

SCAVENGER HUNT



PERPENDICULAR BISECTORS AND THE CIRCUMCENTER

How Far Is the Center From Each Corner?

Distances, right triangles, x and coordinates · the cards check the answers

FREE

ANSWER

19

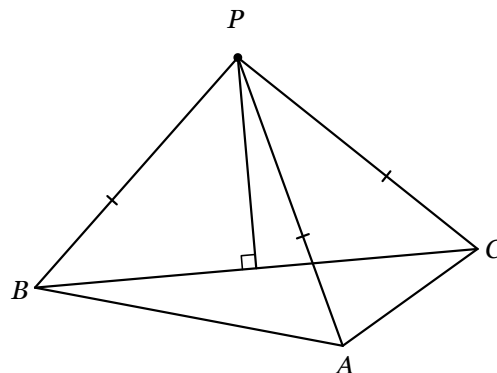
P is the circumcenter of $\triangle ABC$.

$PA = 29$

P is 20 units from side BC.

Find PB.

Draw the three segments from the circumcenter to the corners.



R

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Scavenger Hunt — how to run it

a full class period

1. **Tape the 16 cards around the room**, in any order. Each card has a large letter in its bottom left corner. Start each group at a *different* one of these eight cards: *R H K P C M A L*.
2. **Say this once:** *“The circumcenter is the same distance from all three corners. In a right triangle it is the middle of the longest side. Eight right answers bring you back to your first card.”*
3. **Each group finds the number, then walks to the card with that number at the top.** Nothing else is needed.

16 cards	pages 3–10, one copy. There are two cards on each sheet. Cut the whole stack once.
Record sheet	page 11, one per student. It shows a perpendicular bisector and the circumcenter of three kinds of triangle as small drawings.
Answers	pages 12–13, for you only. Do not put them up in the room.

Every card asks for a distance or a coordinate that the circumcenter decides. On two ring cards, the distance from the circumcenter to one corner and to one side are both given, and the answer is the distance to another corner. On three, the triangle has a right angle, so the circumcenter is the middle of the longest side, and each corner is half of that side away. On two, two distances are written with x : they are equal, so the student solves for x and then finds the distance. On one, the triangle is given by its corners, and the student finds the middle of the horizontal side.

Eight of the sixteen cards make the ring. A group reaches the other eight only by a wrong answer, but all the cards look the same. Each ring card has one common mistake: the distance to a side instead of a corner, the whole longest side instead of half, x instead of the distance, or half the side's length instead of its middle. That mistake gives the number at the top of a card off the ring. The group does that card, and its answer sends the group back to the card it got wrong. If the group goes wrong there too, it goes one more card back and still does not reach the ring.

The check is the length of the group's list. A clean run is **eight lines**. Each wrong answer adds two. Every card has a line in small type under the question. It tells a group that arrived by a wrong answer what to check. A group that arrived by a right answer skips it. If a group keeps coming back to the same cards, do not give the answer. Ask: *“Which of your lengths ends at a corner?”*



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ANSWER

30

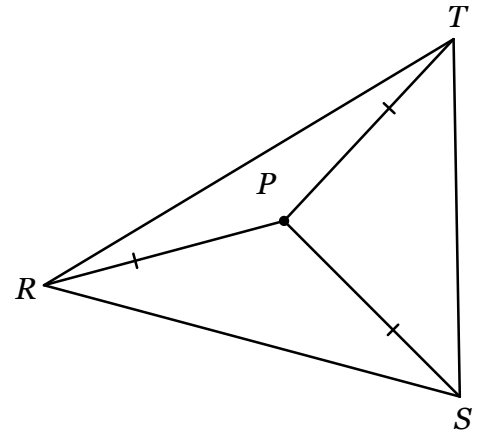
P is the circumcenter of $\triangle RST$.

$$PR = 3x - 5$$

$$PS = x + 13$$

Find PT.

Does your answer make every distance a positive number?



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A

ANSWER

15

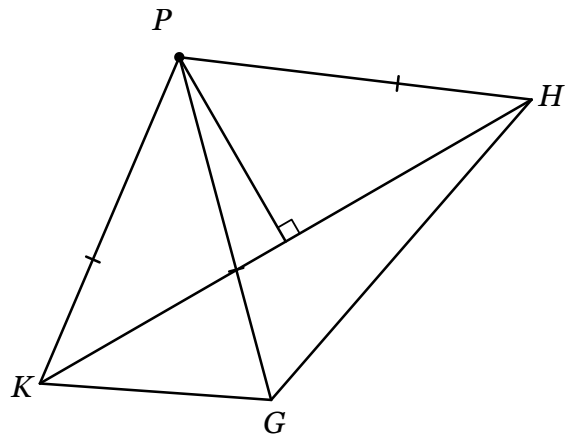
P is the circumcenter of $\triangle GHK$.

$$PG = 10$$

P is 6 units from side HK.

Find PH.

From the middle of the longest side, each end is half of that side away.



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B

ANSWER

12

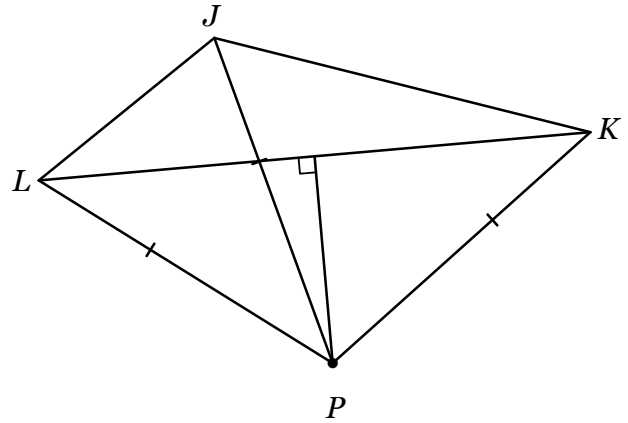
P is the circumcenter of $\triangle JKL$.

$PJ = 10$

P is 6 units from side KL.

Find PK.

Check that all three distances to the corners agree.



C

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ANSWER

20

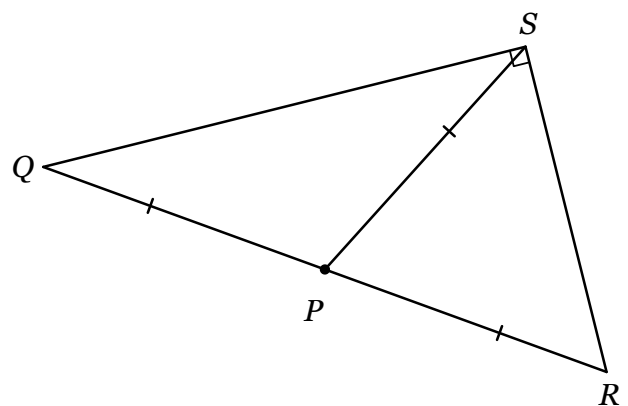
$\triangle QRS$ has a right angle at S.

$QR = 38$

P is the circumcenter.

Find PS.

A circumcenter is the same distance from all three corners, not from the sides.



D

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ANSWER

38

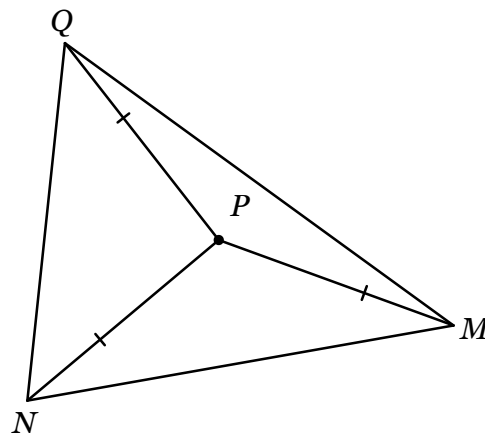
P is the circumcenter of $\triangle MNQ$.

$$PM = 3x - 5$$

$$PN = x + 13$$

Find PQ.

Half of the longest side is the distance to each corner.



E

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ANSWER

7

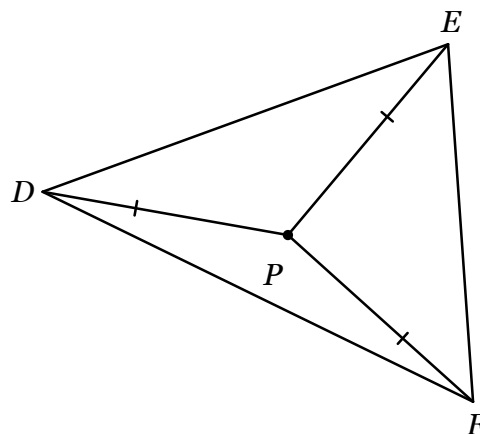
P is the circumcenter of $\triangle DEF$.

$$PD = 2x + 5$$

$$PE = 4x - 11$$

Find PF.

A midpoint is the average of the two ends, not half the length.



F

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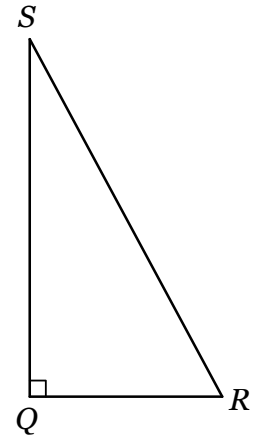
ANSWER

6

$Q(5, 1)$, $R(19, 1)$, $S(5, 27)$

What is the x-coordinate of the circumcenter of $\triangle QRS$?

Which segment ends at a corner? That one is the distance.



G

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ANSWER

29

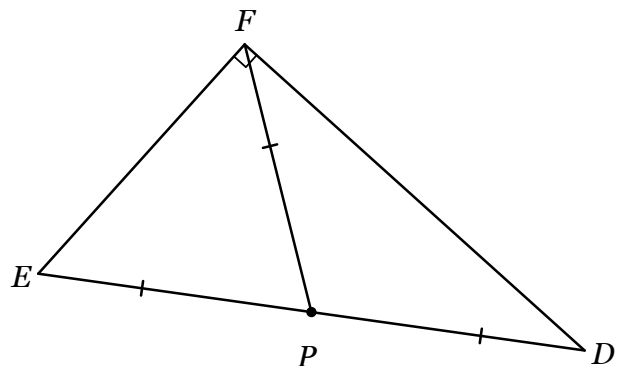
$\triangle DEF$ has a right angle at F.

$DE = 34$

P is the circumcenter.

Find PF.

The circle through the three corners has its center at the circumcenter.



H

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ANSWER

8

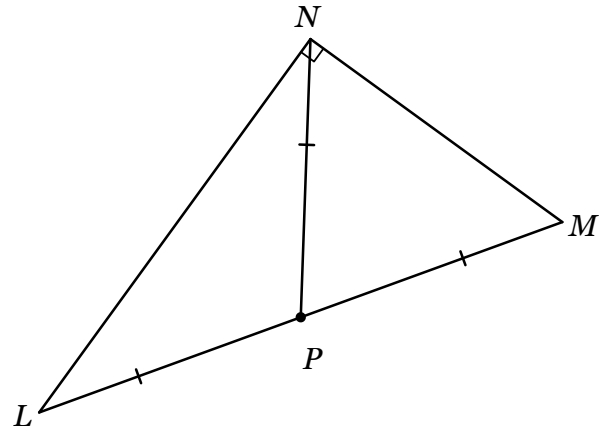
$\triangle LMN$ has a right angle at N .

$$LM = 34$$

P is the circumcenter.

Find PN .

Is your answer a length, or is it x ?



J

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ANSWER

17

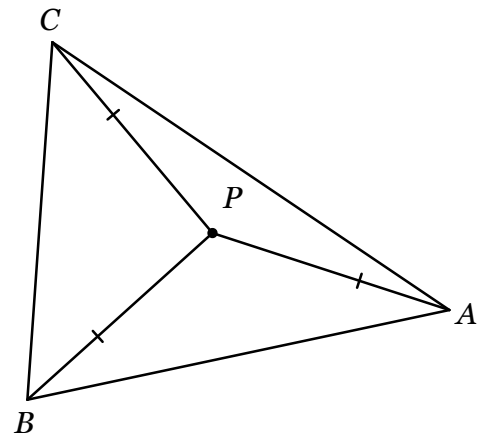
P is the circumcenter of $\triangle ABC$.

$$PA = 2x + 5$$

$$PB = 4x - 11$$

Find PC .

Two distances from the circumcenter are equal. Set them equal and solve.



K

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ANSWER

22

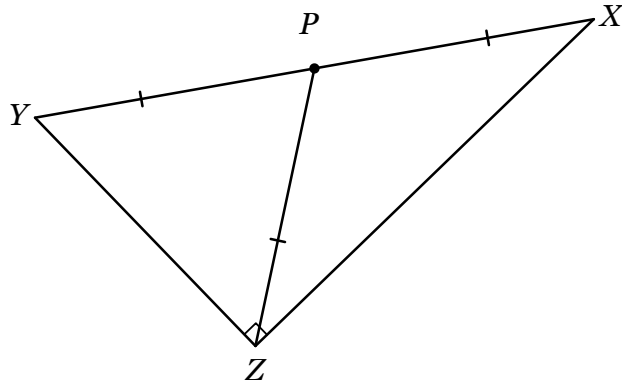
$\triangle XYZ$ has a right angle at Z.

$$XY = 38$$

P is the circumcenter.

Find PZ.

Which side of a right triangle is the longest? Half of it is the radius.



L

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ANSWER

10

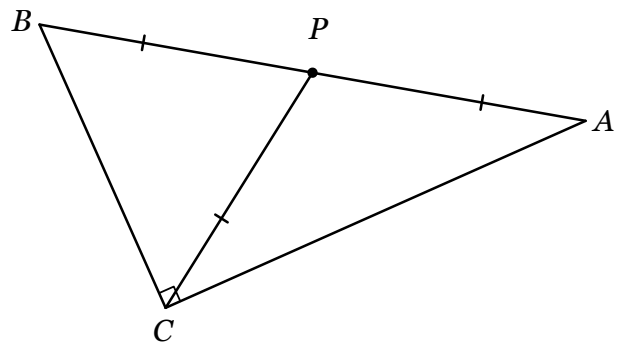
$\triangle ABC$ has a right angle at C.

P is the circumcenter.

$$PC = 15$$

Find AB.

Is the triangle acute, right or obtuse? Predict where the circumcenter is.



M

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ANSWER

9

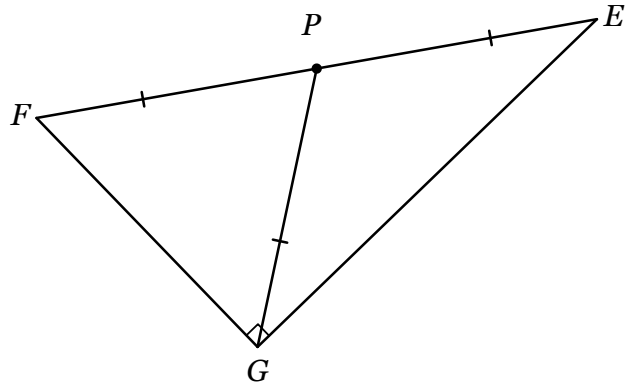
$\triangle EFG$ has a right angle at G .

P is the circumcenter.

$PG = 15$

Find EF .

Put your value of x into the expression to find the distance.



N

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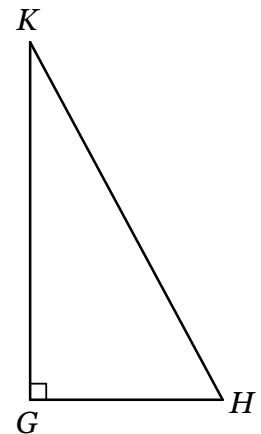
ANSWER

21

$G(5, 1)$, $H(19, 1)$, $K(5, 27)$

What is the x-coordinate of the circumcenter of $\triangle GHK$?

A horizontal side has a vertical perpendicular bisector through its midpoint.



P

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ANSWER

34

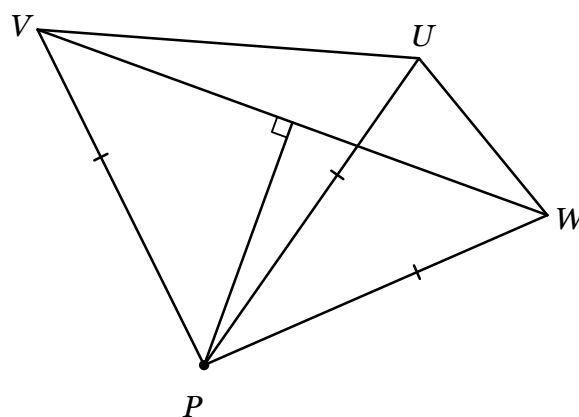
P is the circumcenter of $\triangle UVW$.

$PU = 29$

P is 20 units from side VW .

Find PV .

In a right triangle, the circumcenter is the middle of the longest side.



Q

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ANSWER

19

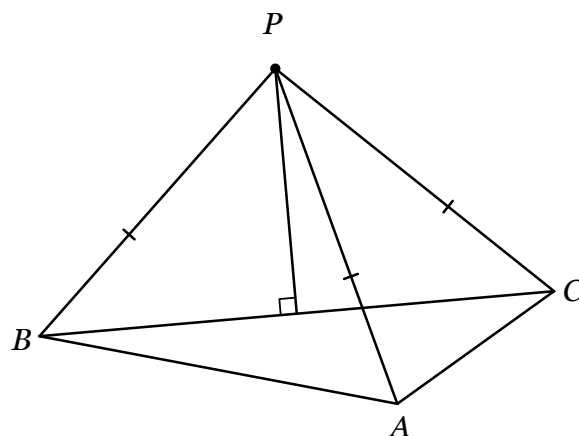
P is the circumcenter of $\triangle ABC$.

$PA = 29$

P is 20 units from side BC .

Find PB .

Draw the three segments from the circumcenter to the corners.



R

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Scavenger Hunt — where I went

The circumcenter

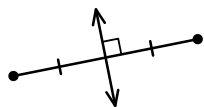
Name _____

Period _____

Date _____

Do the card you are at. Fill in one line. Then find the card with your answer at the top. A clean run is eight lines. Each card has a line in small type under the question. **If you are on a card because you went wrong, its line says what to check. If you were right, skip it.**

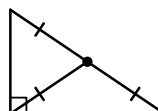
The **circumcenter** (the dot) is the one point that is the same distance from all three **corners**. The ticks show the three equal distances.



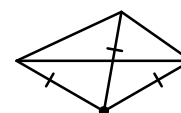
Perpendicular bisector of a side: a right-angle square **and** equal ticks.



Acute triangle: the circumcenter is **inside**.



Right triangle: it is the **middle of the longest side**.



Obtuse triangle: it is **outside**.

Card letter

My work

My answer

1

2

3

4

5

6

7

8

9

10

Why do the three corners never argue about the circumcenter?

Every number stands for a letter. Write the letter for each of your answers in the box above that number. Leave the other boxes empty. Then read the boxes you filled, left to right. **If they do not spell words, one answer is wrong.**

6 = L 7 = A 8 = Y 9 = F 10 = F 12 = A 15 = A 17 = I 19 = R 20 = I 21 = P 22 = L 29 = A 30 = Y 34 = R 38 = P

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

6

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38



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Scavenger Hunt — answers

For the teacher

The ring in order. A group may begin anywhere on it. Wherever it begins, eight right answers bring it back to that card.

R → H → K → P → C → M → A → L → R

Why do the three corners never argue about the circumcenter? The answer is *FAIR PLAY*. The ring's answers, in the order of the boxes on the record sheet (smallest number first), with their letters: **10 F • 12 A • 17 I • 19 R • 21 P • 22 L • 29 A • 30 Y**.

	Card	The card	Answer, and why	The common wrong answer	Joke letter for the number at its top
1	R	P is the circumcenter of $\triangle ABC$. $PA = 29$ P is 20 units from side BC. Find PB.	29. P is the same distance from all three corners, so $PB = PA = 29$. The 20 is the distance to a side.	20, from giving the distance to a side instead of to a corner	R
2	H	$\triangle DEF$ has a right angle at F. $DE = 34$ P is the circumcenter. Find PF.	17. The circumcenter of a right triangle is the middle of the longest side, DE. It is the same distance from all three corners, so $PF = 34 \div 2 = 17$.	34, from giving the whole longest side instead of half of it	A
3	K	P is the circumcenter of $\triangle ABC$. $PA = 2x + 5$ $PB = 4x - 11$ Find PC.	21. $PA = PB$, so $2x + 5 = 4x - 11$, and $x = 8$. Then $PA = 21$, and PC is the same.	8, from giving x instead of the distance	I
4	P	G(5, 1), H(19, 1), K(5, 27) What is the x-coordinate of the circumcenter of $\triangle GHK$?	12. Side GH is horizontal. Its perpendicular bisector is the vertical line through its middle, $x = (5 + 19) \div 2 = 12$.	7, from halving the length of the side instead of finding its middle	P
5	C	P is the circumcenter of $\triangle JKL$. $PJ = 10$ P is 6 units from side KL. Find PK.	10. P is the same distance from all three corners, so $PK = PJ = 10$. The 6 is the distance to a side.	6, from giving the distance to a side instead of to a corner	A
6	M	$\triangle ABC$ has a right angle at C. P is the circumcenter. $PC = 15$ Find AB.	30. The circumcenter is the middle of the longest side, and $PA = PB = PC = 15$. So $AB = 2 \times 15 = 30$.	15, from copying the distance instead of doubling it	F
7	A	P is the circumcenter of $\triangle RST$. $PR = 3x - 5$ $PS = x + 13$ Find PT.	22. $PR = PS$, so $3x - 5 = x + 13$, and $x = 9$. Then $PR = 22$, and PT is the same.	9, from giving x instead of the distance	Y
8	L	$\triangle XYZ$ has a right angle at Z. $XY = 38$ P is the circumcenter. Find PZ.	19. The circumcenter of a right triangle is the middle of the longest side, XY. It is the same distance from all three corners, so $PZ = 38 \div 2 = 19$.	38, from giving the whole longest side instead of half of it	L

Each wrong answer in the table is the number at the top of a card off the ring. Those cards are on the next page.

Scavenger Hunt — the cards off the ring

For the teacher

A group reaches these eight cards only by a wrong answer. The number at the top of each one is the wrong answer from one ring card. Its right answer is the number at the top of that ring card, so the group goes back to the card it got wrong. If the group makes the same kind of mistake there, it goes to another card off the ring. So a second wrong answer never moves a group forward.

Card	The card	Answer, and why	The common wrong answer	Joke letter for the number at its top
D	$\triangle QRS$ has a right angle at S. $QR = 38$ P is the circumcenter. Find PS.	19. The circumcenter of a right triangle is the middle of the longest side, QR. It is the same distance from all three corners, so $PS = 38 \div 2 = 19$.	38, from giving the whole longest side instead of half of it	I
Q	P is the circumcenter of $\triangle UVW$. $PU = 29$ P is 20 units from side VW. Find PV.	29. P is the same distance from all three corners, so $PV = PU = 29$. The 20 is the distance to a side.	20, from giving the distance to a side instead of to a corner	R
J	$\triangle LMN$ has a right angle at N. $LM = 34$ P is the circumcenter. Find PN.	17. The circumcenter of a right triangle is the middle of the longest side, LM. It is the same distance from all three corners, so $PN = 34 \div 2 = 17$.	34, from giving the whole longest side instead of half of it	Y
F	P is the circumcenter of $\triangle DEF$. $PD = 2x + 5$ $PE = 4x - 11$ Find PF.	21. $PD = PE$, so $2x + 5 = 4x - 11$, and $x = 8$. Then $PD = 21$, and PF is the same.	8, from giving x instead of the distance	A
G	$Q(5, 1)$, $R(19, 1)$, $S(5, 27)$ What is the x-coordinate of the circumcenter of $\triangle QRS$?	12. Side QR is horizontal. Its perpendicular bisector is the vertical line through its middle, $x = (5 + 19) \div 2 = 12$.	7, from halving the length of the side instead of finding its middle	L
B	P is the circumcenter of $\triangle GHK$. $PG = 10$ P is 6 units from side HK. Find PH.	10. P is the same distance from all three corners, so $PH = PG = 10$. The 6 is the distance to a side.	6, from giving the distance to a side instead of to a corner	A
N	$\triangle EFG$ has a right angle at G. P is the circumcenter. $PG = 15$ Find EF.	30. The circumcenter is the middle of the longest side, and $PE = PF = PG = 15$. So $EF = 2 \times 15 = 30$.	15, from copying the distance instead of doubling it	F
E	P is the circumcenter of $\triangle MNQ$. $PM = 3x - 5$ $PN = x + 13$ Find PQ.	22. $PM = PN$, so $3x - 5 = x + 13$, and $x = 9$. Then $PM = 22$, and PQ is the same.	9, from giving x instead of the distance	P

Other mistakes lead to no ring card. For example, a group might add the two given distances, or double a longest side, or give the y-coordinate. Its answer is then not the number at the top of any ring card.

Guessing does not work. Every simple rule that uses only the printed numbers, such as copying the first number, answers at most four of the sixteen cards correctly.