

MYSTERY PICTURE



SEGMENT LENGTHS IN CIRCLES: POWER OF A POINT

Power of a Point: The Right Equation

25 drawings · secants and tangents · nothing to solve · a picture appears

FREE

1 $4 \times (4+5) = 3 \times (3+x)$	2 $x^2 = 4 \times (4+5)$	3 $x \times (x+5) = 6 \times (6+8)$	4 $6^2 = 3 \times (3+x)$	5 $3 \times 9 = 4 \times x$
6 $(4+x) \times 4 = (5+7) \times 5$	7 $x \times (x+15) = 10^2$	8 $x = 2 \times (2+6)$	9 $9 \times (9+3) = x \times (x+5)$	10 $8^2 = 4 \times x$
11 $7 \times (7+x) = 8 \times (8+6)$	12 $x \times x = 4 \times (4+12)$	13 $7 \times (7+9) = x \times (x+6)$	14 $x^2 = 5 \times 15$	15 $7 \times 9 = 8 \times x$
16 $4^2 = x \times (x+6)$	17 $6 \times (6+x) = 5 \times (5+13)$	18 $6 = 4 \times (4+x)$	19 $11 \times (11+4) = x \times (x+7)$	20 $6^2 = x \times 5$
21 $3 \times (3+9) = x^2$	22 $5 \times (5+x) = 6 \times (6+9)$	23 $5 \times 13 = x \times 9$	24 $10 = 5 \times (5+x)$	25 $5 \times (5+7) = 6 \times (6+x)$

Mystery Picture — how to run it

25–35 minutes

1. **Give each student page 3** and a pencil. No calculator is needed.
2. **Say this once:** *“Do not solve anything. Look at the drawing. Which number is an outside piece, which is an inside piece, and which is a tangent? If the equation under the drawing is the right one, fill the square with slanted lines. If not, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet page 3, one per student. A box at the bottom shows the rules on lettered drawings.
Answers pages 4–5, for you.

Each square shows a circle and two lines from one outside point, drawn to scale. The lines are two secants, or a tangent and a secant. A number stands beside each piece and x on one piece. On this sheet, a secant runs from the outside point through the circle. Its outside piece runs from the point to the circle, and its inside piece is the chord. Some textbooks use other definitions, so tell the class which ones this sheet uses. The rule: for each secant, outside piece $\times$ whole length gives the same number, and the tangent times itself gives that number too. Thirteen squares are filled.

Each white square holds a common mistake. Some multiply the outside piece by the inside piece. Some use the inside piece as if it were the outside piece. Some set the tangent itself, not the tangent times itself, equal to the product. The student does not solve anything: the only question is whether the right pieces are in the equation.

If part of a student’s picture looks wrong, the mistake is in that square. Ask: *“Which number is the outside piece? What is the whole length of that line?”* Guessing does not work. Every simple guessing rule, such as filling every square with brackets, gets at least six squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture — the right equation

Power of a point

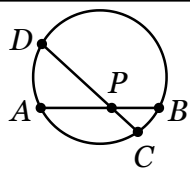
Name _____

Period _____

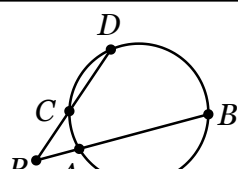
Date _____

Each number is the length of the piece beside it. Do not solve. Is the equation under the drawing the right one for it? **If yes, fill the square with slanted lines. If no, leave it white.** A picture appears. If part of it is missing, check that square again. The box at the bottom helps.

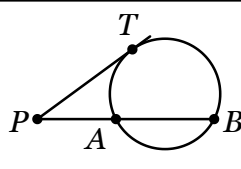
1 $4 \times (4+5) = 3 \times (3+x)$	2 $x^2 = 4 \times (4+5)$	3 $x \times (x+5) = 6 \times (6+8)$	4 $6^2 = 3 \times (3+x)$	5 $3 \times 9 = 4 \times x$
6 $(4+x) \times 4 = (5+7) \times 5$	7 $x \times (x+15) = 10^2$	8 $x = 2 \times (2+6)$	9 $9 \times (9+3) = x \times (x+5)$	10 $8^2 = 4 \times x$
11 $7 \times (7+x) = 8 \times (8+6)$	12 $x \times x = 4 \times (4+12)$	13 $7 \times (7+9) = x \times (x+6)$	14 $x^2 = 5 \times 15$	15 $7 \times 9 = 8 \times x$
16 $4^2 = x \times (x+6)$	17 $6 \times (6+x) = 5 \times (5+13)$	18 $6 = 4 \times (4+x)$	19 $11 \times (11+4) = x \times (x+7)$	20 $6^2 = x \times 5$
21 $3 \times (3+9) = x^2$	22 $5 \times (5+x) = 6 \times (6+9)$	23 $5 \times 13 = x \times 9$	24 $10 = 5 \times (5+x)$	25 $5 \times (5+7) = 6 \times (6+x)$



P inside
 $PA \cdot PB = PC \cdot PD$



P outside
 $PA \cdot PB = PC \cdot PD$



A tangent
 $PT \cdot PT = PA \cdot PB$

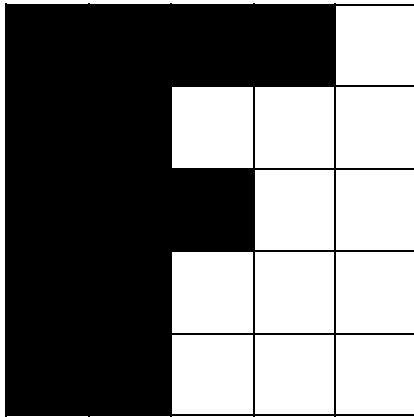
P is where the lines meet. Outside the circle, PB is the whole length: $PB = PA + AB$.



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Mystery Picture — answers

For the teacher



The finished picture is **the letter F**. Thirteen squares are filled.

To check a square, find the outside piece and the inside piece of each secant. The equation is right when each side is outside piece $\times$ whole length of one line, or the tangent times itself. The table gives the reason for every square. It goes on over the page.

	The square	Answer	Why
1	Two secants from one outside point. One line has outside piece 4 and inside piece 5. The other has outside piece 3 and inside piece x . The square says $4 \times (4+5) = 3 \times (3+x)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 9$).
2	A tangent of x and a secant with outside piece 4 and inside piece 5, from one outside point. The square says $x^2 = 4 \times (4+5)$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 6$).
3	Two secants from one outside point. One line has outside piece 6 and inside piece 8. The other has outside piece x and inside piece 5. The square says $x \times (x+5) = 6 \times (6+8)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 7$).
4	A tangent of 6 and a secant with outside piece 3 and inside piece x , from one outside point. The square says $6^2 = 3 \times (3+x)$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 9$).
5	Two secants from one outside point. One line has outside piece 3 and inside piece 9. The other has outside piece 4 and inside piece x . The square says $3 \times 9 = 4 \times x$.	white	It multiplies each outside piece by the inside piece. The rule needs outside piece $\times$ whole length: $3 \times (3+9)$.
6	Two secants from one outside point. One line has outside piece 5 and inside piece 7. The other has outside piece 4 and inside piece x . The square says $(4+x) \times 4 = (5+7) \times 5$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 11$).
7	A tangent of 10 and a secant with outside piece x and inside piece 15, from one outside point. The square says $x \times (x+15) = 10^2$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 5$).
8	A tangent of x and a secant with outside piece 2 and inside piece 6, from one outside point. The square says $x = 2 \times (2+6)$.	white	The tangent appears once. The rule needs the tangent times itself: $x^2 = 2 \times (2+6)$.
9	Two secants from one outside point. One line has outside piece 3 and inside piece 9. The other has outside piece x and inside piece 5. The square says $9 \times (9+3) = x \times (x+5)$.	white	The 9 is the inside piece, not the outside piece. The right side for that line is $3 \times (3+9)$.
10	A tangent of 8 and a secant with outside piece 4 and inside piece x , from one outside point. The square says $8^2 = 4 \times x$.	white	The tangent squared is set equal to outside piece $\times$ inside piece. The rule needs the whole length: $8^2 = 4 \times (4+x)$.
11	Two secants from one outside point. One line has outside piece 8 and inside piece 6. The other has outside piece 7 and inside piece x . The square says $7 \times (7+x) = 8 \times (8+6)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 9$).
12	A tangent of x and a secant with outside piece 4 and inside piece 12, from one outside point. The square says $x \times x = 4 \times (4+12)$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 8$).

Mystery Picture — answers, squares 13 to 25

For the teacher

	The square	Answer	Why
13	Two secants from one outside point. One line has outside piece 7 and inside piece 9. The other has outside piece x and inside piece 6. The square says $7 \times (7+9) = x \times (x+6)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 8$).
14	A tangent of x and a secant with outside piece 5 and inside piece 15, from one outside point. The square says $x^2 = 5 \times 15$.	white	The tangent squared is set equal to outside piece $\times$ inside piece. The rule needs the whole length: $x^2 = 5 \times (5+15)$.
15	Two secants from one outside point. One line has outside piece 7 and inside piece 9. The other has outside piece 8 and inside piece x . The square says $7 \times 9 = 8 \times x$.	white	It multiplies each outside piece by the inside piece. The rule needs outside piece $\times$ whole length: $7 \times (7+9)$.
16	A tangent of 4 and a secant with outside piece x and inside piece 6, from one outside point. The square says $4^2 = x \times (x+6)$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 2$).
17	Two secants from one outside point. One line has outside piece 5 and inside piece 13. The other has outside piece 6 and inside piece x . The square says $6 \times (6+x) = 5 \times (5+13)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 9$).
18	A tangent of 6 and a secant with outside piece 4 and inside piece x , from one outside point. The square says $6 = 4 \times (4+x)$.	white	The tangent appears once. The rule needs the tangent times itself: $6^2 = 4 \times (4+x)$.
19	Two secants from one outside point. One line has outside piece 4 and inside piece 11. The other has outside piece x and inside piece 7. The square says $11 \times (11+4) = x \times (x+7)$.	white	The 11 is the inside piece, not the outside piece. The right side for that line is $4 \times (4+11)$.
20	A tangent of 6 and a secant with outside piece x and inside piece 5, from one outside point. The square says $6^2 = x \times 5$.	white	The tangent squared is set equal to outside piece $\times$ inside piece. The rule needs the whole length: $6^2 = x \times (x+5)$.
21	A tangent of x and a secant with outside piece 3 and inside piece 9, from one outside point. The square says $3 \times (3+9) = x^2$.	fill	The tangent times itself equals outside piece $\times$ whole length, so the equation is right ($x = 6$).
22	Two secants from one outside point. One line has outside piece 6 and inside piece 9. The other has outside piece 5 and inside piece x . The square says $5 \times (5+x) = 6 \times (6+9)$.	fill	Each side is outside piece $\times$ whole length of one line, so the equation is right ($x = 13$).
23	Two secants from one outside point. One line has outside piece 5 and inside piece 13. The other has outside piece x and inside piece 9. The square says $5 \times 13 = x \times 9$.	white	It multiplies each outside piece by the inside piece. The rule needs outside piece $\times$ whole length: $5 \times (5+13)$.
24	A tangent of 10 and a secant with outside piece 5 and inside piece x , from one outside point. The square says $10 = 5 \times (5+x)$.	white	The tangent appears once. The rule needs the tangent times itself: $10^2 = 5 \times (5+x)$.
25	Two secants from one outside point. One line has outside piece 7 and inside piece 5. The other has outside piece 6 and inside piece x . The square says $5 \times (5+7) = 6 \times (6+x)$.	white	The 5 is the inside piece, not the outside piece. The right side for that line is $7 \times (7+5)$.