

ANSWER MAZE

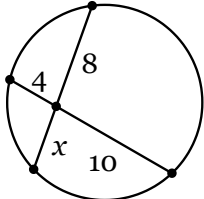


SEGMENT LENGTHS IN CIRCLES: POWER OF A POINT

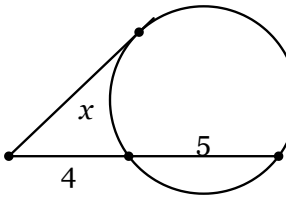
Power of a Point: Chords, Secants and Tangents

Segment lengths in circles · one page · 10–15 minutes · nothing to prepare

FREE



6 Find x .
 $x = 20 \rightarrow$ go to box 3
 $x = 5 \rightarrow$ go to box 2
*Hint: Trace each chord with a pencil.
Which two numbers lie on one chord?*



2 Find x .
 $x = 6 \rightarrow$ go to box 8
 $x = 36 \rightarrow$ go to box 5
*Hint: Read the tangent rule
in the box at the top. What
does x stand for in it?*

1. **Hand out page 3**, one per student, and a calculator. Nothing else is needed.
2. **Say this once:** *“Start at box 6. Every length starts at the point where the two lines meet. Outside the circle, add the two pieces of a line first. Write every box number in the row at the bottom as you go.”*
3. **Check a sheet quickly.** A student with no mistakes writes **6 2 8 4 1 7**: six boxes, then out.

Student sheet page 3, one per student, and scrap paper for working.
Answers page 4.

Every box shows a circle and two lines that meet at one point, drawn to scale. A number stands beside each piece, and the student finds the piece marked x . On each line, the length from the meeting point to the near point times the length to the far point gives the same number for both lines. Inside the circle, those are the two pieces of a chord. Outside, they are the outside piece and the whole length, so the student adds the two pieces first. For a tangent, the tangent times itself gives that number. On this sheet, a secant runs from the outside point through the circle to the far point. Some textbooks use other definitions, so tell the class which ones this sheet uses.

Every wrong answer comes from one of two mistakes. In the first, the student multiplies the wrong two pieces: the outside piece by the inside piece, or a piece of one chord by a piece of the other. These wrong answers send the student to box 3. In the second, the student forgets the square root and gives the tangent times itself. These wrong answers send the student to box 5. Boxes 3 and 5 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more. Guessing does not get a student out: every simple guessing rule, such as always taking the smaller number, goes wrong within the first three boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Answer Maze — power of a point

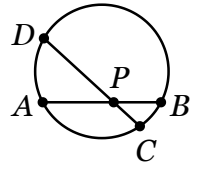
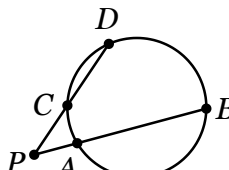
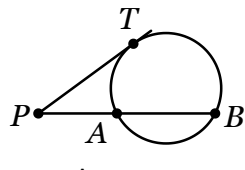
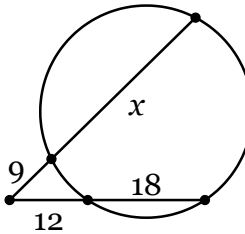
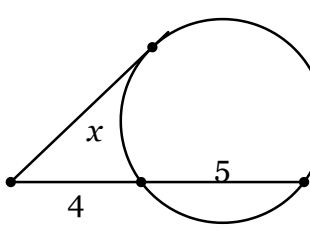
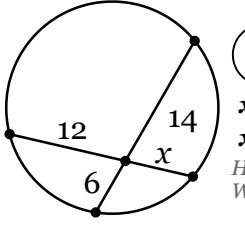
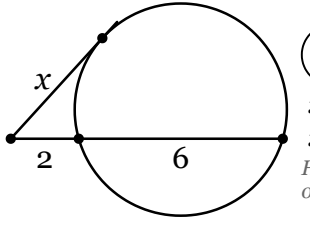
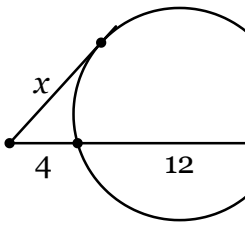
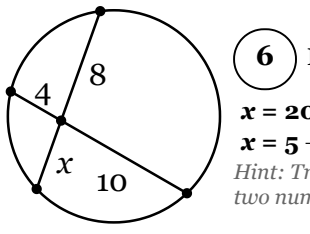
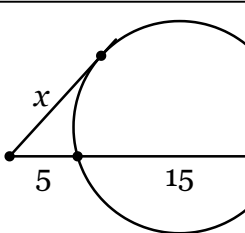
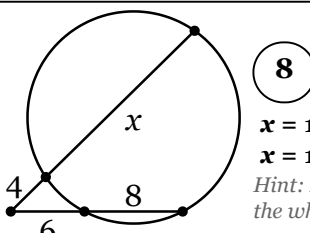
Start at box 6

Name _____

Period _____

Date _____

Start at box 6. Each number is the length of the piece beside it. Find x . Pick one answer and go to the box it names. Write each box number in the row at the bottom as you go.

 P inside $PA \cdot PB = PC \cdot PD$	 P outside $PA \cdot PB = PC \cdot PD$	 A tangent $PT \cdot PT = PA \cdot PB$ P is where the two lines meet. A and C are the near points. B and D are the far points. T is where a line touches the circle. Every length starts at P. Outside the circle, PB is the whole length from P to B: $PB = PA + AB$.
 1 Find x. $x = 24 \rightarrow$ go to box 3 $x = 31 \rightarrow$ go to box 7 <i>Hint: Find the whole length of each line first. Then use the rule in the box.</i>	 2 Find x. $x = 6 \rightarrow$ go to box 8 $x = 36 \rightarrow$ go to box 5 <i>Hint: Read the tangent rule in the box at the top. What does x stand for in it?</i>	
 3 Find x. $x = 7 \rightarrow$ go to box 2 $x = 28 \rightarrow$ go to box 6 <i>Hint: Each chord gives its own product. Which pieces lie on one chord?</i>	 4 Find x. $x = 16 \rightarrow$ go to box 5 $x = 4 \rightarrow$ go to box 1 <i>Hint: Compare the tangent with the other line in the drawing.</i>	
 5 Find x. $x = 64 \rightarrow$ go to box 2 $x = 8 \rightarrow$ go to box 8 <i>Hint: In the tangent rule, x times x is a product. What is x?</i>	 6 Find x. $x = 20 \rightarrow$ go to box 3 $x = 5 \rightarrow$ go to box 2 <i>Hint: Trace each chord with a pencil. Which two numbers lie on one chord?</i>	
 7 Find x. $x = 10 \rightarrow$ you are out $x = 100 \rightarrow$ go to box 5 <i>Hint: Write the tangent rule from the box. Then find x itself, a length.</i>	 8 Find x. $x = 17 \rightarrow$ go to box 4 $x = 12 \rightarrow$ go to box 3 <i>Hint: Each line gives a product. What is the whole length of each line?</i>	

Write every box number here, in order.

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The route, in order. Boxes 3 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table.

6 → 2 → 8 → 4 → 1 → 7 → out

	The box's question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 6 Find x . Two chords cross inside the circle. One has pieces 4 and 10. The other has pieces 8 and x .	5. The two pieces of one chord multiply to the same number as the two pieces of the other: $4 \times 10 = 40 = 8 \times x$, so $x = 5$.	"20" goes to box 3. The chords are mixed: $4 \times x = 8 \times 10$. The 4 and the x are on different chords.
2	Box 2 Find x . From one outside point: a line with outside piece 4 and inside piece 5, and a tangent x .	6. The tangent times itself is outside piece $\times$ whole length: $x \times x = 4 \times (4 + 5) = 36$, so $x = \sqrt{36} = 6$.	"36" goes to box 5. 36 is $x \times x$, not x : the square root is left out. A tangent of 36 would be longer than the whole line, 9.
3	Box 8 Find x . Two lines from one outside point: outside piece 6 and inside piece 8; outside piece 4 and inside piece x .	17. Outside piece $\times$ whole length is the same on both lines. $6 \times (6 + 8) = 84$, so $4 \times (4 + x) = 84$, $4 + x = 21$, and $x = 17$.	"12" goes to box 3. The outside piece is multiplied by the inside piece: $6 \times 8 = 4 \times x$. The rule uses the whole length, $6 + 8 = 14$.
4	Box 4 Find x . From one outside point: a line with outside piece 2 and inside piece 6, and a tangent x .	4. The tangent times itself is outside piece $\times$ whole length: $x \times x = 2 \times (2 + 6) = 16$, so $x = \sqrt{16} = 4$.	"16" goes to box 5. 16 is $x \times x$, not x : the square root is left out. A tangent of 16 would be longer than the whole line, 8.
5	Box 1 Find x . Two lines from one outside point: outside piece 12 and inside piece 18; outside piece 9 and inside piece x .	31. Outside piece $\times$ whole length is the same on both lines. $12 \times (12 + 18) = 360$, so $9 \times (9 + x) = 360$, $9 + x = 40$, and $x = 31$.	"24" goes to box 3. The outside piece is multiplied by the inside piece: $12 \times 18 = 9 \times x$. The rule uses the whole length, $12 + 18 = 30$.
6	Box 7 Find x . From one outside point: a line with outside piece 5 and inside piece 15, and a tangent x .	10. The tangent times itself is outside piece $\times$ whole length: $x \times x = 5 \times (5 + 15) = 100$, so $x = \sqrt{100} = 10$.	"100" goes to box 5. 100 is $x \times x$, not x : the square root is left out. A tangent of 100 would be longer than the whole line, 20.
	Box 3 Find x . Two chords cross inside the circle. One has pieces 6 and 14. The other has pieces 12 and x .	7. The two pieces of one chord multiply to the same number as the two pieces of the other: $6 \times 14 = 84 = 12 \times x$, so $x = 7$.	"28" goes to box 6. The chords are mixed: $6 \times x = 12 \times 14$. The 6 and the x are on different chords.
	Box 5 Find x . From one outside point: a line with outside piece 4 and inside piece 12, and a tangent x .	8. The tangent times itself is outside piece $\times$ whole length: $x \times x = 4 \times (4 + 12) = 64$, so $x = \sqrt{64} = 8$.	"64" goes to box 2. 64 is $x \times x$, not x : the square root is left out. A tangent of 64 would be longer than the whole line, 16.

How to check any box. On each line, multiply the length from the meeting point to the near point by the length to the far point. Both lines give the same number. Outside the circle, the length to the far point is the outside piece plus the inside piece. A tangent times itself gives that number, so the tangent is its square root, and it is always shorter than the other line's whole length.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.