

MYSTERY PICTURE

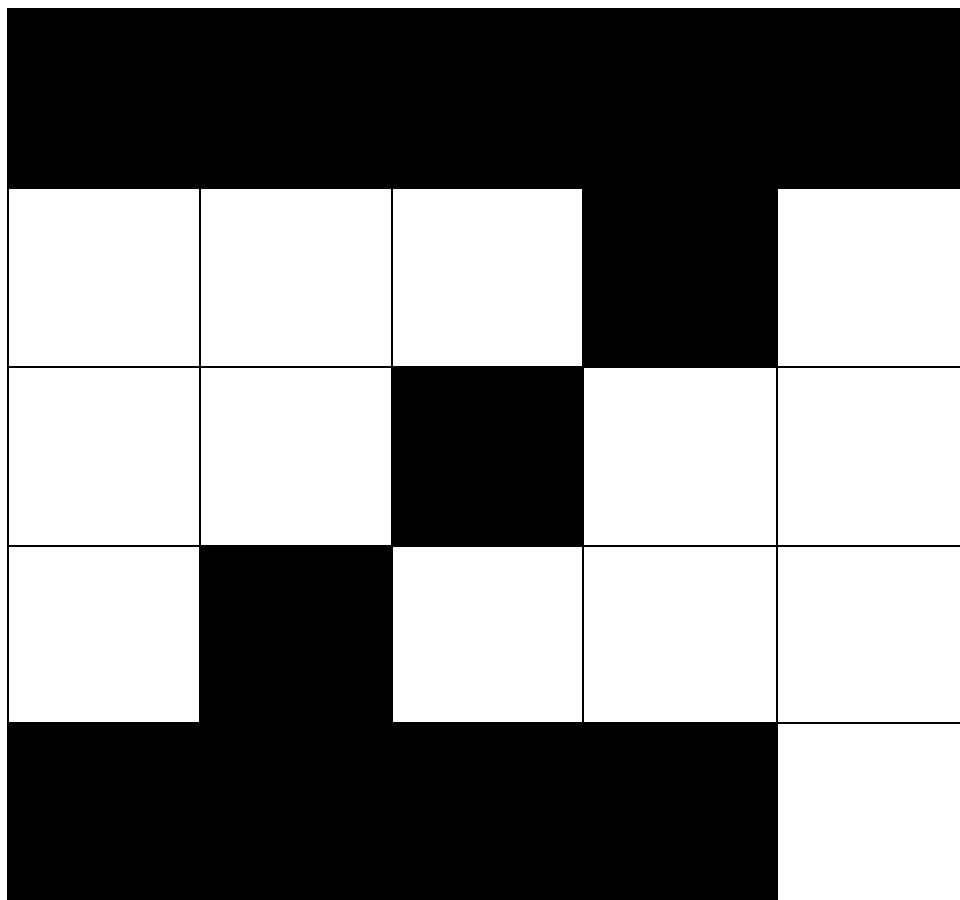


MEDIANS AND THE CENTROID

Is the Long Piece at the Corner?

25 sentences about the centroid · true or false?

FREE



25 squares · 1 student sheet · answer key · 4 pages

Geometry · Grades 8–10 · chooseyourlesson.com

1. **Hand out page 3**, one per student. Pencils. Nothing else is needed.
2. **Say this once:** *“In each square, find the corner across from the midpoint. The piece of the median at that corner is the long piece. Then read the bold sentence. If it is true, fill the square with slanted lines. If not, leave it white.”*
3. **When a student finishes, look at the picture.** If it is not the letter Z, a square is wrong. The student finds it and checks it again.

Student sheet	page 3, one per student. At the top it shows a median and the centroid as two small lettered drawings, with the rule in three forms.
Answers	page 4: the finished picture and the reason for every square.

This sheet practices one fact, said in five ways. The three medians of a triangle meet at the centroid. The centroid cuts each median into two pieces. The piece at the corner is twice the piece at the midpoint. So the corner piece is two thirds of the whole median, the midpoint piece is one third, and the whole median is three times the midpoint piece. Students often learn one of these sentences and do not recognize the others. Each square names a triangle, its centroid and the midpoint of one side, and prints one of these sentences.

The first step is to find the corner. If L is the midpoint of BC, the median is AL, because A is the corner across from side BC. So AG is the long piece and GL is the short piece. The letters are sometimes written in the other order (GA for AG), so the student must look at which letter is the corner.

Most white squares turn the ratio upside down. In nine white squares the sentence calls the short piece the long one, or takes a fraction of the wrong piece. In two white squares the point is only “a point on” the median, not the centroid, so nothing says it cuts the median 2 to 1. In two white squares the sentence says the centroid is the same distance from two corners or from two midpoints. It is not. Unlike the other two centers of this unit, the centroid is not defined by equal distances.

Twelve squares are filled and thirteen are white. Guessing does not work. Every simple guessing rule, such as filling every square with a fraction, gets at least ten squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture

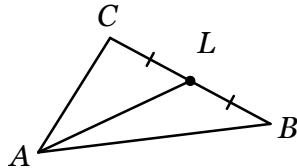
Is the sentence true?

Name _____

Period _____

Date _____

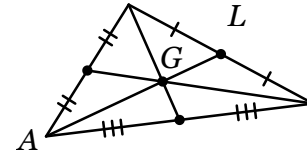
Each square tells you about a triangle and gives one sentence in bold. Is the bold sentence true? If yes, fill the square with slanted lines. If no, leave it white. When you finish, look at the picture. If part of it is missing, or one square does not fit, check that square again.



A **median** joins a corner to the midpoint of the side across.

On every median, the piece at the corner is **long** and the piece at the midpoint is **short**.

$$AG = 2 \times GL \cdot AG = \frac{2}{3} \text{ of } AL \cdot GL = \frac{1}{3} \text{ of } AL$$



The three medians meet at one point, the **centroid** G.

First find the corner across from the midpoint. The median joins them. The piece at that corner is the long piece. The piece at the midpoint is the short piece. A point that is only "a point on" a median is not the centroid.

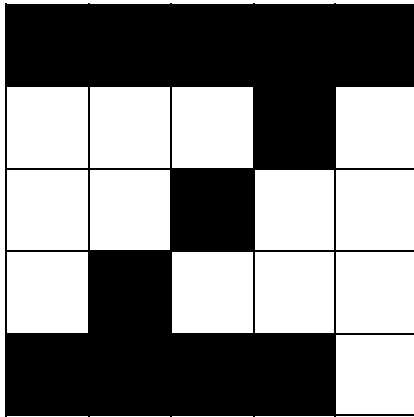
1 G is the centroid of triangle ABC. L is the midpoint of BC. AG = 2 × GL	2 K is the centroid of triangle PQR. M is the midpoint of PR. QM = 3 × KM	3 G is the centroid of triangle DEF. N is the midpoint of DE. GN = $\frac{1}{3}$ of FN	4 P is the centroid of triangle JKL. M is the midpoint of KL. JP : PM = 2 : 1	5 G is the centroid of triangle STU. D is the midpoint of SU. DG = $\frac{1}{2}$ of GT
6 G is the centroid of triangle XYZ. M is the midpoint of XY. GM = 2 × ZG	7 In triangle ABC, M is the midpoint of AC. P is a point on BM. BP = 2 × PM	8 G is the centroid of triangle RST. N is the midpoint of RS. TG = $\frac{1}{3}$ of TN	9 K is the centroid of triangle EFH. M is the midpoint of EH. KF = $\frac{2}{3}$ of MF	10 G is the centroid of triangle ABC. GA = GB
11 G is the centroid of triangle KLM. D is the midpoint of LM. GD : KG = 2 : 1	12 P is the centroid of triangle DEF. L is the midpoint of EF. DL = 3 × DP	13 G is the centroid of triangle PQR. N is the midpoint of QR. GN : PG = 1 : 2	14 G is the centroid of triangle UVW. M is the midpoint of UW. GV = $\frac{1}{2}$ of MG	15 K is the centroid of triangle ABC. N is the midpoint of AB. KN = $\frac{2}{3}$ of CN
16 In triangle XYZ, L is the midpoint of YZ. P is a point on XL. XP = $\frac{2}{3}$ of XL	17 G is the centroid of triangle JKL. N is the midpoint of JL. GK = 2 × NG	18 K is the centroid of triangle STU. M is the midpoint of TU. SK : KM = 1 : 2	19 G is the centroid of triangle DEF. M and L are the midpoints of DF and FE. GM = GL	20 G is the centroid of triangle PQR. D is the midpoint of PQ. DR = 3 × GR
21 P is the centroid of triangle HJK. M is the midpoint of JK. MH = 3 × MP	22 G is the centroid of triangle RST. L is the midpoint of ST. RG = $\frac{2}{3}$ of RL	23 K is the centroid of triangle ABC. N is the midpoint of AC. NK = $\frac{1}{3}$ of NB	24 G is the centroid of triangle XYZ. D is the midpoint of XZ. GD = $\frac{1}{2}$ of YG	25 P is the centroid of triangle KLM. N is the midpoint of KM. NP = 2 × PL



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Mystery Picture — answers

For the teacher



The finished picture is **the letter Z**: a bar across the top, a slant from top right to bottom left, and a bar across the bottom.

A square is filled when its bold sentence is true. The median runs from the corner across from the named side to the midpoint of that side. Its piece at the corner is the long piece, twice the piece at the midpoint.

To check a square with a student, ask: “Which letter is the corner? Which piece is long?” Then read the sentence together.

1–5	1 · FILL · AG = 2 × GL A is across from BC, so on median AL, AG is the long piece and GL the short piece. The long piece is twice the short one: true.	2 · FILL · QM = 3 × KM Q is across from PR, so on median QM, QK is the long piece and KM the short piece. The whole is three short pieces: true.	3 · FILL · GN = $\frac{1}{3}$ of FN F is across from DE, so on median FN, FG is the long piece and GN the short piece. The short piece is one third of the whole: true.	4 · FILL · JP : PM = 2 : 1 J is across from KL, so on median JM, JP is the long piece and PM the short piece. Long to short is 2 to 1: true.	5 · FILL · DG = $\frac{1}{2}$ of GT T is across from SU, so on median TD, TG is the long piece and GD the short piece. The short piece is half of the long one: true.
6–10	6 · white · GM = 2 × ZG Z is across from XY, so on median ZM, ZG is the long piece and GM the short piece. The short piece is not twice the long one. The ratio is upside down.	7 · white · BP = 2 × PM P is only a point on the median BM, not the centroid. Nothing says BP is twice PM.	8 · white · TG = $\frac{1}{3}$ of TN T is across from RS, so on median TN, TG is the long piece and GN the short piece. The long piece is two thirds of the whole, not one third.	9 · FILL · KF = $\frac{2}{3}$ of MF F is across from EH, so on median FM, FK is the long piece and KM the short piece. The long piece is two thirds of the whole: true.	10 · white · GA = GB The centroid is not the same distance from the corners. That is true of the circumcenter, not the centroid.
11–15	11 · white · GD : KG = 2 : 1 K is across from LM, so on median KD, KG is the long piece and GD the short piece. Short to long is 1 to 2, not 2 to 1.	12 · white · DL = 3 × DP D is across from EF, so on median DL, DP is the long piece and PL the short piece. The whole is three short pieces, not three long ones.	13 · FILL · GN : PG = 1 : 2 P is across from QR, so on median PN, PG is the long piece and GN the short piece. Short to long is 1 to 2: true.	14 · white · GV = $\frac{1}{2}$ of MG V is across from UW, so on median VM, VG is the long piece and GM the short piece. The long piece is not half of the short one. It is upside down.	15 · white · KN = $\frac{2}{3}$ of CN C is across from AB, so on median CN, CK is the long piece and KN the short piece. The short piece is one third of the whole, not two thirds.
16–20	16 · white · XP = $\frac{2}{3}$ of XL P is only a point on the median XL, not the centroid. Nothing says XP is two thirds of XL.	17 · FILL · GK = 2 × NG K is across from JL, so on median KN, KG is the long piece and GN the short piece. The long piece is twice the short one: true.	18 · white · SK : KM = 1 : 2 S is across from TU, so on median SM, SK is the long piece and KM the short piece. Long to short is 2 to 1, not 1 to 2.	19 · white · GM = GL The centroid is not the same distance from the midpoints. GM and GL are pieces of two different medians.	20 · white · DR = 3 × GR R is across from PQ, so on median RD, RG is the long piece and GD the short piece. The whole is three short pieces, not three long ones.
21–25	21 · FILL · MH = 3 × MP H is across from JK, so on median HM, HP is the long piece and PM the short piece. The whole is three short pieces: true.	22 · FILL · RG = $\frac{2}{3}$ of RL R is across from ST, so on median RL, RG is the long piece and GL the short piece. The long piece is two thirds of the whole: true.	23 · FILL · NK = $\frac{1}{3}$ of NB B is across from AC, so on median BN, BK is the long piece and KN the short piece. The short piece is one third of the whole: true.	24 · FILL · GD = $\frac{1}{2}$ of YG Y is across from XZ, so on median YD, YG is the long piece and GD the short piece. The short piece is half of the long one: true.	25 · white · NP = 2 × PL L is across from KM, so on median LN, LP is the long piece and PN the short piece. The short piece is not twice the long one. The ratio is upside down.