

CUT & GLUE SORT



TANGENT SEGMENTS AND INSCRIBED AND CIRCUMSCRIBED CIRCLES

How Many Tangent Lines Through a Point?

Twelve pieces · two, one or none · inscribed and circumscribed circles · three rows

FREE

1	2	3
The circle has center O and radius 6. $OP = 10$.	Triangle STU is inscribed in the circle. P is the corner T.	Triangle STU is inscribed in the circle. P is the middle of side ST.

Row 1. Two tangent lines pass through P. P is outside the circle.

Row 2. Exactly one tangent line passes through P. P is on the circle.

Row 3. No tangent line passes through P. P is inside the circle.

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. Nothing is measured.
2. **Say this once:** *“Find where P is: outside the circle, on it, or inside it. Outside gives two tangent lines, on gives one, inside gives none. Read the words inscribed and circumscribed carefully.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and the three cases as pictures.

Answers page 5.

From a point outside a circle there are exactly two tangent lines, and the two tangent segments are equal. A point on the circle has one tangent line, and a point inside has none. Students who have only seen points outside think “two tangents” is always true. Row 3 is where that rule has nothing to say.

Two kinds of piece need care. Two pieces give the diameter, not the radius. A student who compares OP with the diameter puts them in the wrong row. Seven pieces use the words inscribed or circumscribed. A triangle inscribed in a circle has its corners on the circle. A circle inscribed in a triangle is inside it, so the triangle’s corners are outside the circle. Students who think “inscribed means inside” mix these up. On piece 10, a right triangle is inscribed in a circle and P is the middle of its hypotenuse. The hypotenuse is a diameter, so P is the center, inside the circle.

Guessing does not work: a simple guessing rule, such as comparing the two printed numbers, places at most six pieces correctly. This sheet calls a circle inscribed in a triangle when it is inside the triangle and each side is tangent to it, and a triangle inscribed in a circle when its three corners are on the circle. Some textbooks use other definitions, so tell the class which ones this sheet uses.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — how many tangent lines?

Tangent lines through a point

Name _____

Period _____

Date _____

Each piece on the next page tells you where a point P is. **How many tangent lines to the circle pass through P? Glue the piece in its row.**

Row 1. Two tangent lines pass through P. P is outside the circle.

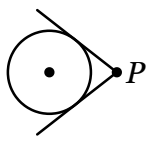
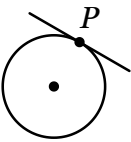
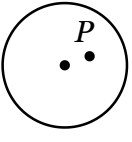
Row 2. Exactly one tangent line passes through P. P is on the circle.

Row 3. No tangent line passes through P. P is inside the circle.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. The box below shows the three rows as pictures.

 P outside: two tangent lines.	 P on the circle: one tangent line.	 P inside: no tangent line.	The dot in the middle of each circle is its center O. Compare OP with the radius. The radius is half the diameter. A triangle inscribed in a circle has its three corners on the circle. A circle inscribed in a triangle is inside the triangle, and each side is tangent to it. The triangle is then circumscribed about the circle.
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1 The circle has center O and radius 6. $OP = 10$.	2 Triangle STU is inscribed in the circle. P is the corner T.	3 Triangle STU is inscribed in the circle. P is the middle of side ST.	4 The circle has center O and diameter 14. $OP = 7$.
5 The circle has center O and radius 9. $OP = 4$.	6 The circle is inscribed in triangle STU. P is a point outside the triangle.	7 The circle has center O and diameter 10. $OP = 6$.	8 The circle is inscribed in triangle STU. It touches side ST at P.
9 P is the center O of the circle.	10 A right triangle is inscribed in the circle. P is the middle of its hypotenuse.	11 Triangle STU is circumscribed about the circle. P is the corner U.	12 A right triangle is inscribed in the circle. P is the corner with the right angle.



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Every piece is listed by the number printed on its corner, with the reason. Each row holds four pieces.

1 **Two tangent lines pass through P. P is outside the circle.**

1. The circle has center O and radius 6. $OP = 10$. *Why:* $OP = 10$ is more than the radius 6, so P is outside the circle.

6. The circle is inscribed in triangle STU. P is a point outside the triangle. *Why:* The circle is inside the triangle. A point outside the triangle is outside the circle too.

7. The circle has center O and diameter 10. $OP = 6$. *Why:* The radius is half of 10, which is 5. $OP = 6$ is more than 5, so P is outside. Comparing 6 with 10 puts it inside, which is the mistake.

11. Triangle STU is circumscribed about the circle. P is the corner U. *Why:* A triangle circumscribed about a circle has the circle inside it, touching each side. A corner is outside the circle, so two tangent lines pass through it: the two sides at that corner.

2 **Exactly one tangent line passes through P. P is on the circle.**

2. Triangle STU is inscribed in the circle. P is the corner T. *Why:* A triangle inscribed in a circle has its three corners on the circle. So P is on the circle.

4. The circle has center O and diameter 14. $OP = 7$. *Why:* The radius is half of 14, which is 7. $OP = 7$ is the radius, so P is on the circle. Comparing 7 with 14 puts it inside, which is the mistake.

8. The circle is inscribed in triangle STU. It touches side ST at P. *Why:* P is a point of tangency, so it is on the circle. The one tangent line through it is the side ST.

12. A right triangle is inscribed in the circle. P is the corner with the right angle. *Why:* All three corners of a triangle inscribed in a circle are on the circle, the right-angle corner too.

3 **No tangent line passes through P. P is inside the circle.**

3. Triangle STU is inscribed in the circle. P is the middle of side ST. *Why:* The corners S and T are on the circle, so ST is a chord. The middle of a chord is inside the circle.

5. The circle has center O and radius 9. $OP = 4$. *Why:* $OP = 4$ is less than the radius 9, so P is inside the circle.

9. P is the center O of the circle. *Why:* The center is inside the circle. Every line through it crosses the circle twice.

10. A right triangle is inscribed in the circle. P is the middle of its hypotenuse. *Why:* The hypotenuse of a right triangle inscribed in a circle is a diameter. Its middle is the center, which is inside.

To check a piece with a student, ask: “Is P outside the circle, on it, or inside it? What is the radius?”