

CUT & GLUE SORT

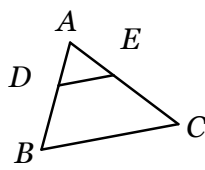
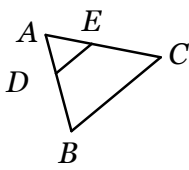
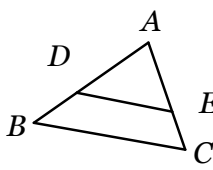
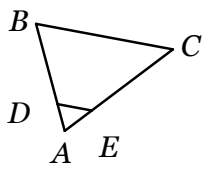


THE TRIANGLE PROPORTIONALITY THEOREM

Is the Cut Parallel, or Can You Not Tell?

Four lengths on every piece · decide from the numbers

FREE

1  $AD = 6, DB = 9$ $AE = 8, EC = 12$	2  $AD = 6, DB = 9$ $DE = 4, BC = 10$	3  $AD = 15, DB = 9$ $AE = 20, EC = 11$	4  $AE = 4, EC = 12$ $DE = 3, BC = 12$
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Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. Nothing else is needed.
2. **Say this once:** *“Nothing on these pieces says the lines are parallel. Use the numbers, not the drawing. Compare AD over DB with AE over EC. If you cannot find both, the answer is cannot tell.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 shows the four segments, the two triangles, and the test for parallel as three small drawings.
Answers	page 5.

The sheet practices the converse. If a line cuts two sides of a triangle into segments that are in proportion, it is parallel to the third side. So $DE \parallel BC$ exactly when $AD/DB = AE/EC$ (or, the same test, $AD/AB = AE/AC$). The student finds both fractions from the four numbers and compares them.

The “not parallel” row holds the common mistakes. In two of its pieces the drawing looks parallel, and it is not. In two of them a student who compares a segment over a segment with a segment over a whole side gets two equal fractions and says “parallel”.

The last row is the most important. The converse needs the segments of both cut sides. In two pieces the lengths of DE and BC are given instead, and DE/BC even equals a segment over its whole side. That does not make DE parallel: with that length, the free end of DE can stand at two places on its side. In the other two, one cut side has only one length given, so its fraction cannot be found. A student who mixes the two kinds of fraction and never uses the last row gets four of the twelve pieces right. Other guessing rules place at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Parallel, not parallel, or cannot tell?

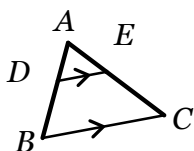
The converse of the triangle proportionality theorem

Name _____

Period _____

Date _____

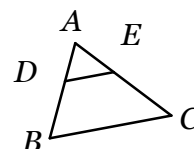
Cut out the twelve pieces on the next page. Each piece shows a triangle with a segment DE across it and four lengths. Nothing says $DE \parallel BC$. **Decide: is DE parallel to BC? Glue the piece in that row.** The drawings are not to scale. Decide from the numbers only.



DE $\parallel$ BC. The four **pieces**: $\frac{AD}{DB} = \frac{AE}{EC}$

DE goes with BC and the **whole** sides:

$$\frac{DE}{BC} = \frac{AD}{AB}$$



No arrowheads: is DE parallel to BC?

DE $\parallel$ BC exactly when $\frac{AD}{DB}$ and $\frac{AE}{EC}$ are equal. Find both fractions first. A whole side is its two pieces added together. DE and BC are not pieces.

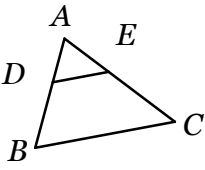
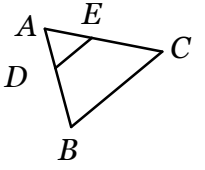
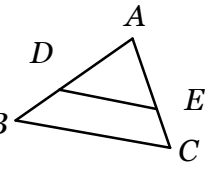
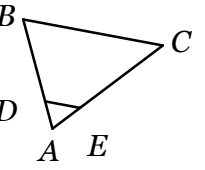
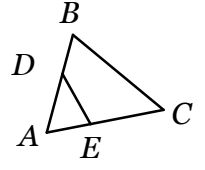
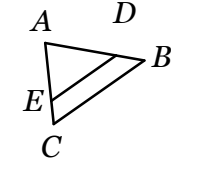
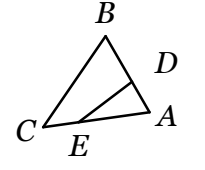
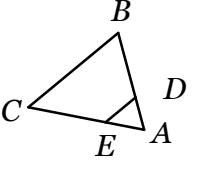
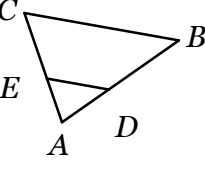
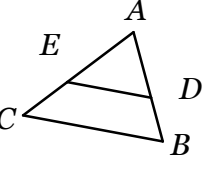
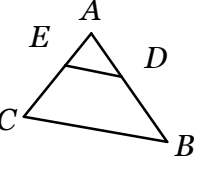
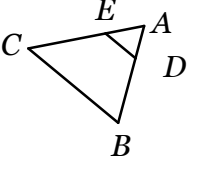
Parallel. The pieces of the two cut sides are in proportion.

Not parallel. The pieces of the two cut sides are not in proportion.

Cannot tell. The four pieces of the two cut sides are not all known.

The twelve pieces

Cut along the dotted lines

1  $AD = 6, DB = 9$ $AE = 8, EC = 12$	2  $AD = 6, DB = 9$ $DE = 4, BC = 10$	3  $AD = 15, DB = 9$ $AE = 20, EC = 11$	4  $AE = 4, EC = 12$ $DE = 3, BC = 12$
5  $AD = 6, AB = 10$ $AE = 9, AC = 24$	6  $AD = 10, DB = 4$ $AE = 15, EC = 6$	7  $AD = 4, DB = 6$ $AE = 6, EC = 3$	8  $AD = 5, AB = 15$ $AE = 4, AC = 12$
9  $AD = 4, DB = 6$ $AB = 10, AE = 6$	10  $AE = 9, EC = 6$ $AD = 12, DB = 8$	11  $AD = 8, DB = 12$ $AE = 10, EC = 16$	12  $AE = 5, EC = 10$ $AC = 15, AD = 3$



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Cut & Glue Sort — answers

For the teacher

Parallel	1 · AD = 6, DB = 9, AE = 8, EC = 12 AD/DB = $6/9 = 0.667$ and AE/EC = $8/12 = 0.667$. Equal, so DE $\parallel$ BC.	6 · AD = 10, DB = 4, AE = 15, EC = 6 AD/DB = $10/4 = 2.5$ and AE/EC = $15/6 = 2.5$. Equal, so DE $\parallel$ BC.	8 · AD = 5, AB = 15, AE = 4, AC = 12 AD/AB = $5/15 = 0.333$ and AE/AC = $4/12 = 0.333$. Equal, so DE $\parallel$ BC.	10 · AE = 9, EC = 6, AD = 12, DB = 8 AD/DB = $12/8 = 1.5$ and AE/EC = $9/6 = 1.5$. Equal, so DE $\parallel$ BC.
Not parallel	3 · AD = 15, DB = 9, AE = 20, EC = 11 AD/DB = $15/9 = 1.667$ and AE/EC = $20/11 = 1.818$. Not equal, so DE is not parallel to BC. The drawing looks parallel, but it is not to scale.	5 · AD = 6, AB = 10, AE = 9, AC = 24 AD/AB = $6/10 = 0.6$ and AE/AC = $9/24 = 0.375$. Not equal, so DE is not parallel to BC. A student who compares AD/AB = $6/10$ with AE/EC = $9/15$ mixes a segment over a whole side with a segment over a segment, and wrongly gets “equal”.	7 · AD = 4, DB = 6, AE = 6, EC = 3 AD/DB = $4/6 = 0.667$ and AE/EC = $6/3 = 2$. Not equal, so DE is not parallel to BC. A student who compares AD/DB = $4/6$ with AE/AC = $6/9$ mixes the two kinds of fraction and wrongly gets “equal”.	11 · AD = 8, DB = 12, AE = 10, EC = 16 AD/DB = $8/12 = 0.667$ and AE/EC = $10/16 = 0.625$. Not equal, so DE is not parallel to BC. The drawing looks parallel, but it is not to scale.
Cannot tell	2 · AD = 6, DB = 9, DE = 4, BC = 10 DE and BC are not segments of the cut sides. AE and EC are not given, so AE/EC cannot be found. DE/BC = 0.4 does not decide it: E could stand at two places on AC with this length of DE.	4 · AE = 4, EC = 12, DE = 3, BC = 12 DE and BC are not segments of the cut sides. AD and DB are not given, so AD/DB cannot be found. DE/BC = 0.25 does not decide it: D could stand at two places on AB with this length of DE.	9 · AD = 4, DB = 6, AB = 10, AE = 6 Only one length on AC is given, so the fraction for AC cannot be found. The three lengths on the other side do not help.	12 · AE = 5, EC = 10, AC = 15, AD = 3 Only one length on AB is given, so the fraction for AB cannot be found. The three lengths on the other side do not help.

Every row holds four pieces. A piece is parallel when $AD/DB = AE/EC$: the two cut sides are divided in the same ratio. It is not parallel when the two fractions differ. It is “cannot tell” when the four segments of the two cut sides are not all known, so one of the two fractions cannot be found.

To check a piece with a student, ask: “Which two numbers are on AB? Which two are on AC? What fraction does each side give?”