

ANSWER MAZE

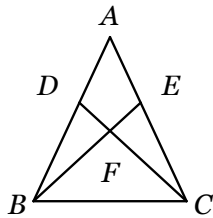


MULTI-STEP AND OVERLAPPING TRIANGLE PROOFS

One Figure, Two Congruences

One page · 10–15 minutes · nothing to prepare

FREE



D is on AB, and E is on AC. BE and CD cross at F.

Given: $AB \cong AC$ and $AD \cong AE$.

To prove: $FB \cong FC$.

Each box asks about one step of this proof. Choose an answer, and go to the box it names.

3

$\triangle ABE$ and $\triangle ACD$. Which angle is an angle of both triangles?

No angle. $\angle BAE$ and $\angle CAD$ have different names. → go to 2

$\angle BAC$. E is on AC, and D is on AB. → go to 6

Follow ray AE from A. Then follow ray AD from A. Where does each ray run?

6

So $\triangle ABE \cong \triangle ACD$ by SAS. Does CPCTC now give $FB \cong FC$?

No. FB is not a side of either triangle. → go to 1

Yes. CPCTC gives $FB \cong FC$. → go to 5

Find the three corners of $\triangle ABE$. Look for F among them.

1. **Hand out page 3**, one per student. Nothing else is needed.
2. **Say this once:** *“Start at box 3. Every box is about the figure at the top. Write every box number in the row at the bottom as you go.”*
3. **Check a sheet quickly.** A student with no mistakes writes **3 6 1 8 4 7**: six boxes, then out.

Student sheet	page 3, one per student. The figure is printed once at the top, with what is given and what is to be proved.
Answers	page 4, with the whole proof written out.

The route walks one proof that needs two congruences. First $\triangle ABE \cong \triangle ACD$ by SAS, using the angle at A that both triangles have. Then CPCTC gives $\angle ABE \cong \angle ACD$. That angle, $DB \cong EC$ and the vertical angles at F prove $\triangle DBF \cong \triangle ECF$ by AAS. CPCTC on the second congruence gives $FB \cong FC$.

Every wrong answer comes from one of two mistakes. The first is reading the figure wrongly: taking $\angle BAE$ and $\angle CAD$ as two angles because their letters differ, taking the vertical angles at F as one shared angle, or reading $DB \cong EC$ off the drawing. These wrong answers send the student to box 2. The second is losing the chain: stopping after the first congruence, using $FB \cong FC$ to prove itself, or counting one congruence where two are needed. These send the student to box 5. Boxes 2 and 5 ask a question of the same kind, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more. Guessing does not get a student out: every simple guessing rule, such as always taking the first answer, goes wrong within the first two boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
chooseyourlesson.com — 135 free geometry lessons.

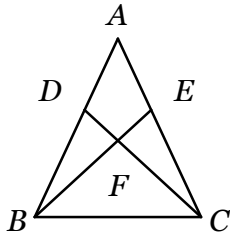
Answer Maze

Start at box 3 · one figure, two congruences

Name _____

Period _____

Date _____



D is on AB, and E is on AC. BE and CD cross at F.

Given: $AB \cong AC$ and $AD \cong AE$.

To prove: $FB \cong FC$.

Each box asks about one step of this proof. Choose an answer, and go to the box it names.

1

$\triangle DBF$ and $\triangle ECF$ need $DB \cong EC$. How is it known?

DB and EC look the same length. → go to 2
Take AD from AB, and take AE from AC. → go to 8

Read the points along AB in order. Then read the points along AC.

2

$\triangle DBC$ and $\triangle ECB$. Which part is in both triangles?

BC. It is a side of each triangle. → go to 6
∠B. Each triangle has a corner at B. → go to 3

Write the three sides of each triangle. Then follow the arms at B in each.

3

$\triangle ABE$ and $\triangle ACD$. Which angle is an angle of both triangles?

No angle. ∠BAE and ∠CAD have different names. → go to 2
∠BAC. E is on AC, and D is on AB. → go to 6

Follow ray AE from A. Then follow ray AD from A. Where does each ray run?

4

$\triangle DBF$ and $\triangle ECF$ meet at F. What do they have at F?

One angle that is in both triangles. → go to 2
Two vertical angles, ∠DFB and ∠EFC. → go to 7

Name the angle of $\triangle DBF$ at F. Then name the angle of $\triangle ECF$ at F.

5

Which two triangles have FB and FC as sides?

$\triangle ABE$ and $\triangle ACD$. → go to 6
 $\triangle DBF$ and $\triangle ECF$. → go to 1

Look for triangle names that hold F and B, and F and C.

6

So $\triangle ABE \cong \triangle ACD$ by SAS. Does CPCTC now give $FB \cong FC$?

No. FB is not a side of either triangle. → go to 1
Yes. CPCTC gives $FB \cong FC$. → go to 5

Find the three corners of $\triangle ABE$. Look for F among them.

7

How many triangle congruences does the proof of $FB \cong FC$ need?

Two. First $\triangle ABE \cong \triangle ACD$, then $\triangle DBF \cong \triangle ECF$. → you are out
One. $\triangle ABE \cong \triangle ACD$ is enough. → go to 5

List each triangle pair the proof uses, from the start to $FB \cong FC$.

8

$\triangle DBF$ and $\triangle ECF$ need $\angle DBF \cong \angle ECF$. Where does it come from?

From $\triangle ABE \cong \triangle ACD$. ∠DBF is ∠ABE, and ∠ECF is ∠ACD. → go to 4
From $FB \cong FC$, the fact to be proved. → go to 5

Check which facts are known so far, and which fact is the goal.

Write each box number as you reach it:

--	--	--	--	--	--	--	--	--



135 free geometry lessons
chooseyourlesson.com

The route, in order. Boxes 2 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table.

3 → 6 → 1 → 8 → 4 → 7 → out

	Box	Right answer, and why	Wrong answer, and why a student picks it
1	Box 3	∠BAC. E is on AC, and D is on AB. At A, $\triangle ABE$ uses rays AB and AE, and $\triangle ACD$ uses rays AC and AD. E is on AC and D is on AB, so these are the same two rays. $\angle BAE$ and $\angle CAD$ are two names for $\angle BAC$.	“No angle. $\angle BAE$ and $\angle CAD$ have different names.” goes to box 2. A student who compares letters instead of looking at the rays picks this.
2	Box 6	No. FB is not a side of either triangle. CPCTC only gives parts of $\triangle ABE$ and $\triangle ACD$. FB is only part of side BE, so the first congruence says nothing about it.	“Yes. CPCTC gives $FB \cong FC$.” goes to box 5. A student who stops after the first congruence picks this.
3	Box 1	Take AD from AB, and take AE from AC. D is between A and B, so $DB = AB - AD$. E is between A and C, so $EC = AC - AE$. Equal lengths taken from equal lengths leave equal lengths.	“DB and EC look the same length.” goes to box 2. A student who reads the fact off the drawing picks this. A drawing can never show that two lengths are equal.
4	Box 8	From $\triangle ABE \cong \triangle ACD$. $\angle DBF$ is $\angle ABE$, and $\angle ECF$ is $\angle ACD$. CPCTC on $\triangle ABE \cong \triangle ACD$ gives $\angle ABE \cong \angle ACD$. D is on BA and F is on BE, so $\angle DBF$ is $\angle ABE$. In the same way $\angle ECF$ is $\angle ACD$.	“From $FB \cong FC$, the fact to be proved.” goes to box 5. A student who uses the goal to prove itself picks this. That is circular.
5	Box 4	Two vertical angles, $\angle DFB$ and $\angle EFC$. The angle of $\triangle DBF$ at F is $\angle DFB$. The angle of $\triangle ECF$ at F is $\angle EFC$. They are across from each other at the crossing: vertical angles, so they are congruent.	“One angle that is in both triangles.” goes to box 2. A student who sees one crossing and thinks of one angle picks this.
6	Box 7	Two. First $\triangle ABE \cong \triangle ACD$, then $\triangle DBF \cong \triangle ECF$. The first congruence gives $\angle ABE \cong \angle ACD$. That angle, $DB \cong EC$ and the vertical angles at F prove $\triangle DBF \cong \triangle ECF$ by AAS. CPCTC on that second congruence gives $FB \cong FC$.	“One. $\triangle ABE \cong \triangle ACD$ is enough.” goes to box 5. A student who does not see that the first congruence is only a step picks this.
	Box 2	BC. It is a side of each triangle. B and C are corners of both, so BC is a side of both. At B, $\triangle DBC$ uses rays BD and BC, but $\triangle ECB$ uses rays BE and BC: two different angles.	“ $\angle B$. Each triangle has a corner at B.” goes to box 3. A student who takes a corner in common for an angle in common picks this.
	Box 5	$\triangle DBF$ and $\triangle ECF$. F and B are corners of $\triangle DBF$, and F and C are corners of $\triangle ECF$. $\triangle ABE$ and $\triangle ACD$ have no corner at F.	“ $\triangle ABE$ and $\triangle ACD$.” goes to box 6. A student who looks only at the big triangles picks this.

The whole proof, for reading aloud.

1	$AB \cong AC$	Given
2	$AD \cong AE$	Given
3	$\angle BAE \cong \angle CAD$	The same angle (reflexive): E is on AC and D is on AB
4	$\triangle ABE \cong \triangle ACD$	SAS (rows 1, 3, 2)
5	$\angle ABE \cong \angle ACD$	CPCTC (row 4)
6	$DB \cong EC$	Subtraction: $DB = AB - AD$ and $EC = AC - AE$ (rows 1, 2)
7	$\angle DFB \cong \angle EFC$	Vertical angles
8	$\triangle DBF \cong \triangle ECF$	AAS (rows 5, 7, 6); $\angle DBF$ is $\angle ABE$ and $\angle ECF$ is $\angle ACD$
9	$FB \cong FC$	CPCTC (row 8)

A student can find the last four boxes of the route by following the arrows backward from “you are out.” So ask a student with a clean row to explain one box.