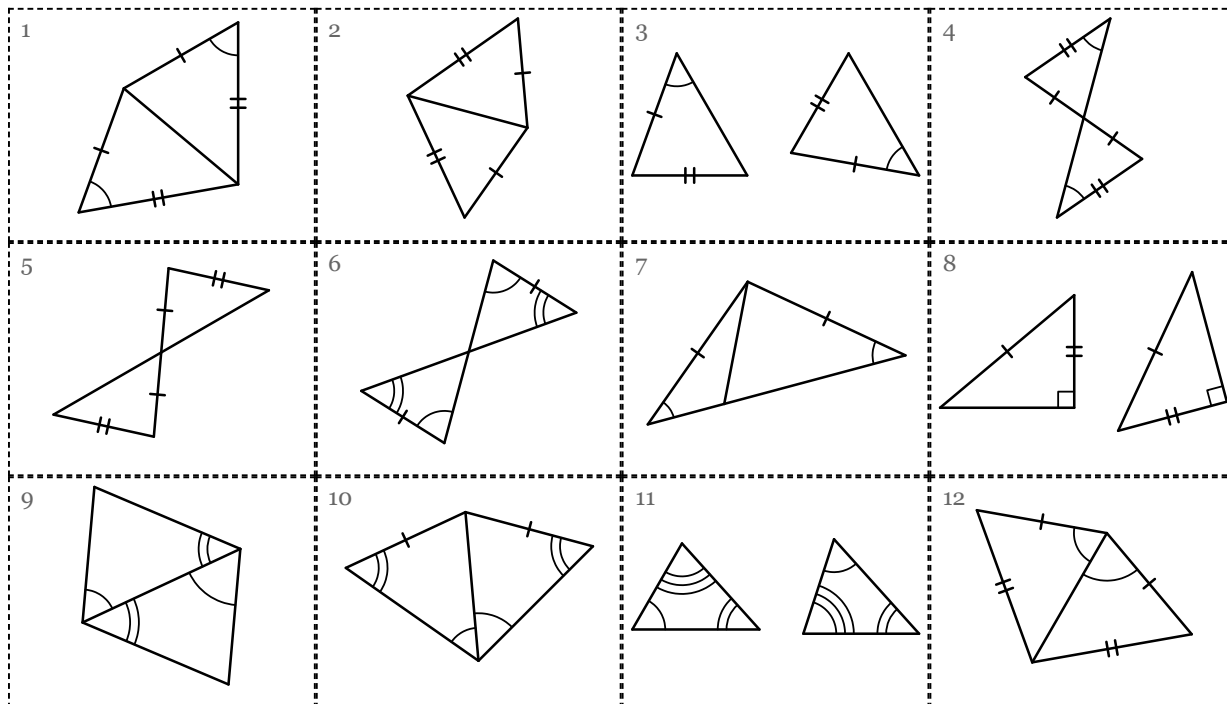
Row 1. Proved by the marks alone. The marks already fit SSS, SAS, ASA, AAS or HL.

Row 2. Proved only with a free pair. The marks alone are not enough. A side in both triangles, or the facing angles where two lines cross, makes it enough.

Row 3. Not proved. Even with the free pairs, no criterion fits.

The twelve pieces

Cut along the dotted lines



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Cut & Glue Sort — answers

For the teacher

Row 1	1 — Two pairs of sides and the angle between them are marked: SAS. The shared side is not needed.	6 — Two pairs of angles are marked, and the marked side is between them: ASA. The facing angles are not needed.	8 — Right angles, the long sides, and one more pair of sides are marked: HL.	10 — Two pairs of angles and a side that is not between them are marked: AAS. The shared side is not needed.
Row 2	2 — Two pairs of sides are marked. The side in both triangles is the third pair: SSS.	4 — The marks alone are two sides and an angle that is not between them. The facing angles at the crossing add a second pair of angles: AAS.	9 — Two pairs of angles are marked. The side in both triangles is between them: ASA.	12 — The marks alone are two sides and an angle that is not between them. The side in both triangles is the third pair of sides: SSS.
Row 3	3 — Two sides and an angle that is not between them (SSA). The triangles look the same, but the marks do not prove it.	5 — The facing angles give a pair of angles, but that angle is not between the two marked sides (SSA). The triangles look the same, but the marks do not prove it.	7 — The side in both triangles gives a second pair of sides, but the marked angle is not between the two sides (SSA).	11 — Three pairs of angles and no side. The triangles look the same, but they could be different sizes.

Every row holds four pieces. Sides with the same number of ticks are equal. Angles with the same number of arcs are equal. A small square marks a right angle.

Row 1: the marks alone fit SSS, SAS, ASA, AAS or HL. **Row 2:** the marks alone do not, but they do once the side in both triangles, or the facing angles at the crossing, are counted as a pair. **Row 3:** nothing fits, even with those pairs.

To check a piece with a student, ask: “Which pairs are marked? Is there a pair with no mark? Where does the angle sit: between the two sides or not?”