

CUT & GLUE SORT



SINE AND COSINE OF COMPLEMENTARY ANGLES

Where Is x ? Add to 90 or Set Equal?

Twelve sine-cosine statements · nothing to solve · three rows

FREE

1	2	3
$\sin A = \frac{4x}{13}$ $\cos B = \frac{2x+6}{13}$	$\sin(2x) =$ $\cos(x+30)$	$\tan A = \frac{5x}{25}$ $\tan B = \frac{2x+6}{25}$

Row 1. A sine and a cosine, with x inside the angles. The two angles add to 90° .

Row 2. A sine and a cosine of A and B , with x in the values. The two values are equal.

Row 3. It is not the sine of one acute angle and the cosine of the other. The rule says nothing.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. Nothing else is needed.
2. **Say this once:** *“Do not solve anything. First ask: is it the sine of one acute angle and the cosine of the other? If not, the piece goes in row 3. If it is, ask: where is x ? Inside the angles, in the brackets after sin and cos? Or in the values, the fractions that sin and cos are equal to?”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a small drawing of why $\sin A = \cos B$.
Answers	page 5.

The same equals sign means two different moves. When x is inside the angles, as in $\sin(2x) = \cos(x + 30)$, the two angles add to 90° . When x is in the values, as in $\sin A = 4x/13$ and $\cos B = (2x + 6)/13$, A and B already add to 90° , so the two values are equal. Many students learn one of these as “the rule” and use it everywhere. This sort asks only which move fits, before anyone solves.

The last row is the most important. The rule is only about a sine and a cosine of the two different acute angles. In two row-3 pieces both ratios are tangents: $\tan A$ and $\tan B$ are not equal, because one is the other turned upside down. In two pieces the sine and the cosine are of the same angle. These pieces use the numbers of a row-2 piece with one letter changed, so read the letters. A student who sorts only by where x is places eight pieces correctly. A student who also checks for one sine and one cosine, but does not read the letters, places ten. Guessing rules place at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Cut & Glue Sort — where is x ?

sine and cosine

Name _____

Period _____

Date _____

Each piece on the next page has an unknown x . **Do not solve it. Decide which row the piece belongs in, and glue it there.** On every piece, A and B are the two acute angles of one right triangle. All angles are in degrees.

Row 1. A sine and a cosine, with x inside the angles. The two angles add to 90° .

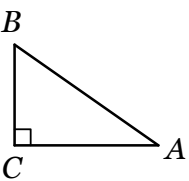
Row 2. A sine and a cosine of A and B , with x in the values. The two values are equal.

Row 3. It is not the sine of one acute angle and the cosine of the other. The rule says nothing.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet.



C is the right angle. A and B are the two acute angles, so $A + B = 90^\circ$.

BC is opposite A and adjacent to B. So $\sin A = \frac{BC}{AB}$ and $\cos B = \frac{BC}{AB}$.

$\sin A = \cos B$ and $\cos A = \sin B$. The sine of one acute angle is the cosine of the other.

1 $\sin A = \frac{4x}{13}$ $\cos B = \frac{2x+6}{13}$	2 $\sin(2x) =$ $\cos(x+30)$	3 $\tan A = \frac{5x}{25}$ $\tan B = \frac{2x+6}{25}$	4 $\cos(5x) =$ $\sin(x+12)$
5 $\sin A = \frac{4x}{13}$ $\cos A = \frac{2x+6}{13}$	6 $\cos A = \frac{3x}{17}$ $\sin B = \frac{x+10}{17}$	7 $\sin(4x-5) =$ $\cos(2x+5)$	8 $\tan(3x) =$ $\tan(x+10)$
9 $\sin B = \frac{3x+1}{25}$ $\cos A = \frac{x+5}{25}$	10 $\cos B = \frac{5x}{29}$ $\sin A = \frac{2x+12}{29}$	11 $\cos(x+20) =$ $\sin(2x+10)$	12 $\sin B = \frac{3x+1}{25}$ $\cos B = \frac{x+5}{25}$



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Each row holds four pieces. Every piece is written again here, with the reason. The value of x is given for rows 1 and 2, in case a student solves it.

1 **A sine and a cosine, with x inside the angles. The two angles add to 90° .**

2. $\sin(2x) = \cos(x + 30)$. *Why:* x is inside the two angles, a sine and a cosine. The sine of an angle equals the cosine of the angle that makes 90° with it, because the side across from one is next to the other. So the two angles add to 90° : $x = 20$, and the angles are 40° and 50° .

4. $\cos(5x) = \sin(x + 12)$. *Why:* x is inside the two angles, a sine and a cosine. The sine of an angle equals the cosine of the angle that makes 90° with it, because the side across from one is next to the other. So the two angles add to 90° : $x = 13$, and the angles are 65° and 25° .

7. $\sin(4x - 5) = \cos(2x + 5)$. *Why:* x is inside the two angles, a sine and a cosine. The sine of an angle equals the cosine of the angle that makes 90° with it, because the side across from one is next to the other. So the two angles add to 90° : $x = 15$, and the angles are 55° and 35° .

11. $\cos(x + 20) = \sin(2x + 10)$. *Why:* x is inside the two angles, a sine and a cosine. The sine of an angle equals the cosine of the angle that makes 90° with it, because the side across from one is next to the other. So the two angles add to 90° : $x = 20$, and the angles are 40° and 50° .

2 **A sine and a cosine of A and B , with x in the values. The two values are equal.**

1. $\sin A = (4x)/13$ and $\cos B = (2x + 6)/13$. *Why:* x is in the values, and A and B are the two acute angles. $\sin A = \cos B$, so the values are equal: $x = 3$, and both are $12/13$.

6. $\cos A = (3x)/17$ and $\sin B = (x + 10)/17$. *Why:* x is in the values, and A and B are the two acute angles. $\cos A = \sin B$, so the values are equal: $x = 5$, and both are $15/17$.

9. $\sin B = (3x + 1)/25$ and $\cos A = (x + 5)/25$. *Why:* x is in the values, and B and A are the two acute angles. $\sin B = \cos A$, so the values are equal: $x = 2$, and both are $7/25$.

10. $\cos B = (5x)/29$ and $\sin A = (2x + 12)/29$. *Why:* x is in the values, and B and A are the two acute angles. $\cos B = \sin A$, so the values are equal: $x = 4$, and both are $20/29$.

3 **It is not the sine of one acute angle and the cosine of the other. The rule says nothing.**

3. $\tan A = (5x)/25$ and $\tan B = (2x + 6)/25$. *Why:* Two tangents of A and B . $\tan A$ and $\tan B$ are not equal: one is the other turned upside down.

5. $\sin A = (4x)/13$ and $\cos A = (2x + 6)/13$. *Why:* Both ratios are of the same angle, A . The sine and the cosine of one angle are not equal (unless it is 45°).

8. $\tan(3x) = \tan(x + 10)$. *Why:* Two tangents, not a sine and a cosine. Adding the angles to 90° gives a false statement here.

12. $\sin B = (3x + 1)/25$ and $\cos B = (x + 5)/25$. *Why:* Both ratios are of the same angle, B . The sine and the cosine of one angle are not equal (unless it is 45°).

To check a piece with a student, ask two questions. First: is it the sine of one acute angle and the cosine of the other acute angle? Two tangents, two sines, or a sine and a cosine of the same letter are not, and go in row 3. Second: where is x ? Inside the brackets after sin and cos means row 1. In the fractions means row 2.