

MYSTERY PICTURE

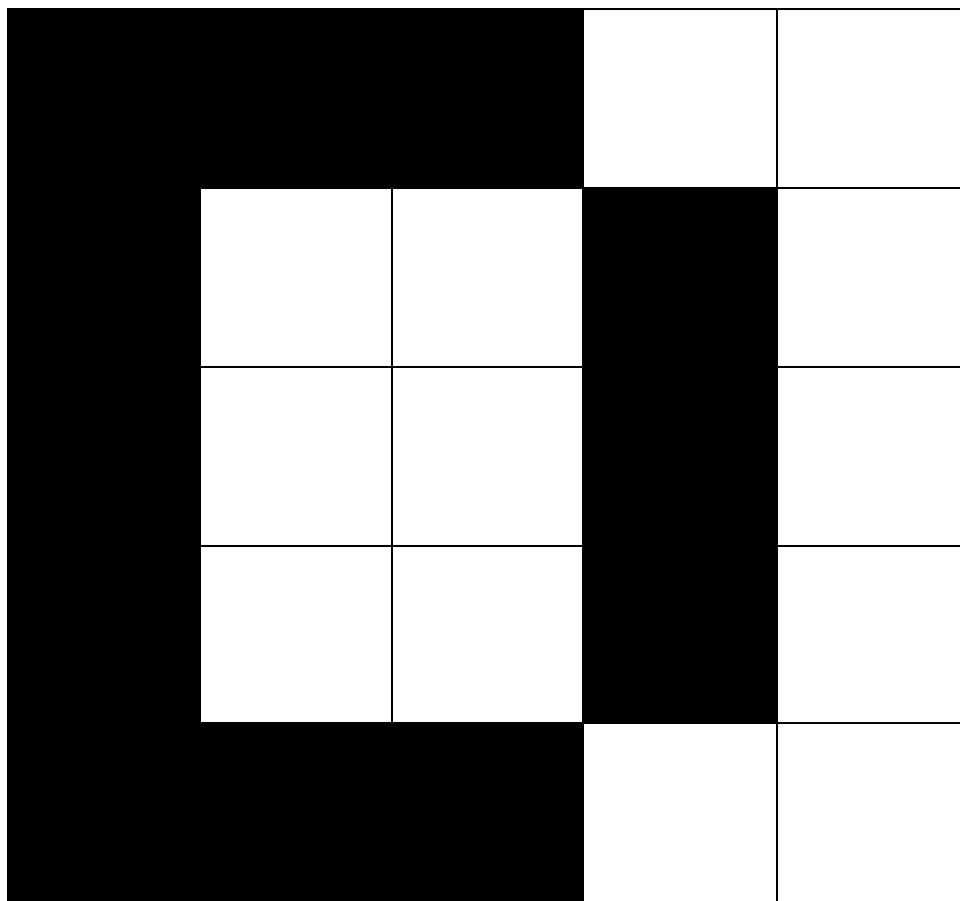


MULTI-STEP AND OVERLAPPING TRIANGLE PROOFS

Is It in Both Triangles?

25 overlapping figures · the shared side and the shared angle

FREE



1. **Hand out page 3**, one per student. Pencils. Nothing else is needed.
2. **Say this once:** *“Each square names two triangles and one part. Is that part a side of both triangles, or an angle of both? If yes, fill the square with slanted lines. If no, leave it white.”*
3. **When a student finishes, look at the picture.** If it is not the letter D, a square is wrong. The student finds it and checks it again.

Student sheet	page 3, one per student. Four small examples at the top show a side in both triangles, an angle in both, and two angles that only look shared.
Answers	page 4: the finished picture and the reason for every square.

This sheet practices one skill: finding what two overlapping triangles really have in common. When one triangle is drawn on top of another, the eye sees one big shape. Students then miss the side the two triangles share, or they name a part that only looks shared.

How to decide a square. A segment is a side of a triangle when both of its letters are corners of that triangle. So a student can write the three sides of each triangle and look for the segment on both lists. An angle is an angle of a triangle when its corner is a corner of the triangle and its two arms run along the triangle’s two sides from that corner. In six filled squares the angle has letters that are not in both triangle names. For example, $\angle BAC$ is an angle of $\triangle ABE$, because E is on AC. Students must look at the drawing for these.

The white squares are the usual mistakes. A diagonal that crosses the shared side is not shared. An angle made of two smaller angles is an angle of neither small triangle. Where two triangles cross at one point, they have two different angles there (vertical angles), not one shared angle. A segment that is only part of a side is not a side.

Twelve squares are filled and thirteen are white; eleven squares ask about a side. Guessing does not work. Every simple guessing rule, such as filling every square with a heavy segment, gets at least five squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

chooseyourlesson.com — 135 free geometry lessons.

Mystery Picture

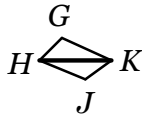
Is it in both triangles?

Name _____

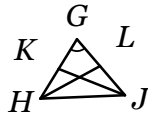
Period _____

Date _____

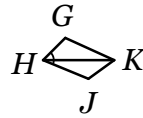
Each square names two triangles and one part. The part is drawn heavy, or marked with an arc. Is the part a side of both triangles, or an angle of both? If yes, fill the square with slanted lines. If no, leave it white.



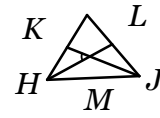
HK is a side of $\triangle GHK$ and a side of $\triangle JHK$. It is in both.



$\angle HGL$ and $\angle JGK$ are one angle. It is in $\triangle GHL$ and in $\triangle GJK$.



$\angle GHJ$ is made of two angles. It is in neither $\triangle GHK$ nor $\triangle JHK$.



$\angle KMH$ is only in $\triangle KHM$. $\triangle LJM$ has $\angle LMJ$, a different angle.

1 $\triangle ABD$ and $\triangle CBD$ BD	2 $\triangle ABE$ and $\triangle ACD$ $\angle BAC$	3 $\triangle PRS$ and $\triangle PST$ PS	4 $\triangle DBF$ and $\triangle ECF$ $\angle DFB$	5 $\triangle ABD$ and $\triangle ABC$ AD
6 $\triangle ABD$ and $\triangle ABC$ $\angle ABD$	7 $\triangle ABD$ and $\triangle CBD$ AC	8 $\triangle DBC$ and $\triangle ECB$ $\angle DBC$	9 $\triangle ABC$ and $\triangle DCB$ BC	10 $\triangle PQS$ and $\triangle PRT$ $\angle RPS$
11 $\triangle ABC$ and $\triangle DBC$ $\angle DBC$	12 $\triangle PQS$ and $\triangle PRT$ RS	13 $\triangle ABD$ and $\triangle CBD$ $\angle ABC$	14 $\triangle ABE$ and $\triangle EBC$ BE	15 $\triangle ABD$ and $\triangle ADC$ $\angle ADB$
16 $\triangle PQR$ and $\triangle PQT$ $\angle PQT$	17 $\triangle ABE$ and $\triangle ACD$ AE	18 $\triangle ABC$ and $\triangle DCB$ $\angle ACB$	19 $\triangle ABD$ and $\triangle ADC$ AD	20 $\triangle ABC$ and $\triangle ADC$ $\angle BAD$
21 $\triangle ABC$ and $\triangle EBC$ $\angle ACB$	22 $\triangle DBC$ and $\triangle ECB$ BC	23 $\triangle ABE$ and $\triangle ACD$ $\angle DAE$	24 $\triangle ABE$ and $\triangle DCE$ $\angle DEC$	25 $\triangle ABC$ and $\triangle ADC$ BD

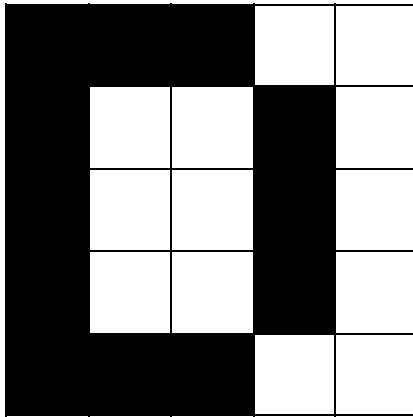
When you finish, look at the picture. If part of it is missing, or one square does not fit, check that square again.



135 free geometry lessons
chooseyourlesson.com

Mystery Picture — answers

For the teacher



The finished picture is **the letter D**: a bar down the left side, short bars along the top and the bottom, and a bar down the right side between them.

A square is filled when the marked part is a side of both triangles, or an angle of both. It is white when the part belongs to one triangle only, or to neither.

To check a side with a student, write the three sides of each triangle and look for it on both lists. To check an angle, put a finger on its corner and follow its two arms. Then do the same for each triangle.

1–5	1 · FILL BD is a side of $\triangle ABD$ and of $\triangle CBD$. B and D are corners of both triangles.	2 · FILL $\angle BAC$ is an angle of $\triangle ABE$ and of $\triangle ACD$. At A, $\triangle ABE$ uses rays AB and AE, and $\triangle ACD$ uses rays AC and AD. E is on AC and D is on AB, so these are the same two rays.	3 · FILL PS is a side of $\triangle PRS$ and of $\triangle PST$. P and S are corners of both triangles.	4 · white $\angle DFB$ is an angle of $\triangle DBF$, but not of $\triangle ECF$. The two triangles meet only at F. $\triangle ECF$ has $\angle EFC$ there, which is the vertical angle across from $\angle DFB$: a different angle.	5 · white AD is a side of $\triangle ABD$, but not of $\triangle ABC$. In $\triangle ABC$, AD is a segment drawn inside the triangle. It joins a corner to a point of side BC.
6–10	6 · FILL $\angle ABD$ is an angle of $\triangle ABD$ and of $\triangle ABC$. D is on BC, so ray BD is ray BC. The angle at B uses rays BA and BC in both triangles.	7 · white AC is a side of neither triangle. AC crosses the shared side BD. C is not a corner of $\triangle ABD$, and A is not a corner of $\triangle CBD$.	8 · white $\angle DBC$ is an angle of $\triangle DBC$, but not of $\triangle ECB$. At B, $\triangle ECB$ has the angle between BE and BC. That is only part of $\angle DBC$.	9 · FILL BC is a side of $\triangle ABC$ and of $\triangle DCB$. B and C are corners of both triangles.	10 · white $\angle RPS$ is an angle of neither triangle. At P, $\triangle PQS$ has $\angle QPS$ and $\triangle PRT$ has $\angle RPT$. $\angle RPS$ is only the part where they overlap.
11–15	11 · FILL $\angle DBC$ is an angle of $\triangle ABC$ and of $\triangle DBC$. D is on BA, so ray BD is ray BA. $\angle DBC$ is the same angle as $\angle ABC$.	12 · white RS is a side of neither triangle. RS is only part of side QS and part of side RT. It is not a whole side of either.	13 · white $\angle ABC$ is an angle of neither triangle. The angle at B is made of two angles: $\angle ABD$ in $\triangle ABD$ and $\angle CBD$ in $\triangle CBD$.	14 · FILL BE is a side of $\triangle ABE$ and of $\triangle EBC$. B and E are corners of both triangles.	15 · white $\angle ADB$ is an angle of $\triangle ABD$, but not of $\triangle ADC$. At D, $\triangle ADC$ has $\angle ADC$, the angle next to $\angle ADB$ on the other side of AD.
16–20	16 · FILL $\angle PQT$ is an angle of $\triangle PQR$ and of $\triangle PQT$. R is on QT, so ray QR is ray QT. The angle at Q uses rays QP and QT in both triangles.	17 · white AE is a side of $\triangle ABE$, but not of $\triangle ACD$. In $\triangle ACD$, AE is only part of side AC.	18 · white $\angle ACB$ is an angle of $\triangle ABC$, but not of $\triangle DCB$. At C, $\triangle DCB$ has $\angle DCB$, between CD and CB. $\angle ACB$ is only part of it.	19 · FILL AD is a side of $\triangle ABD$ and of $\triangle ADC$. A and D are corners of both triangles.	20 · white $\angle BAD$ is an angle of neither triangle. The angle at A is made of two angles: $\angle BAC$ in $\triangle ABC$ and $\angle CAD$ in $\triangle ADC$.
21–25	21 · FILL $\angle ACB$ is an angle of $\triangle ABC$ and of $\triangle ECB$. E is on CA, so ray CE is ray CA. The angle at C uses rays CA and CB in both triangles.	22 · FILL BC is a side of $\triangle DBC$ and of $\triangle ECB$. B and C are corners of both triangles.	23 · FILL $\angle DAE$ is an angle of $\triangle ABE$ and of $\triangle ACD$. D is on AB and E is on AC. So $\angle DAE$ is the angle at A of both triangles.	24 · white $\angle DEC$ is an angle of $\triangle DCE$, but not of $\triangle ABE$. The two triangles meet only at E. $\triangle ABE$ has $\angle AEB$ there, which is the vertical angle across from $\angle DEC$: a different angle.	25 · white BD is a side of neither triangle. BD crosses the shared side AC. B is not a corner of $\triangle ADC$, and D is not a corner of $\triangle ABC$.