

ANSWER MAZE



GEOMETRIC MEAN AND THE ALTITUDE TO THE HYPOTENUSE

The Geometric Mean, Not the Average

One page · 10–15 minutes · nothing to prepare

FREE

3

AD = 9 and DB = 25. How long is CD?

CD = 17 → go to 2

CD = 15 → go to 6

Which rule goes with the altitude CD? What does that rule say about CD?

6

AD = 9 and DB = 16. How long is AC?

AC = 15 → go to 1

AC = 12 → go to 5

AC is a leg. Which segment is the whole hypotenuse?

1. **Hand out page 3**, one per student. Nothing else is needed.
2. **Say this once:** “Start at box 3. Before any arithmetic, say which rule you need: the altitude rule or a leg rule. Write every box number in the row at the bottom as you go.”
3. **Check a sheet quickly.** A student with no mistakes writes **3 6 1 8 4 7**: six boxes, then out.

Student sheet	page 3, one per student. One right triangle ABC at the top, with the altitude CD, is used by every box. It is drawn twice, once for each rule.
Answers	page 4.

The two rules. In right triangle ABC with the right angle at C, the altitude CD cuts the hypotenuse AB into two pieces, AD and DB. The altitude rule: $CD^2 = AD \times DB$, so CD is the *geometric mean* of the two pieces. The leg rule: a leg squared is its own piece times the whole hypotenuse, so $AC^2 = AD \times AB$ and $BC^2 = DB \times AB$. A leg's own piece is the piece that touches the same end of the hypotenuse.

Every wrong answer comes from one of two mistakes. The first is taking the average of the two pieces instead of the geometric mean, or, backwards, treating the altitude as halfway between the two pieces. These wrong answers send the student to box 2. The second is using the wrong piece with a leg: the piece that does not touch the leg, or the two pieces multiplied together where the whole hypotenuse belongs. These send the student to box 5. Boxes 2 and 5 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more. Guessing does not get a student out: every simple guessing rule, such as always taking the smaller number, goes wrong within the first two boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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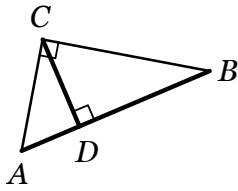
Answer Maze

Start at box 3 · the altitude to the hypotenuse

Name _____

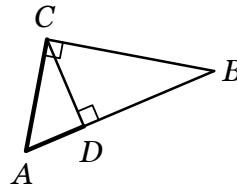
Period _____

Date _____



Altitude rule. The altitude squared is one piece times the other piece.

$$CD^2 = AD \cdot DB, \text{ so } \frac{AD}{CD} = \frac{CD}{DB}$$



Leg rule. A leg squared is its own piece times the whole hypotenuse.

$$AC^2 = AD \cdot AB, \text{ so } \frac{AD}{AC} = \frac{AC}{AB}$$

The **altitude** CD goes from the right angle to the hypotenuse AB. It cuts AB into two **pieces**, AD and DB. $AD + DB = AB$. A leg's **own piece** is the piece that touches the same end of the hypotenuse: AD for the leg AC, DB for the leg BC.

Every box uses the triangle ABC above: the right angle is at C and CD is the altitude. The drawing is not to scale. **Choose an answer, and go to the box it names.**

1

CD = 6 and AD = 4. How long is DB?

DB = 9 → go to 8

DB = 8 → go to 2

*The altitude is the geometric mean of the two pieces.
Write the rule with DB in it.*

2

AD = 8 and DB = 18. How long is CD?

CD = 12 → go to 6

CD = 13 → go to 3

CD is the altitude. Put each answer into the altitude rule and check it.

3

AD = 9 and DB = 25. How long is CD?

CD = 17 → go to 2

CD = 15 → go to 6

Which rule goes with the altitude CD? What does that rule say about CD?

4

DB = 36 and AD = 16. How long is CD?

CD = 26 → go to 2

CD = 24 → go to 7

Which segment is asked for? Write the rule that goes with it.

5

AC = 10 and AB = 25. How long is AD?

AD = 21 → go to 6

AD = 4 → go to 1

A leg goes with its own piece and the whole hypotenuse. Which piece touches AC?

6

AD = 9 and DB = 16. How long is AC?

AC = 15 → go to 1

AC = 12 → go to 5

AC is a leg. Which segment is the whole hypotenuse?

7

AB = 50 and AD = 18. How long is AC?

AC = 30 → you are out

AC = 40 → go to 5

Put a finger on the leg AC. Which piece of AB shares the point A with it?

8

BC = 20 and AB = 25. How long is DB?

DB = 9 → go to 5

DB = 16 → go to 4

Which piece touches the leg BC? Is it the piece asked for?

Write each box number as you reach it:

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The route, in order. Boxes 2 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table.

3 → 6 → 1 → 8 → 4 → 7 → out

	Box and its question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 3 AD = 9 and DB = 25. How long is CD?	CD = 15. CD is the altitude, so the altitude rule: $CD^2 = AD \times DB = 9 \times 25 = 225$, and $CD = 15$.	"CD = 17" goes to box 2. A student who takes the average of the two pieces instead of the geometric mean picks this.
2	Box 6 AD = 9 and DB = 16. How long is AC?	AC = 15. AC is a leg. Its own piece is AD, and the whole hypotenuse is $AB = 9 + 16 = 25$. Leg rule: $AC^2 = 9 \times 25 = 225$, so $AC = 15$.	"AC = 12" goes to box 5. A student who multiplies the two pieces (the altitude rule) instead of using the whole hypotenuse picks this.
3	Box 1 CD = 6 and AD = 4. How long is DB?	DB = 9. CD is the altitude and DB a piece, so the altitude rule: $6^2 = 4 \times DB$, so $DB = 36 \div 4 = 9$.	"DB = 8" goes to box 2. A student who thinks the altitude is halfway between the two pieces ($DB = 2 \times CD - AD$) picks this.
4	Box 8 BC = 20 and AB = 25. How long is DB?	DB = 16. BC is a leg, and its own piece is DB (both touch B). Leg rule: $20^2 = DB \times 25$, so $DB = 400 \div 25 = 16$.	"DB = 9" goes to box 5. A student who pairs BC with AD, the piece that does not touch it, picks this.
5	Box 4 DB = 36 and AD = 16. How long is CD?	CD = 24. CD is the altitude, so the altitude rule: $CD^2 = AD \times DB = 16 \times 36 = 576$, and $CD = 24$.	"CD = 26" goes to box 2. A student who takes the average of the two pieces instead of the geometric mean picks this.
6	Box 7 AB = 50 and AD = 18. How long is AC?	AC = 30. AC is a leg, and its own piece is AD (both touch A). Leg rule: $AC^2 = 18 \times 50 = 900$, so $AC = 30$.	"AC = 40" goes to box 5. A student who pairs AC with DB, the piece that does not touch it, picks this.
	Box 2 AD = 8 and DB = 18. How long is CD?	CD = 12. CD is the altitude, so the altitude rule: $CD^2 = AD \times DB = 8 \times 18 = 144$, and $CD = 12$.	"CD = 13" goes to box 3. A student who takes the average of the two pieces instead of the geometric mean picks this.
	Box 5 AC = 10 and AB = 25. How long is AD?	AD = 4. AC is a leg, and its own piece is AD (both touch A). Leg rule: $10^2 = AD \times 25$, so $AD = 100 \div 25 = 4$.	"AD = 21" goes to box 6. A student who pairs AC with DB, the piece that does not touch it, picks this.

The definition this sheet uses. The geometric mean of two positive numbers is the positive number whose square is their product, so the geometric mean of 4 and 16 is 8. Some textbooks use other definitions, so tell the class which ones this sheet uses.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.