

MYSTERY PICTURE



AREA OF A TRIANGLE USING SINE

Area = $\frac{1}{2}ab \sin C$: Which Two Sides?

25 triangles · three sides, one angle · no calculator · a picture appears

FREE

1 $\frac{1}{2} \times 12 \times 7 \times \sin 101^\circ$	2 $\frac{1}{2} \times 11 \times 14 \times \sin 40^\circ$	3 $\frac{1}{2} \times 13 \times 7 \times \sin 66^\circ$	4 $\frac{1}{2} \times 9 \times 12 \times \sin 75^\circ$	5 $\frac{1}{2} \times 14 \times 17 \times \sin 103^\circ$
6 $\frac{1}{2} \times 14 \times 7 \times \sin 59^\circ$	7 $\frac{1}{2} \times 18 \times 17 \times \sin 40^\circ$	8 $\frac{1}{2} \times 12 \times 21 \times \sin 33^\circ$	9 $\frac{1}{2} \times 17 \times 18 \times \sin 79^\circ$	10 $\frac{1}{2} \times 10 \times 11 \times \sin 108^\circ$
11 $\frac{1}{2} \times 14 \times 10 \times \sin 63^\circ$	12 $\frac{1}{2} \times 12 \times 13 \times \sin 52^\circ$	13 $\frac{1}{2} \times 24 \times 11 \times \sin 103^\circ$	14 $\frac{1}{2} \times 19 \times 14 \times \sin 42^\circ$	15 $\frac{1}{2} \times 20 \times 13 \times \sin 53^\circ$
16 $\frac{1}{2} \times 14 \times 12 \times \sin 70^\circ$	17 $\frac{1}{2} \times 14 \times 17 \times \sin 101^\circ$	18 $\frac{1}{2} \times 15 \times 20 \times \sin 74^\circ$	19 $\frac{1}{2} \times 21 \times 18 \times \sin 59^\circ$	20 $\frac{1}{2} \times 16 \times 12 \times \sin 53^\circ$
21 $\frac{1}{2} \times 21 \times 13 \times \sin 54^\circ$	22 $\frac{1}{2} \times 19 \times 21 \times \sin 50^\circ$	23 $\frac{1}{2} \times 20 \times 16 \times \sin 55^\circ$	24 $\frac{1}{2} \times 20 \times 21 \times \sin 52^\circ$	25 $\frac{1}{2} \times 14 \times 12 \times \sin 68^\circ$

Mystery Picture — how to run it

25–35 minutes

1. **Give each student page 3** and a pencil. No calculator is needed.
2. **Say this once:** *“Find the arc. Which two sides meet at that corner? If the expression under the triangle uses those two sides, fill the square with slanted lines. If not, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet page 3, one per student. A box at the bottom shows the formula on a labeled triangle.
Answers pages 4–5, for you.

Each square shows a triangle drawn to scale, with all three sides labeled and one angle marked. Under it is an expression for the area, such as $\frac{1}{2} \times 12 \times 7 \times \sin 101^\circ$. The area of a triangle is $\frac{1}{2} \times a \times b \times \sin C$, where a and b are the two sides that meet at the angle C . The student does not compute anything. The only question is whether the expression uses the right two sides. Seventeen squares are filled.

Each white square uses the side across from the marked angle in place of one of the two sides that meet there. This is the most common mistake with this formula, and it gives a wrong area with nothing strange about it. The two right sides are sometimes the two longest, sometimes the two shortest, and sometimes the longest and the shortest, so the size of the numbers does not decide. In five squares the marked angle is more than 90° .

If part of a student’s picture looks wrong, the mistake is in that square. Ask: *“Put a finger on the arc. Which two sides touch it? Which side is across from it?”* Guessing does not work. Every simple guessing rule, such as filling every square that uses the two largest numbers, gets at least eight squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture — which two sides?

Area of a triangle with sine

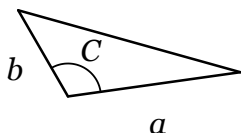
Name _____

Period _____

Date _____

Each square has a triangle and an expression for its area. Do not compute. Find the arc. Which two sides meet there? **If the expression uses those two sides, fill the square with slanted lines. If not, leave it white.** A picture appears. If part of it is missing, check that square again. The box at the bottom helps.

1 $\frac{1}{2} \times 12 \times 7 \times \sin 101^\circ$	2 $\frac{1}{2} \times 11 \times 14 \times \sin 40^\circ$	3 $\frac{1}{2} \times 13 \times 7 \times \sin 66^\circ$	4 $\frac{1}{2} \times 9 \times 12 \times \sin 75^\circ$	5 $\frac{1}{2} \times 14 \times 17 \times \sin 103^\circ$
6 $\frac{1}{2} \times 14 \times 7 \times \sin 59^\circ$	7 $\frac{1}{2} \times 18 \times 17 \times \sin 40^\circ$	8 $\frac{1}{2} \times 12 \times 21 \times \sin 33^\circ$	9 $\frac{1}{2} \times 17 \times 18 \times \sin 79^\circ$	10 $\frac{1}{2} \times 10 \times 11 \times \sin 108^\circ$
11 $\frac{1}{2} \times 14 \times 10 \times \sin 63^\circ$	12 $\frac{1}{2} \times 12 \times 13 \times \sin 52^\circ$	13 $\frac{1}{2} \times 24 \times 11 \times \sin 103^\circ$	14 $\frac{1}{2} \times 19 \times 14 \times \sin 42^\circ$	15 $\frac{1}{2} \times 20 \times 13 \times \sin 53^\circ$
16 $\frac{1}{2} \times 14 \times 12 \times \sin 70^\circ$	17 $\frac{1}{2} \times 14 \times 17 \times \sin 101^\circ$	18 $\frac{1}{2} \times 15 \times 20 \times \sin 74^\circ$	19 $\frac{1}{2} \times 21 \times 18 \times \sin 59^\circ$	20 $\frac{1}{2} \times 16 \times 12 \times \sin 53^\circ$
21 $\frac{1}{2} \times 21 \times 13 \times \sin 54^\circ$	22 $\frac{1}{2} \times 19 \times 21 \times \sin 50^\circ$	23 $\frac{1}{2} \times 20 \times 16 \times \sin 55^\circ$	24 $\frac{1}{2} \times 20 \times 21 \times \sin 52^\circ$	25 $\frac{1}{2} \times 14 \times 12 \times \sin 68^\circ$



Area = $\frac{1}{2} \times a \times b \times \sin C$.

C is the angle between the sides a and b : the corner where the two sides meet.

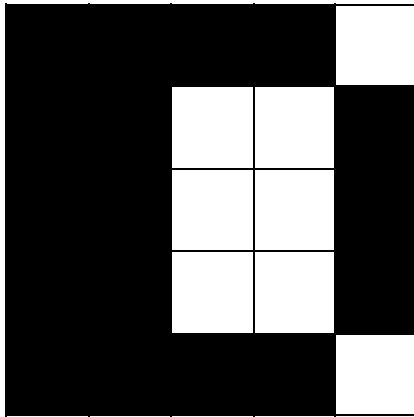
The third side, across from the angle, is not used.



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Mystery Picture — answers

For the teacher



The finished picture is **the letter D**. Seventeen squares are filled.

To check a square, find the arc and the two sides that meet at its corner. The expression is right when it uses those two sides. The table gives the reason for every square. It goes on over the page.

	The square	Answer	Why
1	Sides 7, 12 and 15. The 101° arc is where 7 and 12 meet. The square says $\frac{1}{2} \times 12 \times 7 \times \sin 101^\circ$.	fill	The two sides that meet at the 101° arc are 7 and 12, and the expression uses them.
2	Sides 11, 14 and 9. The 40° arc is where 11 and 14 meet. The square says $\frac{1}{2} \times 11 \times 14 \times \sin 40^\circ$.	fill	The two sides that meet at the 40° arc are 11 and 14, and the expression uses them.
3	Sides 7, 13 and 12. The 66° arc is where 7 and 13 meet. The square says $\frac{1}{2} \times 13 \times 7 \times \sin 66^\circ$.	fill	The two sides that meet at the 66° arc are 7 and 13, and the expression uses them.
4	Sides 9, 12 and 13. The 75° arc is where 9 and 12 meet. The square says $\frac{1}{2} \times 9 \times 12 \times \sin 75^\circ$.	fill	The two sides that meet at the 75° arc are 9 and 12, and the expression uses them.
5	Sides 7, 14 and 17. The 103° arc is where 7 and 14 meet. The square says $\frac{1}{2} \times 14 \times 17 \times \sin 103^\circ$.	white	The 17 is across from the 103° arc and does not touch it. The two sides at the arc are 7 and 14, so the right expression is $\frac{1}{2} \times 7 \times 14 \times \sin 103^\circ$.
6	Sides 7, 14 and 12. The 59° arc is where 7 and 14 meet. The square says $\frac{1}{2} \times 14 \times 7 \times \sin 59^\circ$.	fill	The two sides that meet at the 59° arc are 7 and 14, and the expression uses them.
7	Sides 17, 18 and 12. The 40° arc is where 17 and 18 meet. The square says $\frac{1}{2} \times 18 \times 17 \times \sin 40^\circ$.	fill	The two sides that meet at the 40° arc are 17 and 18, and the expression uses them.
8	Sides 14, 21 and 12. The 33° arc is where 14 and 21 meet. The square says $\frac{1}{2} \times 12 \times 21 \times \sin 33^\circ$.	white	The 12 is across from the 33° arc and does not touch it. The two sides at the arc are 14 and 21, so the right expression is $\frac{1}{2} \times 14 \times 21 \times \sin 33^\circ$.
9	Sides 10, 17 and 18. The 79° arc is where 10 and 17 meet. The square says $\frac{1}{2} \times 17 \times 18 \times \sin 79^\circ$.	white	The 18 is across from the 79° arc and does not touch it. The two sides at the arc are 10 and 17, so the right expression is $\frac{1}{2} \times 10 \times 17 \times \sin 79^\circ$.
10	Sides 10, 11 and 17. The 108° arc is where 10 and 11 meet. The square says $\frac{1}{2} \times 10 \times 11 \times \sin 108^\circ$.	fill	The two sides that meet at the 108° arc are 10 and 11, and the expression uses them.
11	Sides 10, 14 and 13. The 63° arc is where 10 and 14 meet. The square says $\frac{1}{2} \times 14 \times 10 \times \sin 63^\circ$.	fill	The two sides that meet at the 63° arc are 10 and 14, and the expression uses them.
12	Sides 12, 13 and 11. The 52° arc is where 12 and 13 meet. The square says $\frac{1}{2} \times 12 \times 13 \times \sin 52^\circ$.	fill	The two sides that meet at the 52° arc are 12 and 13, and the expression uses them.

Mystery Picture — answers, squares 13 to 25

For the teacher

	The square	Answer	Why
13	Sides 11, 19 and 24. The 103° arc is where 11 and 19 meet. The square says $\frac{1}{2} \times 24 \times 11 \times \sin 103^\circ$.	white	The 24 is across from the 103° arc and does not touch it. The two sides at the arc are 11 and 19, so the right expression is $\frac{1}{2} \times 11 \times 19 \times \sin 103^\circ$.
14	Sides 19, 20 and 14. The 42° arc is where 19 and 20 meet. The square says $\frac{1}{2} \times 19 \times 14 \times \sin 42^\circ$.	white	The 14 is across from the 42° arc and does not touch it. The two sides at the arc are 19 and 20, so the right expression is $\frac{1}{2} \times 19 \times 20 \times \sin 42^\circ$.
15	Sides 13, 20 and 16. The 53° arc is where 13 and 20 meet. The square says $\frac{1}{2} \times 20 \times 13 \times \sin 53^\circ$.	fill	The two sides that meet at the 53° arc are 13 and 20, and the expression uses them.
16	Sides 12, 14 and 15. The 70° arc is where 12 and 14 meet. The square says $\frac{1}{2} \times 14 \times 12 \times \sin 70^\circ$.	fill	The two sides that meet at the 70° arc are 12 and 14, and the expression uses them.
17	Sides 14, 17 and 24. The 101° arc is where 14 and 17 meet. The square says $\frac{1}{2} \times 14 \times 17 \times \sin 101^\circ$.	fill	The two sides that meet at the 101° arc are 14 and 17, and the expression uses them.
18	Sides 15, 18 and 20. The 74° arc is where 15 and 18 meet. The square says $\frac{1}{2} \times 15 \times 20 \times \sin 74^\circ$.	white	The 20 is across from the 74° arc and does not touch it. The two sides at the arc are 15 and 18, so the right expression is $\frac{1}{2} \times 15 \times 18 \times \sin 74^\circ$.
19	Sides 11, 21 and 18. The 59° arc is where 11 and 21 meet. The square says $\frac{1}{2} \times 21 \times 18 \times \sin 59^\circ$.	white	The 18 is across from the 59° arc and does not touch it. The two sides at the arc are 11 and 21, so the right expression is $\frac{1}{2} \times 11 \times 21 \times \sin 59^\circ$.
20	Sides 12, 16 and 13. The 53° arc is where 12 and 16 meet. The square says $\frac{1}{2} \times 16 \times 12 \times \sin 53^\circ$.	fill	The two sides that meet at the 53° arc are 12 and 16, and the expression uses them.
21	Sides 13, 21 and 17. The 54° arc is where 13 and 21 meet. The square says $\frac{1}{2} \times 21 \times 13 \times \sin 54^\circ$.	fill	The two sides that meet at the 54° arc are 13 and 21, and the expression uses them.
22	Sides 19, 21 and 17. The 50° arc is where 19 and 21 meet. The square says $\frac{1}{2} \times 19 \times 21 \times \sin 50^\circ$.	fill	The two sides that meet at the 50° arc are 19 and 21, and the expression uses them.
23	Sides 16, 20 and 17. The 55° arc is where 16 and 20 meet. The square says $\frac{1}{2} \times 20 \times 16 \times \sin 55^\circ$.	fill	The two sides that meet at the 55° arc are 16 and 20, and the expression uses them.
24	Sides 20, 21 and 18. The 52° arc is where 20 and 21 meet. The square says $\frac{1}{2} \times 20 \times 21 \times \sin 52^\circ$.	fill	The two sides that meet at the 52° arc are 20 and 21, and the expression uses them.
25	Sides 12, 13 and 14. The 68° arc is where 12 and 13 meet. The square says $\frac{1}{2} \times 14 \times 12 \times \sin 68^\circ$.	white	The 14 is across from the 68° arc and does not touch it. The two sides at the arc are 12 and 13, so the right expression is $\frac{1}{2} \times 12 \times 13 \times \sin 68^\circ$.