

MYSTERY PICTURE

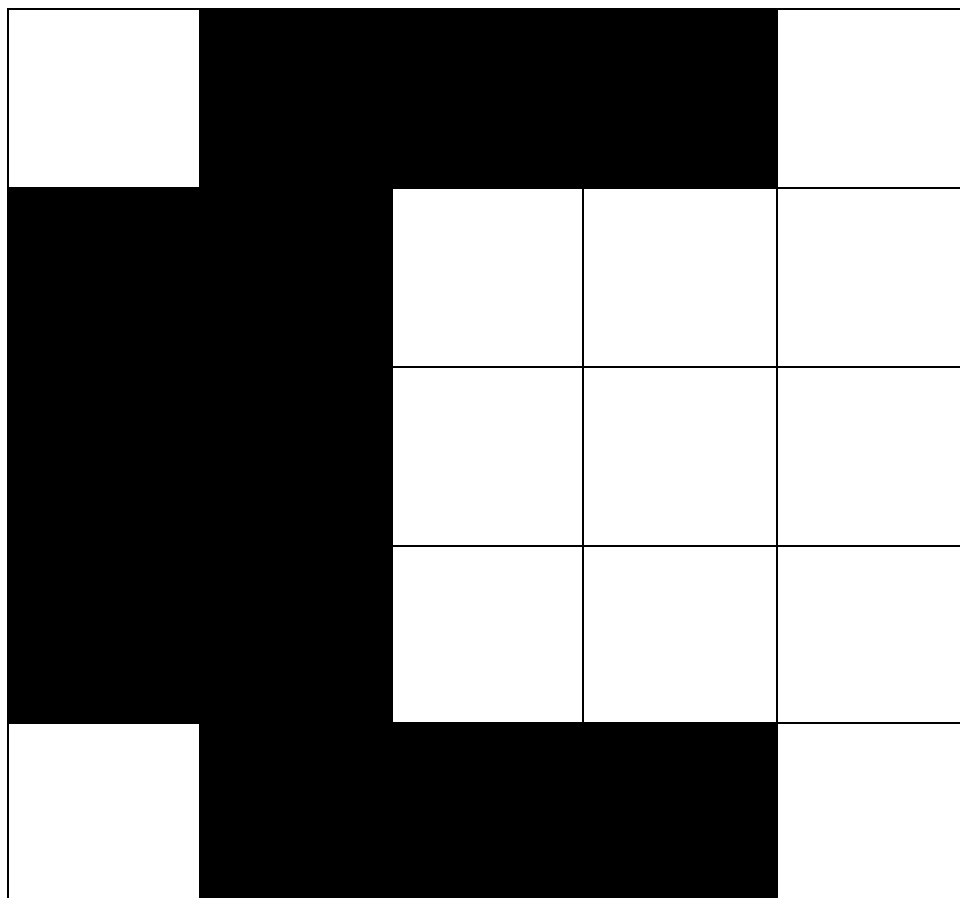


THE TRIANGLE INEQUALITY AND THE HINGE THEOREM

Can You Be Sure Which Side Is Longer?

25 pairs of triangles · does the Hinge Theorem fit?

FREE



1. **Hand out page 3**, one per student. Pencils. Nothing else is needed.
2. **Say this once:** *“The middle side belongs to both triangles, so it is one pair of equal sides. Check that the two arms have the same number, and that the angles are at the top corner. Only then does the bigger angle give the longer bottom side.”*
3. **When a student finishes, look at the picture.** If it is not the letter C, a square is wrong. The student finds it and checks it again.

Student sheet	page 3, one per student. At the top it shows the theorem and the two ways it can fail to fit, as three small drawings.
Answers	page 4: the finished picture and the reason for every square.

The Hinge Theorem. Suppose two sides of one triangle are equal to two sides of another triangle. If the angle between those sides is bigger in the first triangle, then the third side of the first triangle is longer. In every square the two triangles share the middle side, so that side is one pair of equal sides with no number on it. Many students look for two pairs of numbers, find only one, and give up. The shared side is the second pair.

A square is filled only when all three checks pass. The two arms carry the same number. The two angles are at the top corner, between the arm and the middle side. The left angle is the bigger one. In the twelve filled squares the left angle is bigger by only two to seven degrees, so the drawing cannot decide it by eye.

Each white square shows a common mistake. In five squares the angles are at the bottom corners. They are not between the two equal sides, so the theorem does not fit, even though the left number is bigger. This is the most common mistake with this theorem. In four squares the arms have different numbers, so the theorem does not fit. In two squares the right angle is bigger, so the right bottom side is longer. In two squares the top angles are equal. Then the two triangles are congruent and the bottom sides are equal. The reason is congruence (SAS), not the Hinge Theorem.

Guessing does not work. Every simple guessing rule, such as filling every square where the left number is bigger, gets at least four squares wrong. Even a student who measures with a ruler gets four squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture

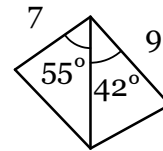
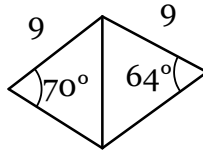
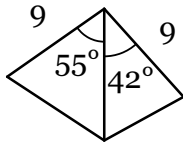
The Hinge Theorem

Name _____

Period _____

Date _____

Each square shows two triangles that share the middle side. Does the Hinge Theorem show that the bottom side of the LEFT triangle is longer? If yes, fill the square with slanted lines. If no, leave it white. When you finish, look at the picture. If part of it is missing, or one square does not fit, check that square again.



The middle side belongs to **both** triangles. The arms are equal. The **bigger top angle** gives the **longer bottom side**.

These angles are at the bottom corners, **not between** the two sides. The theorem does not fit.

The arms are **not equal**. The theorem does not fit.

If the two top angles are equal, the bottom sides are equal. Then the left one is not longer. Do not measure the drawings. Use only the numbers.

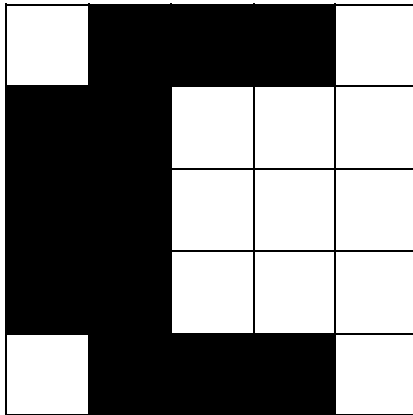
1 10 10 70° 65°	2 9 9 52° 47°	3 7 7 44° 41°	4 11 11 38° 35°	5 10 10 45° 45°
6 8 8 60° 54°	7 10 10 41° 37°	8 8 9 56° 50°	9 9 9 46° 50°	10 9 9 72° 68°
11 6 6 57° 50°	12 12 12 36° 33°	13 9 10 52° 46°	14 7 7 53° 53°	15 8 8 70° 65°
16 9 9 47° 45°	17 8 8 55° 48°	18 10 10 70° 66°	19 8 8 51° 53°	20 6 7 58° 51°
21 10 11 47° 42°	22 10 10 43° 40°	23 7 7 62° 56°	24 12 12 39° 34°	25 7 7 74° 69°



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Mystery Picture — answers

For the teacher



The finished picture is **the letter C**: a thick back on the left, with bars along the top and the bottom.

A square is filled when the two arms have the same number, the angles are at the top corner, and the left angle is bigger. Then the Hinge Theorem says the left bottom side is longer.

To check a square with a student, ask: “Which two pairs of sides are equal? Where are the angles? Which angle is bigger?”

1–5	1 · white The angles are at the bottom corners, not between the equal sides. The theorem does not fit.	2 · FILL Both arms are 9, and the middle side is shared. The left top angle is bigger ($52^\circ > 47^\circ$), so the left bottom side is longer.	3 · FILL Both arms are 7, and the middle side is shared. The left top angle is bigger ($44^\circ > 41^\circ$), so the left bottom side is longer.	4 · FILL Both arms are 11, and the middle side is shared. The left top angle is bigger ($38^\circ > 35^\circ$), so the left bottom side is longer.	5 · white Both arms are 10, and the middle side is shared. Equal top angles make the triangles congruent (SAS), so the bottom sides are equal.
6–10	6 · FILL Both arms are 8, and the middle side is shared. The left top angle is bigger ($60^\circ > 54^\circ$), so the left bottom side is longer.	7 · FILL Both arms are 10, and the middle side is shared. The left top angle is bigger ($41^\circ > 37^\circ$), so the left bottom side is longer.	8 · white The arms are 8 and 9, not equal. The theorem does not fit, even though 56° is bigger.	9 · white Both arms are 9, and the middle side is shared. The right top angle is bigger ($50^\circ > 46^\circ$), so the right bottom side is longer.	10 · white The angles are at the bottom corners, not between the equal sides. The theorem does not fit.
11–15	11 · FILL Both arms are 6, and the middle side is shared. The left top angle is bigger ($57^\circ > 50^\circ$), so the left bottom side is longer.	12 · FILL Both arms are 12, and the middle side is shared. The left top angle is bigger ($36^\circ > 33^\circ$), so the left bottom side is longer.	13 · white The arms are 9 and 10, not equal. The theorem does not fit, even though 52° is bigger.	14 · white Both arms are 7, and the middle side is shared. Equal top angles make the triangles congruent (SAS), so the bottom sides are equal.	15 · white The angles are at the bottom corners, not between the equal sides. The theorem does not fit.
16–20	16 · FILL Both arms are 9, and the middle side is shared. The left top angle is bigger ($47^\circ > 45^\circ$), so the left bottom side is longer.	17 · FILL Both arms are 8, and the middle side is shared. The left top angle is bigger ($55^\circ > 48^\circ$), so the left bottom side is longer.	18 · white The angles are at the bottom corners, not between the equal sides. The theorem does not fit.	19 · white Both arms are 8, and the middle side is shared. The right top angle is bigger ($53^\circ > 51^\circ$), so the right bottom side is longer.	20 · white The arms are 6 and 7, not equal. The theorem does not fit, even though 58° is bigger.
21–25	21 · white The arms are 10 and 11, not equal. The theorem does not fit, even though 47° is bigger.	22 · FILL Both arms are 10, and the middle side is shared. The left top angle is bigger ($43^\circ > 40^\circ$), so the left bottom side is longer.	23 · FILL Both arms are 7, and the middle side is shared. The left top angle is bigger ($62^\circ > 56^\circ$), so the left bottom side is longer.	24 · FILL Both arms are 12, and the middle side is shared. The left top angle is bigger ($39^\circ > 34^\circ$), so the left bottom side is longer.	25 · white The angles are at the bottom corners, not between the equal sides. The theorem does not fit.