

MYSTERY PICTURE



PROPORTIONAL SEGMENTS AND THE ANGLE BISECTOR THEOREM

True From the Marks Alone?

Four figures · 25 equations · read the marks · a picture appears

FREE

1 Figure 1 $\frac{AB}{BC} = \frac{DE}{EF}$	2 Figure 2 $\frac{QS}{SR} = \frac{PQ}{PR}$	3 Figure 4 $UW = WV$	4 Figure 1 $AB \cdot EF = BC \cdot DE$	5 Figure 3 $\frac{GH}{HJ} = \frac{KL}{LM}$
6 Figure 2 $\frac{QS}{PQ} = \frac{SR}{PR}$	7 Figure 1 $\frac{AB}{BC} = \frac{EF}{DE}$	8 Figure 2 $\frac{QS}{SR} = \frac{PR}{PQ}$	9 Figure 4 $\frac{UW}{WV} = \frac{TU}{TV}$	10 Figure 1 $\frac{AB}{DE} = \frac{BC}{EF}$
11 Figure 2 $QS \cdot PR = SR \cdot PQ$	12 Figure 1 $AB = DE$	13 Figure 3 $\frac{GH}{KL} = \frac{HJ}{LM}$	14 Figure 2 $QS = SR$	15 Figure 1 $\frac{AB}{AC} = \frac{DE}{DF}$
16 Figure 2 $\frac{SR}{QS} = \frac{PR}{PQ}$	17 Figure 4 $UW \cdot TV = WV \cdot TU$	18 Figure 1 $\frac{AB}{EF} = \frac{BC}{DE}$	19 Figure 2 $\frac{QS}{PR} = \frac{SR}{PQ}$	20 Figure 1 $\frac{EF}{DE} = \frac{BC}{AB}$
21 Figure 1 $\frac{BC}{AC} = \frac{EF}{DF}$	22 Figure 2 $\frac{PR}{SR} = \frac{PQ}{QS}$	23 Figure 1 $\frac{DE}{AB} = \frac{EF}{BC}$	24 Figure 3 $GH \cdot LM = HJ \cdot KL$	25 Figure 4 $\frac{WV}{UW} = \frac{TV}{TU}$

1. Give each student page 3 and a pencil.
2. Say this once: *“Each square has an equation about one of the four figures. Is it always true, from what the marks and words say? If yes, fill the square with slanted lines. If no, leave it white.”*
3. When a student finishes, look at the picture. If it is not the letter D, a square is wrong. The student finds it and checks it again.

Student sheet	page 3, one per student. The four figures are at the top of it.
Answers	page 4, for you.

The four figures. Figure 1 has three parallel lines cut by two transversals. The top piece over the bottom piece is the same fraction on both transversals, so $AB/BC = DE/EF$. Figure 2 is a triangle with the angle bisector PS. Each piece of QR goes with the side that meets it at the same corner, so $QS/SR = PQ/PR$. Figure 3 looks like figure 1, but nothing says its lines are parallel, so no equation about it is always true. Figure 4 is the same triangle as figure 2 with a median instead of a bisector. Its two pieces are equal, and they are not in the ratio of the sides.

On these sheets, an angle bisector of a triangle is the segment from a corner to the opposite side that cuts the angle at that corner into two equal angles. Some textbooks use other definitions, so tell the class which ones this sheet uses.

Each white square tests a common mistake. Some students compare the pieces crosswise, or think two pieces on different transversals are equal. Some pair a piece with the side across from it instead of the side at its corner. Some mix up the bisector and the median. Some use the parallel-lines rule on figure 3. Thirteen squares are filled.

If part of a student's picture looks wrong, the mistake is in that square. Ask: *“Which corner does this piece touch?”* or *“What on the figure says these lines are parallel?”* Guessing does not work. Every simple guessing rule, such as filling every square that has fractions, gets at least nine squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture — always true?

Proportional pieces

Name _____

Period _____

Date _____

Each square has an equation about one of the four figures. **Is it always true, from what the marks and words say? If yes, fill the square with slanted lines. If no, leave it white.** Do not measure. When you finish, a picture appears. If part of it is missing, or one square does not fit, check that square again.

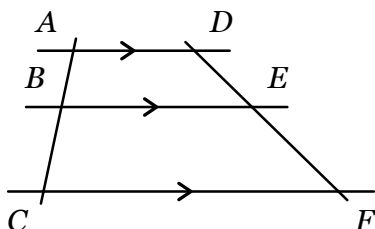


Figure 1. The three lines with arrowheads are parallel.

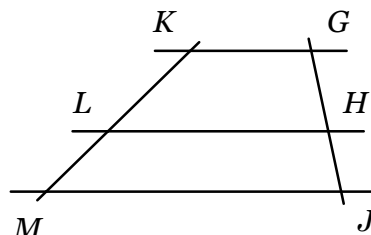


Figure 3.

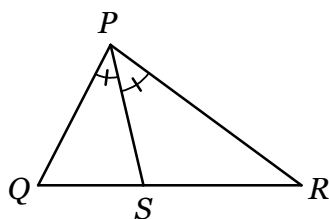


Figure 2. PS cuts angle P into two equal angles.

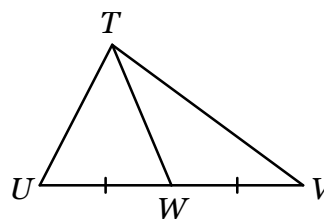


Figure 4. W is the midpoint of UV.

Parallel lines: the top piece over the bottom piece is the same on both lines. Angle bisector: each piece goes with the side that meets it at the same corner. You may know other facts about angle bisectors. This sheet uses only this one.

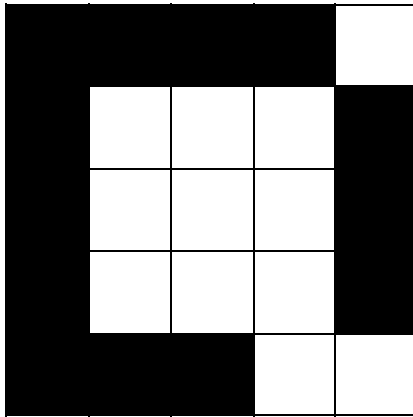
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Mystery Picture — answers

For the teacher



The finished picture is **the letter D**. Thirteen squares are filled.

A square is filled only when its equation is true on every figure the marks and words allow. The table gives the reason for every square.

	The square's equation	Answer	Why
1	Figure 1: $AB/BC = DE/EF$	fill	Both fractions put the top piece over the bottom piece, one on each transversal.
2	Figure 2: $QS/SR = PQ/PR$	fill	Each piece is matched with the side that meets it at the same corner (QS with PQ at Q, SR with PR at R).
3	Figure 4: $UW = WV$	fill	W is the midpoint of UV, so $UW = WV$.
4	Figure 1: $AB \cdot EF = BC \cdot DE$	fill	$AB \cdot EF = BC \cdot DE$ means $AB/BC = DE/EF$. Both fractions put the top piece over the bottom piece, one on each transversal.
5	Figure 3: $GH/HJ = KL/LM$	white	Figure 3 has no arrowheads and no words, so nothing says its lines are parallel. Nothing follows.
6	Figure 2: $QS/PQ = SR/PR$	fill	Each piece is matched with the side that meets it at the same corner (QS with PQ at Q, SR with PR at R).
7	Figure 1: $AB/BC = EF/DE$	white	AB/BC is the top piece over the bottom piece, but EF/DE is the bottom piece over the top piece.
8	Figure 2: $QS/SR = PR/PQ$	white	A piece is matched with the side across from it. QS goes with PQ (both touch Q), and SR goes with PR.
9	Figure 4: $UW/WV = TU/TV$	white	TW is a median, not an angle bisector. $UW = WV$, but TU and TV are not equal, so the two fractions are not equal.
10	Figure 1: $AB/DE = BC/EF$	fill	Both fractions put a part of one transversal over the matching part of the other.
11	Figure 2: $QS \cdot PR = SR \cdot PQ$	fill	$QS \cdot PR = SR \cdot PQ$ means $QS/SR = PQ/PR$. Each piece is matched with the side that meets it at the same corner (QS with PQ at Q, SR with PR at R).
12	Figure 1: $AB = DE$	white	AB and DE are on different transversals. They are equal only if the two transversals are equally steep.
13	Figure 3: $GH/KL = HJ/LM$	white	Figure 3 has no arrowheads and no words, so nothing says its lines are parallel. Nothing follows.
14	Figure 2: $QS = SR$	white	PS cuts the angle in half, not the side. $QS = SR$ only if $PQ = PR$.
15	Figure 1: $AB/AC = DE/DF$	fill	Both fractions put the top piece over the whole, one on each transversal.
16	Figure 2: $SR/QS = PR/PQ$	fill	Each piece is matched with the side that meets it at the same corner (QS with PQ at Q, SR with PR at R).
17	Figure 4: $UW \cdot TV = WV \cdot TU$	white	TW is a median, not an angle bisector. $UW = WV$, but TU and TV are not equal, so the two fractions are not equal.
18	Figure 1: $AB/EF = BC/DE$	white	AB/EF puts a top piece over a bottom piece. Those parts do not match.
19	Figure 2: $QS/PR = SR/PQ$	white	A piece is matched with the side across from it. QS goes with PQ (both touch Q), and SR goes with PR.
20	Figure 1: $EF/DE = BC/AB$	fill	Both fractions put the bottom piece over the top piece, one on each transversal.
21	Figure 1: $BC/AC = EF/DF$	fill	Both fractions put the bottom piece over the whole, one on each transversal.
22	Figure 2: $PR/SR = PQ/QS$	fill	Each piece is matched with the side that meets it at the same corner (QS with PQ at Q, SR with PR at R).
23	Figure 1: $DE/AB = EF/BC$	fill	Both fractions put a part of one transversal over the matching part of the other.
24	Figure 3: $GH \cdot LM = HJ \cdot KL$	white	Figure 3 has no arrowheads and no words, so nothing says its lines are parallel. Nothing follows.
25	Figure 4: $WV/UW = TV/TU$	white	TW is a median, not an angle bisector. $UW = WV$, but TU and TV are not equal, so the two fractions are not equal.