

CUT & GLUE SORT

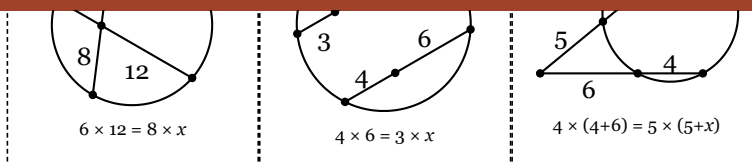


SEGMENT LENGTHS IN CIRCLES: POWER OF A POINT

Power of a Point: Correct, Wrong, or Cannot Tell?

Twelve drawings · chords, secants and tangents · when the rule does
not apply

FREE



Row 1. Correct. The equation follows the rule for this drawing.

Row 2. Wrong. The rule fits this drawing, but the equation multiplies the wrong pieces.

Row 3. Cannot tell. The rule does not fit this drawing: the two drawn lines do not meet, or a line does not meet the circle.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Do not solve. The rule needs two lines through one point, and each line must meet the circle. If the rule fits the drawing, check the equation. If it multiplies the right lengths, row 1. If it multiplies the wrong ones, row 2. If the rule does not fit, the piece goes in row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and the rules on lettered drawings.
Answers	page 5.

Each piece is a drawing, to scale, with an equation under it. The rule: when two lines pass through one point and each meets the circle, multiply the length from the point to the near point of the circle by the length to the far point, on each line. Both lines give the same number. Inside the circle, those are the two pieces of a chord. Outside, they are the outside piece and the whole length. A tangent times itself gives the same number. On this sheet, a secant runs from the outside point through the circle, and its outside piece runs from the point to the circle. Some textbooks use other definitions, so tell the class which ones this sheet uses.

Row 2 holds the common mistakes: a piece of one chord times a piece of the other, the outside piece times the inside piece, the inside piece used as if it were the outside piece, and the tangent squared set equal to the outside piece times the inside piece.

Row 3 is where the rule says nothing. Four pieces look just like the others, and each equation even fits its drawing. But the two lines do not meet at one point, or one line misses the circle, so the rule does not apply. The same printed numbers also fit another drawing with a different x , so nothing makes the equation true. Students who sort by look put these in row 1. Guessing does not work: a simple guessing rule, such as “brackets mean correct,” places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Cut & Glue Sort — is the equation right?

Power of a point

Name _____

Period _____

Date _____

Each piece on the next page is a drawing with an equation under it. Do not solve. **Is the equation correct for the drawing, wrong, or can you not tell? Glue the piece in its row.**

Row 1. Correct. The equation follows the rule for this drawing.

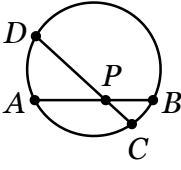
Row 2. Wrong. The rule fits this drawing, but the equation multiplies the wrong pieces.

Row 3. Cannot tell. The rule does not fit this drawing: the two drawn lines do not meet, or a line does not meet the circle.

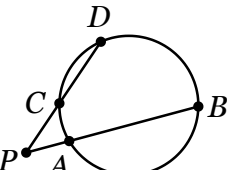
Cut & Glue Sort — the pieces

Cut along the dashed lines

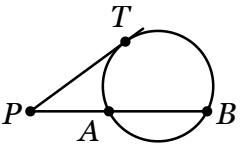
Cut out the twelve pieces. Glue each one in its row on the other sheet. Each number is the length of the piece beside it. A dot marks the end of a piece.



P inside
 $PA \cdot PB = PC \cdot PD$

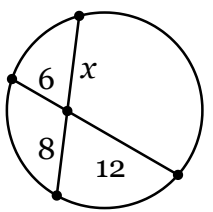
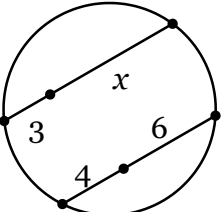
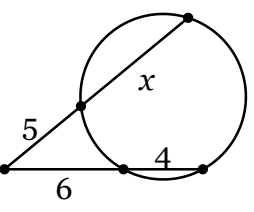
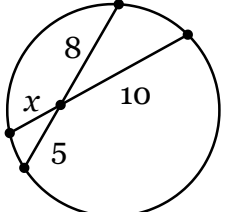
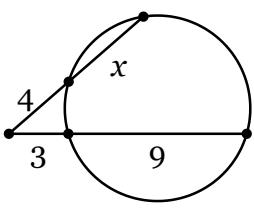
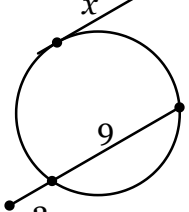
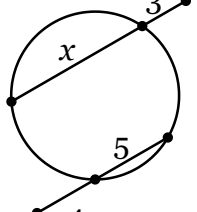
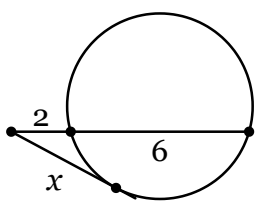
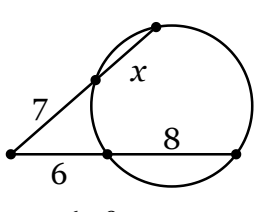
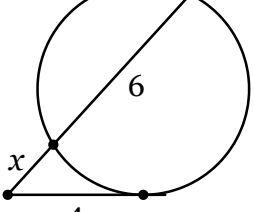
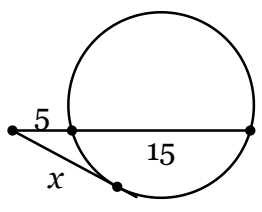
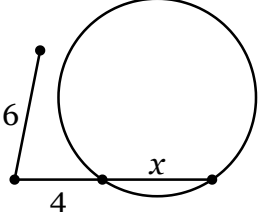


P outside
 $PA \cdot PB = PC \cdot PD$



A tangent
 $PT \cdot PT = PA \cdot PB$

P is where the two lines meet. **A** and **C** are the near points. **B** and **D** are the far points. **T** is where a line touches the circle.
Every length starts at P. Outside the circle, PB is the whole length from P to B: $PB = PA + AB$.

1  $6 \times 12 = 8 \times x$	2  $4 \times 6 = 3 \times x$	3  $4 \times (4+6) = 5 \times (5+x)$	4  $5 \times 10 = 8 \times x$
5  $3 \times (3+9) = 4 \times (4+x)$	6  $x^2 = 3 \times (3+9)$	7  $4 \times (4+5) = 3 \times (3+x)$	8  $x^2 = 2 \times (2+6)$
9  $6 \times 8 = 7 \times x$	10  $4^2 = x \times (x+6)$	11  $x^2 = 5 \times 15$	12  $6^2 = 4 \times (4+x)$



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Every piece is listed by the number printed on its corner, with the reason. Each row holds four pieces. Where the rule decides x , its value is given, though the students do not find it.

1 **Correct. The equation follows the rule for this drawing.**

1. Two chords cross inside the circle. The piece says $6 \times 12 = 8 \times x$. *Why:* Each side multiplies the near length by the far length of one line, measured from the point where the lines meet ($x = 9$).

5. Two secants from one outside point. The piece says $3 \times (3+9) = 4 \times (4+x)$. *Why:* Each side multiplies the near length by the far length of one line, measured from the point where the lines meet ($x = 5$).

8. A tangent and a secant from one outside point. The piece says $x^2 = 2 \times (2+6)$. *Why:* One side is the tangent times itself, and the other is outside piece $\times$ whole length of the secant ($x = 4$).

10. A tangent and a secant from one outside point. The piece says $4^2 = x \times (x+6)$. *Why:* One side is the tangent times itself, and the other is outside piece $\times$ whole length of the secant ($x = 2$).

2 **Wrong. The rule fits this drawing, but the equation multiplies the wrong pieces.**

3. Two secants from one outside point. The piece says $4 \times (4+6) = 5 \times (5+x)$. *Why:* It uses the inside piece 4 as if it were the outside piece. The outside piece is 6, so that line gives $6 \times (6+4)$.

4. Two chords cross inside the circle. The piece says $5 \times 10 = 8 \times x$. *Why:* It multiplies a piece of one chord by a piece of the other. The rule is $5 \times 8 = 10 \times x$.

9. Two secants from one outside point. The piece says $6 \times 8 = 7 \times x$. *Why:* It multiplies each outside piece by the inside piece. The rule uses the whole length: $6 \times (6+8)$.

11. A tangent and a secant from one outside point. The piece says $x^2 = 5 \times 15$. *Why:* It sets the tangent squared equal to outside piece $\times$ inside piece. The rule uses the whole length: $x^2 = 5 \times (5+15)$.

3 **Cannot tell. The rule does not fit this drawing: the two drawn lines do not meet, or a line does not meet the circle.**

2. Two chords that do not cross, each cut by its own point. The piece says $4 \times 6 = 3 \times x$. *Why:* The rule needs two lines through one point, each meeting the circle. Here the two drawn lines do not meet. In this drawing x is 8, so the equation happens to hold. The same printed numbers also fit a drawing where x is 5, so the rule does not make it true.

6. A secant from one point and a tangent from another point. The piece says $x^2 = 3 \times (3+9)$. *Why:* The rule needs two lines through one point, each meeting the circle. Here the two drawn lines do not meet. In this drawing x is 6, so the equation happens to hold. The same printed numbers also fit a drawing where x is 9, so the rule does not make it true.

7. Two secants from two different outside points. The piece says $4 \times (4+5) = 3 \times (3+x)$. *Why:* The rule needs two lines through one point, each meeting the circle. Here the two drawn lines do not meet. In this drawing x is 9, so the equation happens to hold. The same printed numbers also fit a drawing where x is 6, so the rule does not make it true.

12. From one point, a secant and a second line that misses the circle. The piece says $6^2 = 4 \times (4+x)$. *Why:* The rule needs two lines through one point, each meeting the circle. The second line misses the circle, so it is not a tangent. In this drawing x is 5, so the equation happens to hold. The same printed numbers also fit a drawing where x is 8, so the rule does not make it true.

To check a piece with a student, ask: “Do the two lines meet at one point? Does each line meet the circle? Then which number is the near length and which the far length on each line?”