

MYSTERY PICTURE

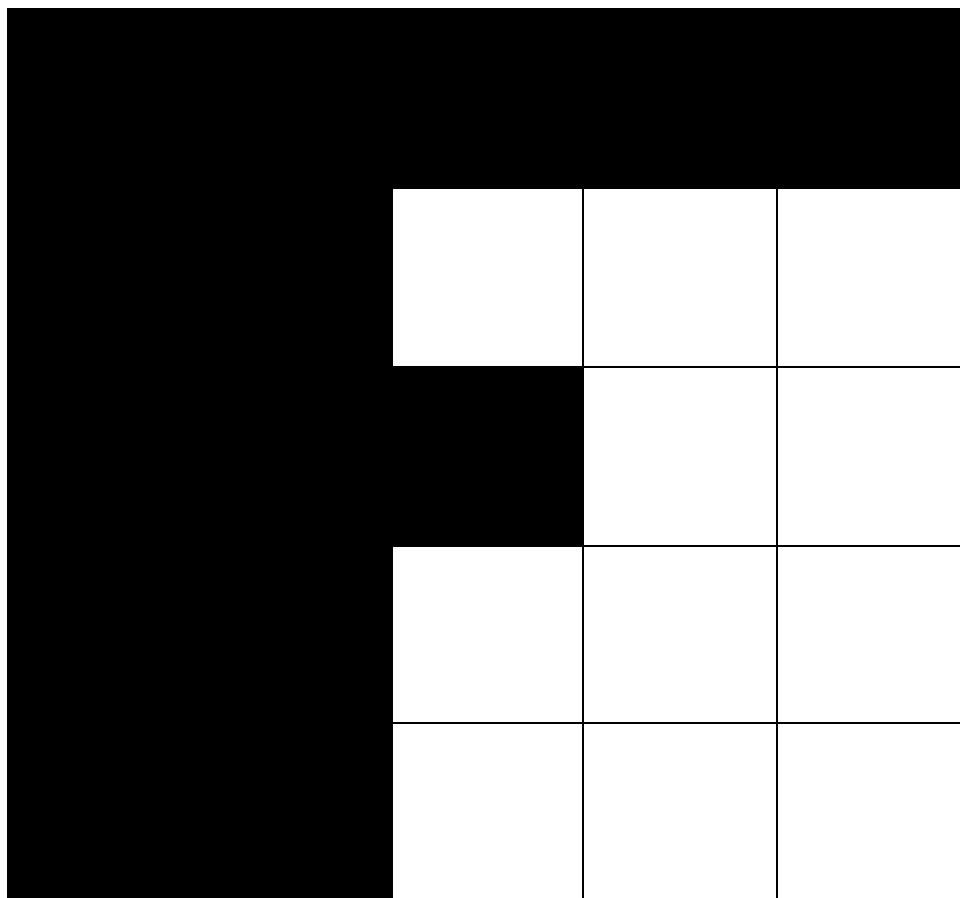


GEOMETRIC MEAN AND THE ALTITUDE TO THE HYPOTENUSE

Is This Equation True?

25 equations · three lettered right triangles · true or not?

FREE



1. **Hand out page 3**, one per student. Pencils. Nothing else is needed.
2. **Say this once:** *“Find the figure with the same letters. Find the altitude, the two pieces and the legs in it. If the equation is true, fill the square with slanted lines. If not, leave it white.”*
3. **When a student finishes, look at the picture.** If it is not the letter F, a square is wrong. The student finds it and checks it again.

Student sheet	page 3, one per student. The three figures and the two rules, as two small drawings, are printed at the top.
Answers	page 4: the finished picture and the reason for every square.

This sheet practices one question: is this equation true for every triangle drawn this way?

In a right triangle, the altitude from the right angle cuts the hypotenuse into two pieces. The altitude rule: the altitude squared is one piece times the other piece. The leg rule: a leg squared is its own piece (the piece that touches the same end of the hypotenuse) times the whole hypotenuse. Ten squares are written as a proportion, such as $RU/RS = RS/RT$ (square 2). Multiplying out a proportion gives the squared form, so the two are the same rule.

Each white square shows a common mistake. Two use the average of the two pieces for the altitude. Two pair a leg with the piece that does not touch it. Six use a piece where the whole hypotenuse belongs, or the reverse. One says the altitude cuts the hypotenuse in half. None of the three figures has equal legs. On a triangle with equal legs, five of the eleven white squares would be true, so such a figure hides these mistakes.

Fourteen squares are filled and eleven are white. Guessing does not work: every simple guessing rule, such as filling every equation with a square in it, gets at least nine squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture

Is the equation true?

Name _____

Period _____

Date _____

Each square is an equation about one of the three figures. Its letters tell you which figure. Is the equation true? If yes, fill the square with slanted lines. If no, leave it white. When you finish, look at the picture. If part of it is missing, or one square does not fit, check that square again.

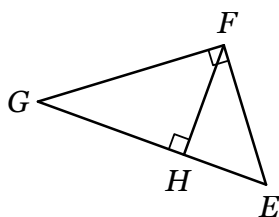


Figure 1. Right angle at F. FH is the altitude.

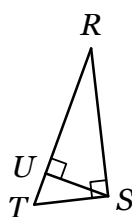


Figure 2. Right angle at S. SU is the altitude.

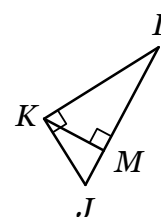
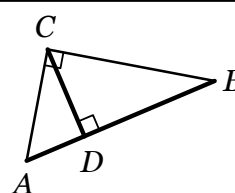
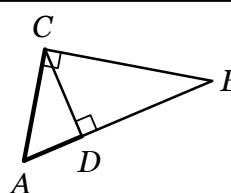


Figure 3. Right angle at K. KM is the altitude.



Altitude rule. The altitude squared is one piece times the other piece.

$$CD^2 = AD \cdot DB, \text{ so } \frac{AD}{CD} = \frac{CD}{DB}$$



Leg rule. A leg squared is its own piece times the whole hypotenuse.

$$AC^2 = AD \cdot AB, \text{ so } \frac{AD}{AC} = \frac{AC}{AB}$$

The **altitude** CD goes from the right angle to the hypotenuse AB. It cuts AB into two **pieces**, AD and DB. $AD + DB = AB$. A leg's **own piece** is the piece that touches the same end of the hypotenuse: AD for the leg AC, DB for the leg BC.

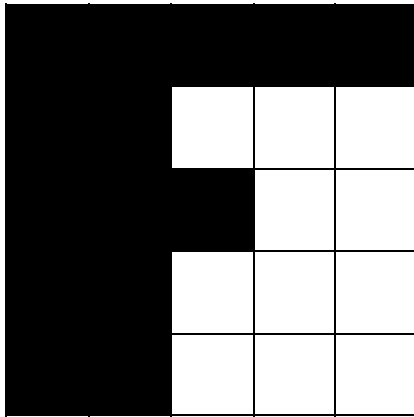
1 $FH^2 = EH \cdot HG$	2 $\frac{RU}{RS} = \frac{RS}{RT}$	3 $\frac{KM}{JM} = \frac{ML}{KM}$	4 $GF^2 = HG \cdot EG$	5 $\frac{UT}{SU} = \frac{SU}{RU}$
6 $\frac{JL}{JK} = \frac{JK}{JM}$	7 $TS^2 = UT \cdot RT$	8 $EF^2 = HG \cdot EG$	9 $SU = \frac{RU + UT}{2}$	10 $JK^2 = JM \cdot ML$
11 $\frac{EH}{FH} = \frac{FH}{HG}$	12 $KM^2 = JM \cdot ML$	13 $RS^2 = RU \cdot RT$	14 $FH^2 = EH \cdot EG$	15 $\frac{JM}{JK} = \frac{JK}{ML}$
16 $\frac{EG}{GF} = \frac{GF}{HG}$	17 $JM + ML = JL$	18 $RU = UT$	19 $\frac{ML}{LK} = \frac{LK}{JM}$	20 $\frac{EH}{FH} = \frac{FH}{EG}$
21 $\frac{HG}{GF} = \frac{GF}{EG}$	22 $LK^2 = ML \cdot JL$	23 $TS^2 = RU \cdot RT$	24 $KM = \frac{JM + ML}{2}$	25 $EF^2 = EH \cdot HG$



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Mystery Picture — answers

For the teacher



The finished picture is **the letter F**: a stem two squares thick, a full top bar and a middle bar three squares long.

A square is filled when its equation is the altitude rule, a leg rule (in either form), or the two pieces adding to the hypotenuse. It is white when it pairs the wrong segments.

To check a square with a student, point at each segment in the equation and ask: “Is this the altitude, a piece, a leg, or the hypotenuse?”

1–5	1 · FILL $FH^2 = EH \cdot HG$ FH is the altitude and EH and HG are the two pieces: the altitude rule.	2 · FILL $RU/RS = RS/RT$ Multiplying out gives $RS^2 = RU \cdot RT$. RS is a leg, RU is its own piece (both touch R) and RT is the whole hypotenuse: the leg rule.	3 · FILL $KM/JM = ML/KM$ Multiplying out gives $KM^2 = JM \cdot ML$. KM is the altitude and JM and ML are the two pieces: the altitude rule.	4 · FILL $GF^2 = HG \cdot EG$ GF is a leg, HG is its own piece (both touch G) and EG is the whole hypotenuse: the leg rule.	5 · FILL $UT/SU = SU/RU$ Multiplying out gives $SU^2 = RU \cdot UT$. SU is the altitude and RU and UT are the two pieces: the altitude rule.
6–10	6 · FILL $JL/JK = JK/JM$ Multiplying out gives $JK^2 = JM \cdot JL$. JK is a leg, JM is its own piece (both touch J) and JL is the whole hypotenuse: the leg rule.	7 · FILL $TS^2 = UT \cdot RT$ TS is a leg, UT is its own piece (both touch T) and RT is the whole hypotenuse: the leg rule.	8 · white $EF^2 = HG \cdot EG$ HG does not touch the leg EF. The leg rule uses its own piece, EH.	9 · white $SU = (RU + UT)/2$ The altitude SU is the geometric mean of the two pieces, not their average. The geometric mean is less than the average unless the two pieces are equal.	10 · white $JK^2 = JM \cdot ML$ That is the altitude rule used on the leg JK. A leg needs its own piece times the whole hypotenuse JL.
11–15	11 · FILL $EH/FH = FH/HG$ Multiplying out gives $FH^2 = EH \cdot HG$. FH is the altitude and EH and HG are the two pieces: the altitude rule.	12 · FILL $KM^2 = JM \cdot ML$ KM is the altitude and JM and ML are the two pieces: the altitude rule.	13 · FILL $RS^2 = RU \cdot RT$ RS is a leg, RU is its own piece (both touch R) and RT is the whole hypotenuse: the leg rule.	14 · white $FH^2 = EH \cdot EG$ The altitude rule uses the two pieces EH and HG, not a piece and the whole hypotenuse.	15 · white $JM/JK = JK/ML$ Multiplying out gives $JK^2 = JM \cdot ML$. That is the altitude rule used on the leg JK. A leg needs its own piece times the whole hypotenuse JL.
16–20	16 · FILL $EG/GF = GF/HG$ Multiplying out gives $GF^2 = HG \cdot EG$. GF is a leg, HG is its own piece (both touch G) and EG is the whole hypotenuse: the leg rule.	17 · FILL $JM + ML = JL$ M is on JL, so the two pieces together make the whole hypotenuse.	18 · white $RU = UT$ The altitude cuts the hypotenuse in half only when the two legs are equal. Nothing marks the legs RS and TS as equal, so $RU = UT$ is not always true.	19 · white $ML/LK = LK/JM$ Multiplying out gives $LK^2 = ML \cdot JM$. That is the altitude rule used on the leg LK. A leg needs its own piece times the whole hypotenuse JL.	20 · white $EH/FH = FH/EG$ Multiplying out gives $FH^2 = EH \cdot EG$. The altitude rule uses the two pieces EH and HG, not a piece and the whole hypotenuse.
21–25	21 · FILL $HG/GF = GF/EG$ Multiplying out gives $GF^2 = HG \cdot EG$. GF is a leg, HG is its own piece (both touch G) and EG is the whole hypotenuse: the leg rule.	22 · FILL $LK^2 = ML \cdot JL$ LK is a leg, ML is its own piece (both touch L) and JL is the whole hypotenuse: the leg rule.	23 · white $TS^2 = RU \cdot RT$ RU does not touch the leg TS. The leg rule uses its own piece, UT.	24 · white $KM = (JM + ML)/2$ The altitude KM is the geometric mean of the two pieces, not their average. The geometric mean is less than the average unless the two pieces are equal.	25 · white $EF^2 = EH \cdot HG$ That is the altitude rule used on the leg EF. A leg needs its own piece times the whole hypotenuse EG.