

ANSWER MAZE



PROVING PARALLELOGRAMS

Which Fact Finishes the Proof?

One page · 10–15 minutes · nothing to prepare

FREE

6

In quadrilateral ABCD, $AB \parallel DC$. Which one more fact makes ABCD a parallelogram?

$AD = BC \rightarrow$ **go to 8**

$AD \parallel BC \rightarrow$ **go to 3**

Sketch the two facts. Check whether each pair of sides has a job, and what that job is.

3

Given: in quadrilateral ABCD, $AB = DC$ and $AD = BC$. Which reason proves that ABCD is a parallelogram?

If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram. $\rightarrow$ **go to 7**

Opposite sides of a parallelogram are congruent. $\rightarrow$ **go to 1**

Which fact is known at the start, and which one is being proved? The reason must start from the known fact.

1. **Hand out page 3**, one per student. Nothing else is needed.
2. **Say this once:** *“Start at box 6. A reason starts from what you know and ends with what you are proving. Write every box number in the row at the bottom as you go.”*
3. **Check a sheet quickly.** A student with no mistakes writes **6 3 7 2 5 4**: six boxes, then out.

Student sheet	page 3, one per student. It shows the five ways to prove a quadrilateral is a parallelogram as small marked drawings.
Answers	page 4.

Every wrong answer comes from one of two mistakes. The first is the direction of the reason. To prove that ABCD is a parallelogram, the reason must end with “it is a parallelogram,” for example “If both pairs of opposite sides are congruent, then it is a parallelogram.” Many students write “Opposite sides of a parallelogram are congruent” instead. That sentence starts from a parallelogram, so it assumes what is being proved. One box turns this around: once ABCD is proved to be a parallelogram, a fact about a parallelogram is the right reason. These wrong answers send the student to box 1.

The second mistake is about pairs. One pair of opposite sides can prove a parallelogram only if the *same* pair is both parallel and congruent. One pair parallel and the *other* pair congruent is also true of an isosceles trapezoid. Students also mix up diagonals that bisect each other with diagonals that are equal. These wrong answers send the student to box 8. Boxes 1 and 8 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more. Guessing does not get a student out: every simple guessing rule, such as always taking the first answer, goes wrong within the first four boxes.

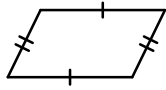


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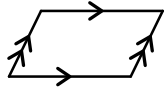
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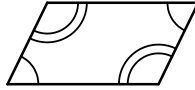
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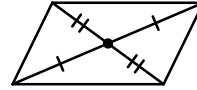
Both pairs of opposite sides are **congruent**.



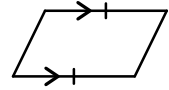
Both pairs of opposite sides are **parallel**.



Both pairs of opposite **angles** are congruent.



The diagonals **bisect** each other.



One pair of opposite sides is both parallel **and** congruent.

Answer the question in the box. Choose an answer, and go to the box it names. A reason starts from what is known and ends with what is being proved. Write each box number in the row at the bottom as you reach it.

1

Given: the diagonals of quadrilateral ABCD bisect each other. Which reason proves that ABCD is a parallelogram?

The diagonals of a parallelogram bisect each other. → **go to 3**

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram. → **go to 7**

Read each sentence from its first fact to its last. Which part is known here?

2

ABCD has already been proved to be a parallelogram. Its diagonals meet at M. Which reason gives $AM = MC$?

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram. → **go to 1**

The diagonals of a parallelogram bisect each other. → **go to 5**

Which fact is known at the start, and which one is being proved? The reason must start from the known fact.

3

Given: in quadrilateral ABCD, $AB = DC$ and $AD = BC$. Which reason proves that ABCD is a parallelogram?

If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram. → **go to 7**

Opposite sides of a parallelogram are congruent. → **go to 1**

Which fact is known at the start, and which one is being proved? The reason must start from the known fact.

4

Given: in quadrilateral ABCD, $\angle A = \angle C$ and $\angle B = \angle D$. Which reason proves that ABCD is a parallelogram?

If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram. → **you are out**

If a quadrilateral is a parallelogram, then its opposite angles are congruent. → **go to 1**

Read each sentence from "If" to "then." Which part is known here?

5

In quadrilateral ABCD, $AD = BC$. Which one more fact makes ABCD a parallelogram?

$AB \parallel DC$ → **go to 8**

$AD \parallel BC$ → **go to 4**

Sketch the two facts. Check whether each pair of sides has a job, and what that job is.

6

In quadrilateral ABCD, $AB \parallel DC$. Which one more fact makes ABCD a parallelogram?

$AD = BC$ → **go to 8**

$AD \parallel BC$ → **go to 3**

Sketch the two facts. Check whether each pair of sides has a job, and what that job is.

7

The diagonals of quadrilateral ABCD meet at M, and $AM = MC$. Which one more fact makes ABCD a parallelogram?

$BM = MD$ → **go to 2**

$AC = BD$ → **go to 8**

Look at what each fact says about the point M.

8

In quadrilateral ABCD, $AB = DC$. Which one more fact makes ABCD a parallelogram?

$AB \parallel DC$ → **go to 3**

$AD \parallel BC$ → **go to 6**

Sketch the two facts. Check whether each pair of sides has a job, and what that job is.

Write each box number as you reach it:

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The route, in order. Boxes 1 and 8 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table.

6 → 3 → 7 → 2 → 5 → 4 → out

Box	Right answer, and why	Wrong answer, and why a student picks it
1 Box 6 In quadrilateral ABCD, $AB \parallel DC$. Which one more fact makes ABCD a parallelogram?	$AD \parallel BC$ With $AD \parallel BC$ too, both pairs of opposite sides are parallel. That is the definition of a parallelogram.	" $AD = BC$ " goes to box 8. One pair parallel and the other pair congruent is also true of an isosceles trapezoid. Students who split the two jobs between two pairs pick this.
2 Box 3 Given: in quadrilateral ABCD, $AB = DC$ and $AD = BC$. Which reason proves that ABCD is a parallelogram?	If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram. The sides are known first and the parallelogram comes last, so the reason must start from the sides.	"Opposite sides of a parallelogram are congruent." goes to box 1. This sentence starts from a parallelogram, which is what is being proved. It argues in a circle.
3 Box 7 The diagonals of quadrilateral ABCD meet at M, and $AM = MC$. Which one more fact makes ABCD a parallelogram?	$BM = MD$ With $BM = MD$, each diagonal cuts the other in half, so the diagonals bisect each other.	" $AC = BD$ " goes to box 8. Equal diagonals are not the same as diagonals that cut each other in half. Students often mix up "bisect" and "congruent."
4 Box 2 ABCD has already been proved to be a parallelogram. Its diagonals meet at M. Which reason gives $AM = MC$?	The diagonals of a parallelogram bisect each other. Here the parallelogram is known first and $AM = MC$ comes last, so the reason starts from the parallelogram.	"If the diagonals of a quadrilateral bisect each other, then it is a parallelogram." goes to box 1. This sentence ends with "it is a parallelogram," which is already known. It does not give $AM = MC$.
5 Box 5 In quadrilateral ABCD, $AD = BC$. Which one more fact makes ABCD a parallelogram?	$AD \parallel BC$ With $AD \parallel BC$, the same pair AD and BC is both parallel and congruent. That proves a parallelogram.	" $AB \parallel DC$ " goes to box 8. $AB \parallel DC$ with $AD = BC$ is an isosceles trapezoid again: the two jobs are on two different pairs.
6 Box 4 Given: in quadrilateral ABCD, $\angle A = \angle C$ and $\angle B = \angle D$. Which reason proves that ABCD is a parallelogram?	If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram. The angles are known first and the parallelogram comes last, so the reason must start from the angles.	"If a quadrilateral is a parallelogram, then its opposite angles are congruent." goes to box 1. This sentence starts from a parallelogram, which is what is being proved. It is the right fact in the wrong direction.
Box 1 Given: the diagonals of quadrilateral ABCD bisect each other. Which reason proves that ABCD is a parallelogram?	If the diagonals of a quadrilateral bisect each other, then it is a parallelogram. The diagonals are known first, so the reason must start from the diagonals and end with the parallelogram.	"The diagonals of a parallelogram bisect each other." goes to box 3. A student who still gives the reason in the wrong direction picks this.
Box 8 In quadrilateral ABCD, $AB = DC$. Which one more fact makes ABCD a parallelogram?	$AB \parallel DC$ With $AB \parallel DC$, the same pair AB and DC is both parallel and congruent. That proves a parallelogram.	" $AD \parallel BC$ " goes to box 6. A student who still splits the two jobs between two pairs picks this.

How to check any box. For a reason, ask: "What do we know at the start, and what are we proving?" The reason must start from what we know. For one more fact, sketch both facts. Then check that both pairs match, or that one pair is both parallel and congruent, or that the diagonals cut each other in half.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.