

CUT & GLUE SORT

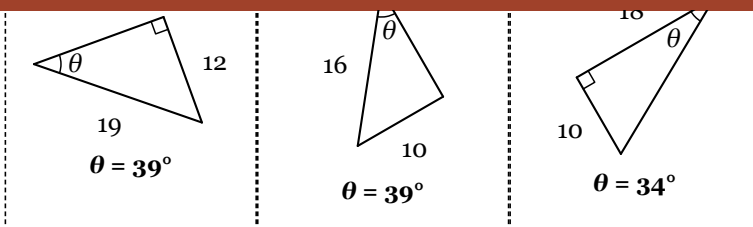


INVERSE TRIG RATIOS: FINDING AN ANGLE

Inverse Trig Ratios: Is θ Right?

Twelve triangles · right, wrong, or nobody can tell · three rows

FREE



Row 1. Correct: the value of θ is right.

--

Row 2. Wrong: the value of θ is not right.

--

Row 3. Cannot tell: no right angle is marked, or only one side has a length.

--

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors, glue and a calculator.
2. **Say this once:** *“First check the marks. Is there a right-angle square, and do two sides have lengths? If yes, find θ to the nearest degree and check the value.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and the three ratios on a labeled triangle.
Answers	page 5.

Each piece is a triangle drawn to scale, with an arc and θ at one corner and a value for θ .

When a right angle is marked and two sides have lengths, the student names the ratio of those two sides, writes it as a decimal, and uses $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$. This sheet writes the keys that way. Read $\sin^{-1}(0.6)$ as “the angle whose sine is 0.6.” Some textbooks write arcsin, arccos and arctan instead. Some textbooks use other definitions, so tell the class which ones this sheet uses.

Row 2 holds the two common wrong answers. One is the other acute angle, 90° minus θ . The other comes from the wrong key: $\sin^{-1}$ on two legs, or $\tan^{-1}$ on a leg and the hypotenuse.

Row 3 matters most. In two of its pieces the triangle looks right-angled, but no right angle is marked. Sine, cosine and tangent only work in a triangle known to have a right angle. In the other two, only one side has a length, and one side does not fix an angle. Every value in row 3 is the one the drawing would give, so the honest answer is “cannot tell.”

A student who trusts the drawing and does no calculating places only six pieces correctly. A student who finds row 3 and calls every other piece correct places eight, and the rows come out uneven. Guessing rules place at most six pieces correctly. The triangles are drawn to scale, so keep protractors away.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
chooseyourlesson.com — 135 free geometry lessons.

Cut & Glue Sort — is θ right?

Inverse trig ratios

Name _____

Period _____

Date _____

Each piece on the next page is a triangle with a value for θ . **First check: is a right angle marked, and do two sides have lengths? If not, glue the piece in row 3. If yes, find θ to the nearest degree. Glue the piece in row 1 if the value is right, or in row 2 if it is not.**

Row 1. Correct: the value of θ is right.

Row 2. Wrong: the value of θ is not right.

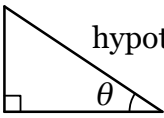
Row 3. Cannot tell: no right angle is marked, or only one side has a length.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. A small square in a corner means a right angle. Put your calculator in degree mode.

opposite

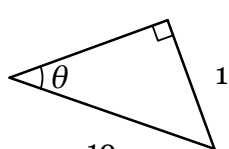
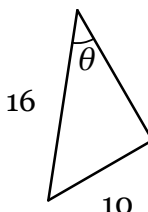
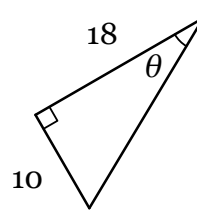
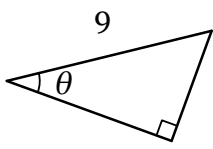
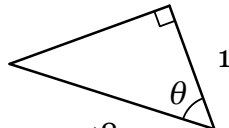
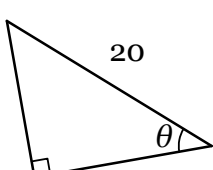
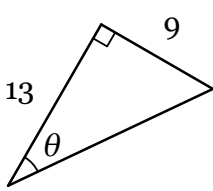
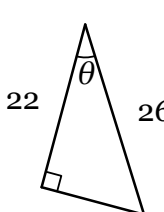
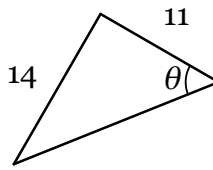
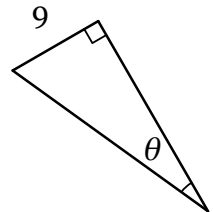
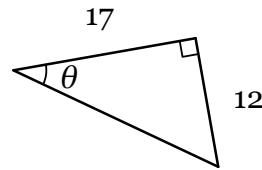
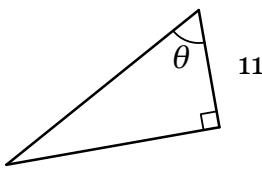


adjacent

$\sin \theta = \text{opposite} \div \text{hypotenuse}$
 $\cos \theta = \text{adjacent} \div \text{hypotenuse}$
 $\tan \theta = \text{opposite} \div \text{adjacent}$

$\sin^{-1}(0.6)$ means **the angle whose sine is 0.6**. $\cos^{-1}$ and $\tan^{-1}$ work the same way.

1. Name the ratio of the two sides you know. 2. Write it as a decimal. 3. Use the key $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

1  $\theta = 39^\circ$	2  $\theta = 39^\circ$	3  $\theta = 34^\circ$	4  $\theta = 56^\circ$
5  $\theta = 52^\circ$	6  $\theta = 41^\circ$	7  $\theta = 35^\circ$	8  $\theta = 40^\circ$
9  $\theta = 52^\circ$	10  $\theta = 24^\circ$	11  $\theta = 55^\circ$	12  $\theta = 61^\circ$



135 free geometry lessons — worksheets, worked answers and lesson plans.
chooseyourlesson.com

Each row holds four pieces. Every piece is described again here, with the reason. Sides are named as seen from θ .

1 Correct: the value of θ is right.

- 1.** The opposite is 12. The hypotenuse is 19. The piece says $\theta = 39^\circ$. *Why:* The 12 is opposite θ and the 19 is the hypotenuse, so they make the sine: $12 \div 19 = 0.6316$, and $\sin^{-1}(0.6316) = 39.17^\circ$, so $\theta = 39^\circ$.
- 5.** The adjacent is 11. The hypotenuse is 18. The piece says $\theta = 52^\circ$. *Why:* The 11 is next to θ (adjacent) and the 18 is the hypotenuse, so they make the cosine: $11 \div 18 = 0.6111$, and $\cos^{-1}(0.6111) = 52.33^\circ$, so $\theta = 52^\circ$.
- 7.** The opposite is 9. The adjacent is 13. The piece says $\theta = 35^\circ$. *Why:* The 9 is opposite θ and the 13 is next to θ (adjacent), so they make the tangent: $9 \div 13 = 0.6923$, and $\tan^{-1}(0.6923) = 34.70^\circ$, so $\theta = 35^\circ$.
- 12.** The opposite is 20. The adjacent is 11. The piece says $\theta = 61^\circ$. *Why:* The 20 is opposite θ and the 11 is next to θ (adjacent), so they make the tangent: $20 \div 11 = 1.8182$, and $\tan^{-1}(1.8182) = 61.19^\circ$, so $\theta = 61^\circ$.

2 Wrong: the value of θ is not right.

- 3.** The opposite is 10. The adjacent is 18. The piece says $\theta = 34^\circ$. *Why:* The 10 is opposite θ and the 18 is next to θ (adjacent), so they make the tangent: $10 \div 18 = 0.5556$, and $\tan^{-1}(0.5556) = 29.05^\circ$, so $\theta = 29^\circ$. The piece gives 34° , from $\sin^{-1}(0.5556)$, the wrong key.
- 4.** The opposite is 5. The hypotenuse is 9. The piece says $\theta = 56^\circ$. *Why:* The 5 is opposite θ and the 9 is the hypotenuse, so they make the sine: $5 \div 9 = 0.5556$, and $\sin^{-1}(0.5556) = 33.75^\circ$, so $\theta = 34^\circ$. The piece gives 56° , the other acute angle.
- 8.** The adjacent is 22. The hypotenuse is 26. The piece says $\theta = 40^\circ$. *Why:* The 22 is next to θ (adjacent) and the 26 is the hypotenuse, so they make the cosine: $22 \div 26 = 0.8462$, and $\cos^{-1}(0.8462) = 32.20^\circ$, so $\theta = 32^\circ$. The piece gives 40° , from $\tan^{-1}(0.8462)$, the wrong key.
- 11.** The opposite is 12. The adjacent is 17. The piece says $\theta = 55^\circ$. *Why:* The 12 is opposite θ and the 17 is next to θ (adjacent), so they make the tangent: $12 \div 17 = 0.7059$, and $\tan^{-1}(0.7059) = 35.22^\circ$, so $\theta = 35^\circ$. The piece gives 55° , the other acute angle.

3 Cannot tell: no right angle is marked, or only one side has a length.

- 2.** No right-angle mark. The opposite is 10. The hypotenuse is 16. The piece says $\theta = 39^\circ$. *Why:* No right angle is marked, so sine, cosine and tangent do not apply. It looks right-angled, but nothing says it is. (With the same 10 and 16, if the corner were 50° or 60° , θ would be about 29° or 33° .)
- 6.** The hypotenuse is 20. The piece says $\theta = 41^\circ$. *Why:* Only one side has a length (the hypotenuse, 20). One side does not fix the angle: a right triangle with that hypotenuse can have $\theta = 30^\circ$ or $\theta = 60^\circ$.
- 9.** No right-angle mark. The opposite is 14. The adjacent is 11. The piece says $\theta = 52^\circ$. *Why:* No right angle is marked, so sine, cosine and tangent do not apply. It looks right-angled, but nothing says it is. (With the same 14 and 11, if the corner were 80° or 100° , θ would be about 58° or 46° .)
- 10.** The opposite is 9. The piece says $\theta = 24^\circ$. *Why:* Only one side has a length (the opposite, 9). One side does not fix the angle: a right triangle with that opposite can have $\theta = 30^\circ$ or $\theta = 60^\circ$.

To check a piece with a student, ask two questions. First: is a right angle marked, and do two sides have lengths? If no, the piece goes in row 3. If yes, which ratio do those two sides make for θ ? Divide, use the matching key, and round to the nearest degree. If the value on the piece matches, it goes in row 1. If not, row 2.