

CUT & GLUE SORT

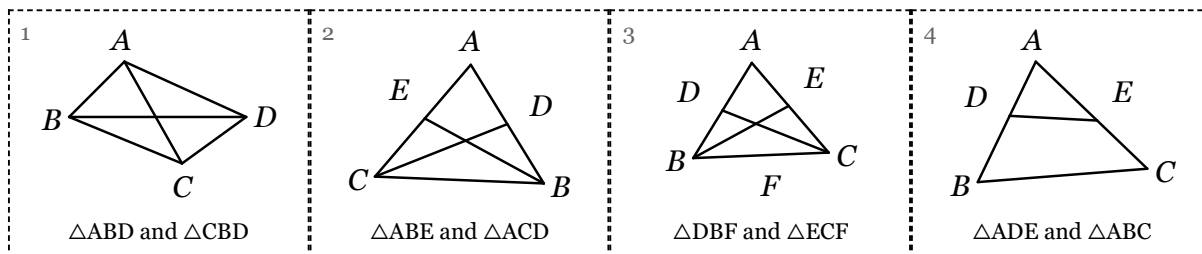


MULTI-STEP AND OVERLAPPING TRIANGLE PROOFS

What Do the Two Triangles Share?

Overlapping triangles · a side, an angle, or nothing

FREE



Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. Nothing else is needed.
2. **Say this once:** *“Each piece names two triangles. Find what they share: a side, an angle, or nothing. A corner in common is not always an angle in common.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 shows four small examples of parts that are shared and parts that only look shared.
Answers	page 5.

The first row is found from the letters. If two letters are in both triangle names, the segment joining them is a side of both. Students write the three sides of each triangle and look for a segment on both lists. This is the habit the lesson builds: when one triangle is drawn over another, the eye sees one big shape and misses the side they share.

The last row is the hardest, and it is the most important. Every piece in it has a corner in common, and students take that corner’s angle as shared. It is not. Where two triangles cross at one point, their angles there are vertical angles: two different angles. Where two triangles share a corner, their angles there can still use different rays. And where two triangles stand on one line and overlap, the overlap is part of a side of each but a side of neither.

How to see the mistake. A student who reads “a corner in common” as “an angle in common” gets eight of the twelve pieces right, and the second row fills up with eight pieces while the last row stays empty. Ask that student to follow the two arms of the angle in each triangle. Guessing does not work: every other simple guessing rule places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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What do the two triangles share?

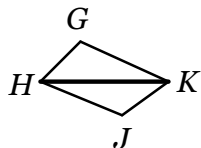
A side, an angle, or nothing

Name _____

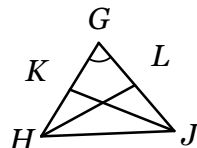
Period _____

Date _____

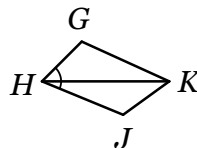
Cut out the twelve pieces on the next page. Each piece names two triangles in one drawing. **What do the two triangles share? Glue the piece in that row.**



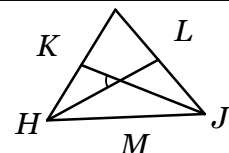
HK is a side of $\triangle GHK$ and a side of $\triangle JHK$. It is in both.



$\angle HGL$ and $\angle JGK$ are one angle. It is in $\triangle GHL$ and in $\triangle GJK$.



$\angle GHJ$ is made of two angles. It is in neither $\triangle GHK$ nor $\triangle JHK$.



$\angle KMH$ is only in $\triangle KHM$. $\triangle LJM$ has $\angle LMJ$, a different angle.

They share a side. *One segment is a side of both triangles.*

They share an angle, but no side. *One angle is an angle of both triangles.*

They share no side and no angle. *Some parts may look shared, but they are not.*

The twelve pieces

Cut along the dotted lines

1 $\triangle ABD$ and $\triangle CBD$	2 $\triangle ABE$ and $\triangle ACD$	3 $\triangle DBF$ and $\triangle ECF$	4 $\triangle ADE$ and $\triangle ABC$
5 $\triangle ABE$ and $\triangle DBC$	6 $\triangle DBC$ and $\triangle ECB$	7 $\triangle ABE$ and $\triangle DCE$	8 $\triangle PQR$ and $\triangle PQT$
9 $\triangle DBF$ and $\triangle ABE$	10 $\triangle ABD$ and $\triangle ADC$	11 $\triangle PQS$ and $\triangle PRT$	12 $\triangle ECF$ and $\triangle ACD$

A side of a triangle joins two of its corners. Write the three sides of each triangle. Is one segment on both lists? For an angle, look at the corner in the drawing and follow its two arms.



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Cut & Glue Sort — answers

For the teacher

They share a side.	1 · $\triangle ABD$ and $\triangle CBD$ B and D are corners of both triangles, so BD is a side of both. AC crosses BD and belongs to neither.	6 · $\triangle DBC$ and $\triangle ECB$ BC is a side of both. Their angles at B are different: $\angle EBC$ is only part of $\angle DBC$.	8 · $\triangle PQR$ and $\triangle PQT$ PQ is a side of both. They also share the angle at Q, because R is on QT.	10 · $\triangle ABD$ and $\triangle ADC$ AD is a side of both. The angles at A and at D are next to each other, not shared.
They share an angle, but no side.	2 · $\triangle ABE$ and $\triangle ACD$ The angle at A uses rays AB and AC in both triangles, because D is on AB and E is on AC. No segment is a side of both.	4 · $\triangle ADE$ and $\triangle ABC$ D is on AB and E is on AC, so the angle at A is the same in both. AD and AE are only parts of sides of $\triangle ABC$.	9 · $\triangle DBF$ and $\triangle ABE$ D is on BA and F is on BE, so the angle at B is the same in both. DB and BF are only parts of sides of $\triangle ABE$.	12 · $\triangle ECF$ and $\triangle ACD$ E is on CA and F is on CD, so the angle at C is the same in both. EC and CF are only parts of sides of $\triangle ACD$.
They share no side and no angle.	3 · $\triangle DBF$ and $\triangle ECF$ The triangles meet only at F. $\angle DFB$ and $\angle EFC$ are vertical angles: two different angles, not one.	5 · $\triangle ABE$ and $\triangle DBC$ B is a corner of both, but the angles at B differ: $\angle ABE$ is between BA and BD, and $\angle DBC$ is between BD and BC.	7 · $\triangle ABE$ and $\triangle DCE$ The triangles meet only at E. $\angle AEB$ and $\angle DEC$ are vertical angles: two different angles.	11 · $\triangle PQS$ and $\triangle PRT$ RS is only part of QS and part of RT, so it is not a side of either. At P, $\angle QPS$ and $\angle RPT$ overlap but are different angles.

Every row holds four pieces. A segment is a side of a triangle when both of its letters are corners of that triangle. An angle is an angle of a triangle when its corner is a corner of the triangle and its two arms run along the triangle's two sides from that corner.

To check a piece with a student, write the three sides of each triangle side by side. If no segment is on both lists, put a finger on the corner the two names have in common. Follow the two arms of that corner in the first triangle, then in the second. If the arms are the same two rays, the angle is shared.