

SCAVENGER HUNT



SEGMENT LENGTHS IN CIRCLES: POWER OF A POINT

Power of a Point: Chords, Secants and Tangents

Segment lengths in circles · whole-number answers · the cards check
the answers

FREE

ANSWER

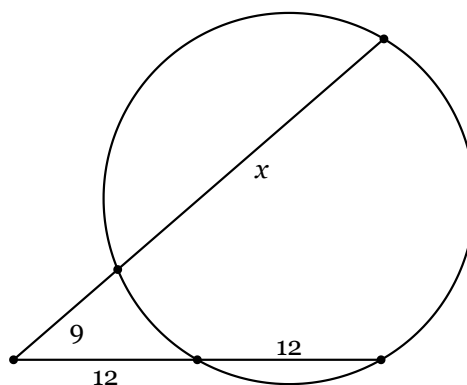
8

Each number is the length of the piece beside
it.

Find x .

Write a whole number, like 11.

*Outside the circle, each line gives outside piece times
whole length.*



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Q

1. **Tape the 16 cards around the room**, in any order. Each card has a large letter in its bottom left corner. Start each group at a *different* one of these eight cards: *Q M F D K C A H*.
2. **Say this once:** *“Every length starts at the point where the two lines meet. Outside the circle, add the two pieces of a line to get its whole length. Every answer is a whole number. Then walk to the card with that answer at the top.”*
3. **Each group writes one line per card.** Eight right answers bring the group back to its first card.

16 cards	pages 3–10, one copy. There are two cards on each sheet. Cut the whole stack once.
Record sheet	page 11, one per student. It shows the three rules on lettered drawings.
Answers	pages 12–13, for you only. Do not put them up in the room.

Every card shows a circle and two lines that meet at a point P, drawn to scale. A number stands beside each piece, and the student finds the piece marked x . On the ring, three cards show two chords crossing inside the circle, two show two secants from a point outside, and three show a tangent and a secant. On this sheet, a secant runs from P through the circle to the far point. Its outside piece runs from P to the near point, and its inside piece is the chord. Some textbooks use other definitions, so tell the class which ones this sheet uses.

The rule is the same inside and outside. On each line, multiply the length from P to the near point by the length from P to the far point. Both lines give the same number, called the power of the point P. Inside, those are the two pieces of a chord. Outside, they are the outside piece and the whole length, which the student must add up first. For a tangent, the tangent times itself gives that number.

Eight of the sixteen cards make the ring. A group reaches the other eight only by a wrong answer. There are three mistakes. Outside, the student multiplies the outside piece by the inside piece. With a tangent, the student forgets the square root. Inside, the student multiplies a piece of one chord by a piece of the other. Each of these lands on a card off the ring, and that card sends the group back to the card it got wrong. A clean run is **eight lines**, and each wrong answer adds two. The line in italics under each card’s question tells a group that arrived by a wrong answer what to check. Guessing does not work: no single rule with the printed numbers, such as multiplying two of them, gets more than six of the cards.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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ANSWER

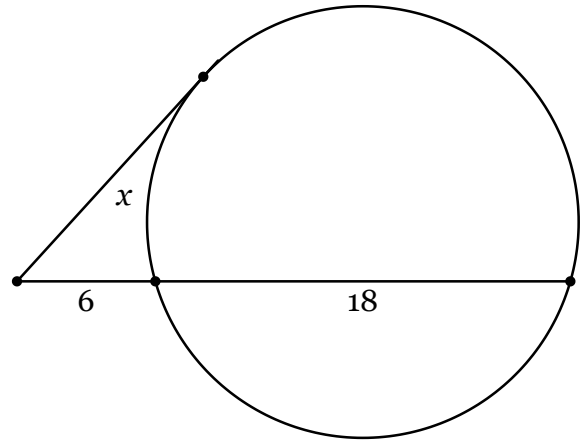
4

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

*A tangent is shorter than the other line's whole length.
Is your x shorter?*



A

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ANSWER

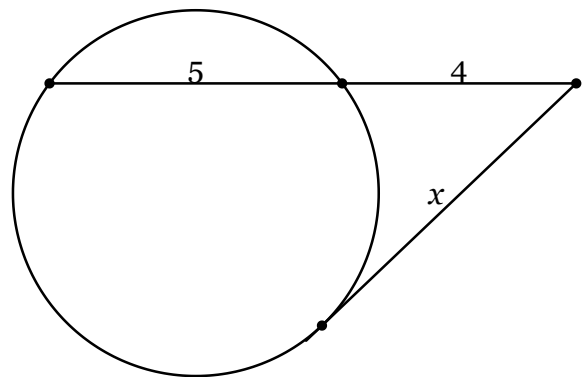
30

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Outside piece times whole length, on each line. Check both lines.



B

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ANSWER

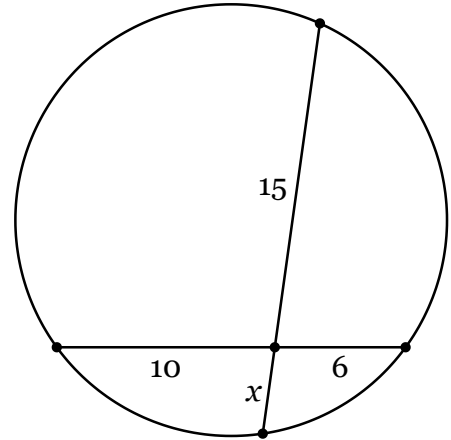
40

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Each chord makes its own product. Do the two products match?



C

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ANSWER

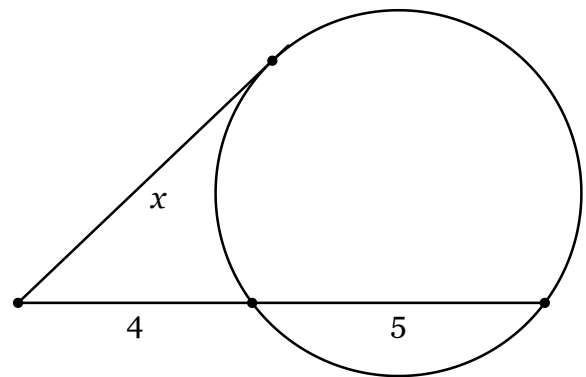
2

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

The tangent times itself makes the product. The tangent alone is x .



D

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ANSWER

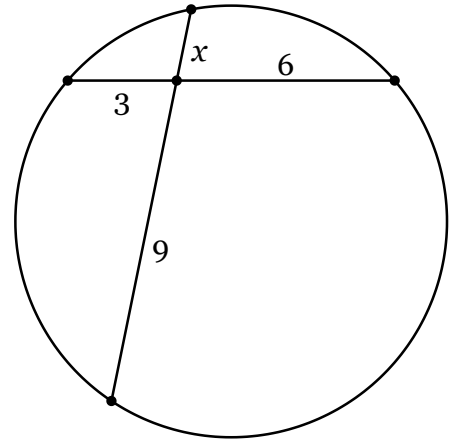
36

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Write x times x for the tangent. Then find x itself.



E

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ANSWER

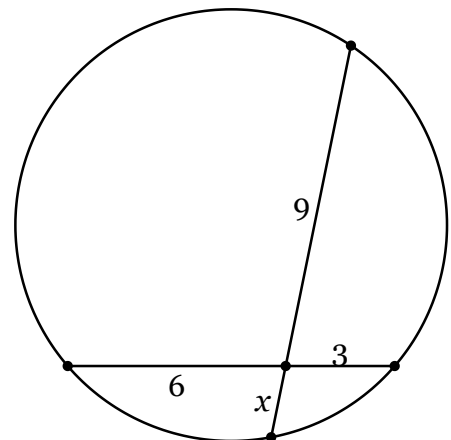
10

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Put a finger on one chord. Its two pieces go together.



F

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ANSWER

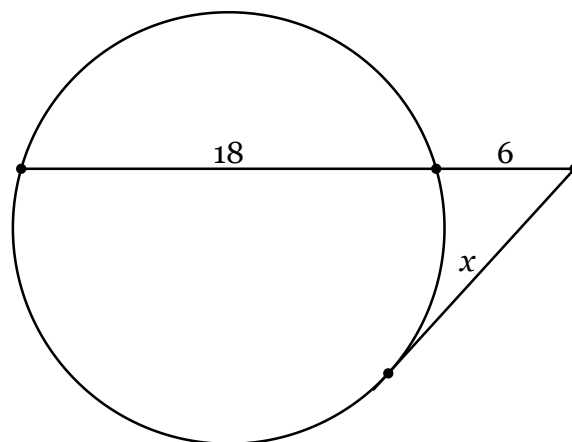
50

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Inside the circle, multiply the two pieces of one chord.



G

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ANSWER

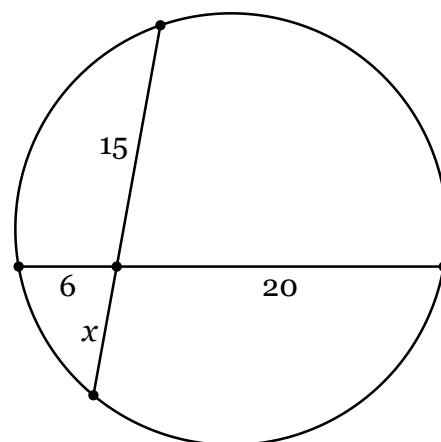
12

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Trace one chord from end to end. Multiply only the pieces on it.



H

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ANSWER

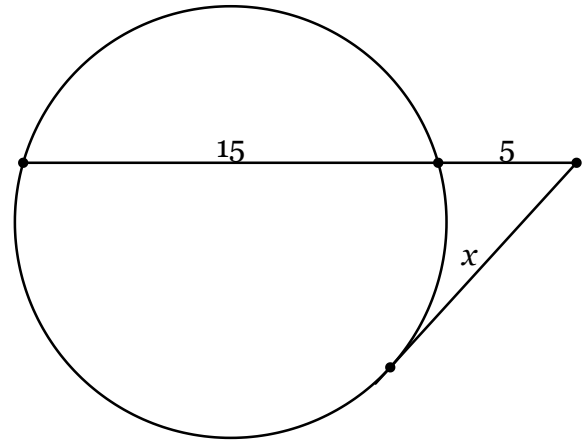
18

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Trace each chord with a pencil. Keep its two pieces together.



J

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ANSWER

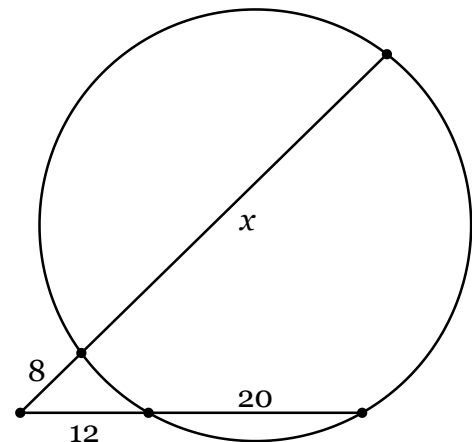
6

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Add the two pieces of a line first. Then multiply by the outside piece.



K

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ANSWER

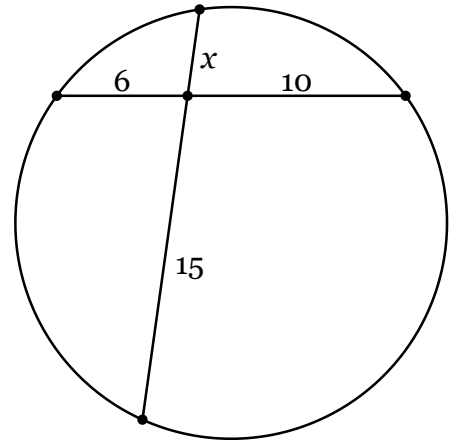
144

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

The tangent times itself is the product. Is x the tangent alone?



L

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ANSWER

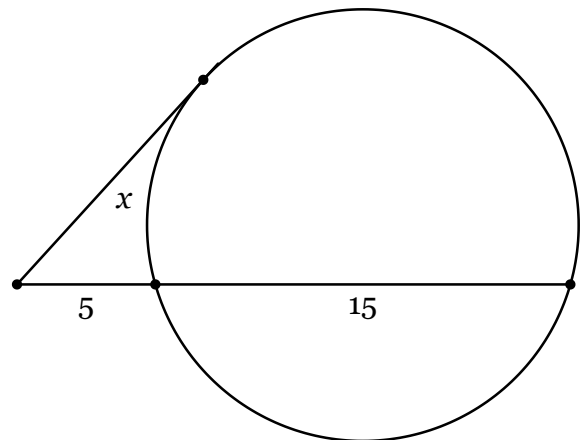
23

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Is x a length, or a length times itself? Check the square root.



M

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ANSWER

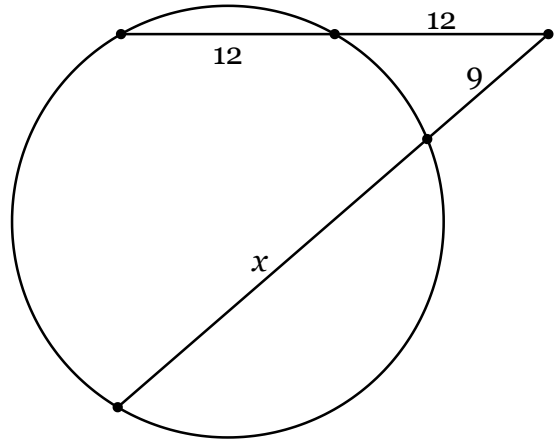
100

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

A tangent times itself gives the product. The tangent is the square root of that product.



N

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ANSWER

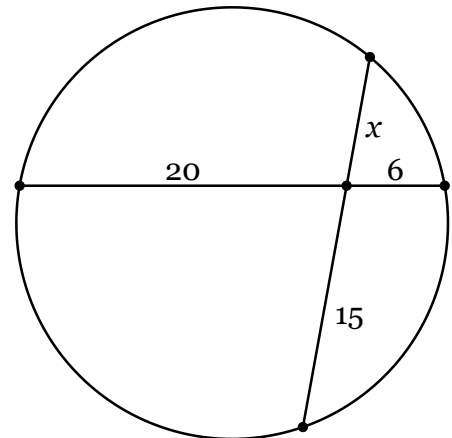
16

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

A whole length is the outside piece plus the inside piece.



P

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ANSWER

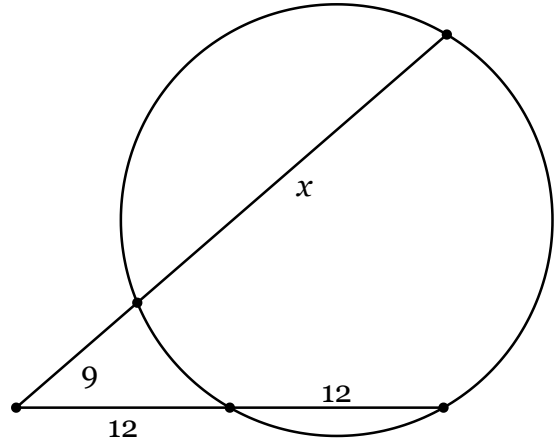
8

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Outside the circle, each line gives outside piece times whole length.



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Q

ANSWER

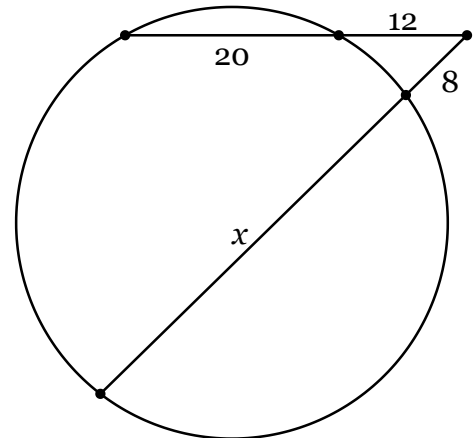
25

Each number is the length of the piece beside it.

Find x .

Write a whole number, like 11.

Which two pieces lie on one chord? Multiply those two.



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R

Scavenger Hunt — where I went

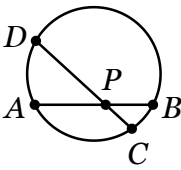
Power of a point

Name _____

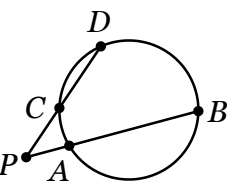
Period _____

Date _____

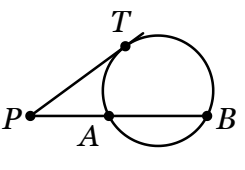
For each card, write the equation, then find x . **Every answer is a whole number, like 11.** Then find the card with your answer at the top. A clean run is eight lines. Each card has a line in italics under its question. **If you are on a card because you went wrong, its line says what to check. If you were right, skip it.**



P inside
PA · PB = PC · PD



P outside
PA · PB = PC · PD



A tangent
PT · PT = PA · PB

P is where the two lines meet. **A** and **C** are the near points. **B** and **D** are the far points. **T** is where a line touches the circle.
Every length starts at P. Outside the circle, PB is the whole length from P to B: $PB = PA + AB$.

Card letter	My equation	x
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Why is point P so strong?

Every number stands for a letter. Write the letter for each answer in the box above its number. Leave the other boxes empty. Then read the boxes you filled, left to right. **If they do not spell the answer, one of your answers is wrong.**

2 = P 4 = O 6 = W 8 = E 10 = R 12 = F 16 = O 18 = L 23 = U 25 = P 30 = R 36 = W 40 = L 50 = U 100 = F
144 = E

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

2 4 6 8 10 12 16 18 23 25 30 36 40 50 100 144



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Scavenger Hunt — answers

For the teacher

The ring in order, with the answer found on each card. A group may begin anywhere on it. Wherever it begins, eight right answers bring it back to that card.

Q (23) → M (10) → F (2) → D (6) → K (40) → C (4) → A (12) → H (8) → Q

Why is point P so strong? The answer is *POWERFUL!* It is read along the boxes of the record sheet, which run from the smallest number to the largest.

Card	What the card shows	Answer, and why	The common wrong answer on this card, and why	Letter of its top
1 Q	Two lines from P outside the circle. One has outside piece 12 and inside piece 12. The other has outside piece 9; x is its inside piece.	23. Outside the circle, each line gives outside piece $\times$ whole length. $12 \times (12 + 12) = 12 \times 24 = 288$. So $9 \times (9 + x) = 288$, $9 + x = 32$, and $x = 23$.	16. The outside piece is multiplied by the inside piece: $12 \times 12 = 9 \times x$ gives $x = 16$. The rule uses the whole length from P, $12 + 12 = 24$.	E
2 M	From P outside the circle, a line with outside piece 5 and inside piece 15, and a tangent x.	10. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 5 \times (5 + 15) = 5 \times 20 = 100$. So $x = \sqrt{100} = 10$.	100. 100 is $x \times x$, not x. The square root is left out. A tangent of 100 would be longer than the whole line from P, which is 20.	U
3 F	Two chords cross at P inside the circle. One chord has pieces 6 and 3. The other has pieces 9 and x.	2. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $6 \times 3 = 18$, so $9 \times x = 18$ and $x = 2$.	18. The pieces of the two chords are mixed: $3 \times x = 9 \times 6$ gives $x = 18$. The 3 and the x are on different chords.	R
4 D	From P outside the circle, a line with outside piece 4 and inside piece 5, and a tangent x.	6. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 4 \times (4 + 5) = 4 \times 9 = 36$. So $x = \sqrt{36} = 6$.	36. 36 is $x \times x$, not x. The square root is left out. A tangent of 36 would be longer than the whole line from P, which is 9.	P
5 K	Two lines from P outside the circle. One has outside piece 12 and inside piece 20. The other has outside piece 8; x is its inside piece.	40. Outside the circle, each line gives outside piece $\times$ whole length. $12 \times (12 + 20) = 12 \times 32 = 384$. So $8 \times (8 + x) = 384$, $8 + x = 48$, and $x = 40$.	30. The outside piece is multiplied by the inside piece: $12 \times 20 = 8 \times x$ gives $x = 30$. The rule uses the whole length from P, $12 + 20 = 32$.	W
6 C	Two chords cross at P inside the circle. One chord has pieces 10 and 6. The other has pieces 15 and x.	4. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $10 \times 6 = 60$, so $15 \times x = 60$ and $x = 4$.	25. The pieces of the two chords are mixed: $6 \times x = 15 \times 10$ gives $x = 25$. The 6 and the x are on different chords.	L
7 A	From P outside the circle, a line with outside piece 6 and inside piece 18, and a tangent x.	12. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 6 \times (6 + 18) = 6 \times 24 = 144$. So $x = \sqrt{144} = 12$.	144. 144 is $x \times x$, not x. The square root is left out. A tangent of 144 would be longer than the whole line from P, which is 24.	O
8 H	Two chords cross at P inside the circle. One chord has pieces 6 and 20. The other has pieces 15 and x.	8. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $6 \times 20 = 120$, so $15 \times x = 120$ and $x = 8$.	50. The pieces of the two chords are mixed: $6 \times x = 15 \times 20$ gives $x = 50$. The 6 and the x are on different chords.	F

Scavenger Hunt — the cards off the ring

For the teacher

A group reaches these eight cards only by a wrong answer. The number at the top of each one is the wrong answer from one ring card. Its right answer is the number at the top of that ring card, so the group goes to the card it got wrong and tries it again. If the group makes the same mistake here, it goes to another card off the ring. So a second wrong answer never moves a group forward.

Card	What the card shows	Answer, and why	The common wrong answer on this card, and why	Letter of its top
P	Two chords cross at P inside the circle. One chord has pieces 6 and 20. The other has pieces 15 and x.	8. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $6 \times 20 = 120$, so $15 \times x = 120$ and $x = 8$.	50. The pieces of the two chords are mixed: $6 \times x = 15 \times 20$ gives $x = 50$. The 6 and the x are on different chords.	O
N	Two lines from P outside the circle. One has outside piece 12 and inside piece 12. The other has outside piece 9; x is its inside piece.	23. Outside the circle, each line gives outside piece $\times$ whole length. $12 \times (12 + 12) = 12 \times 24 = 288$. So $9 \times (9 + x) = 288$, $9 + x = 32$, and $x = 23$.	16. The outside piece is multiplied by the inside piece: $12 \times 12 = 9 \times x$ gives $x = 16$. The rule uses the whole length from P, $12 + 12 = 24$.	F
J	From P outside the circle, a line with outside piece 5 and inside piece 15, and a tangent x.	10. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 5 \times (5 + 15) = 5 \times 20 = 100$. So $x = \sqrt{100} = 10$.	100. 100 is $x \times x$, not x. The square root is left out. A tangent of 100 would be longer than the whole line from P, which is 20.	L
E	Two chords cross at P inside the circle. One chord has pieces 6 and 3. The other has pieces 9 and x.	2. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $6 \times 3 = 18$, so $9 \times x = 18$ and $x = 2$.	18. The pieces of the two chords are mixed: $3 \times x = 9 \times 6$ gives $x = 18$. The 3 and the x are on different chords.	W
B	From P outside the circle, a line with outside piece 4 and inside piece 5, and a tangent x.	6. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 4 \times (4 + 5) = 4 \times 9 = 36$. So $x = \sqrt{36} = 6$.	36. 36 is $x \times x$, not x. The square root is left out. A tangent of 36 would be longer than the whole line from P, which is 9.	R
R	Two lines from P outside the circle. One has outside piece 12 and inside piece 20. The other has outside piece 8; x is its inside piece.	40. Outside the circle, each line gives outside piece $\times$ whole length. $12 \times (12 + 20) = 12 \times 32 = 384$. So $8 \times (8 + x) = 384$, $8 + x = 48$, and $x = 40$.	30. The outside piece is multiplied by the inside piece: $12 \times 20 = 8 \times x$ gives $x = 30$. The rule uses the whole length from P, $12 + 20 = 32$.	P
L	Two chords cross at P inside the circle. One chord has pieces 10 and 6. The other has pieces 15 and x.	4. Inside the circle, the two pieces of one chord multiply to the same number as the two pieces of the other. $10 \times 6 = 60$, so $15 \times x = 60$ and $x = 4$.	25. The pieces of the two chords are mixed: $6 \times x = 15 \times 10$ gives $x = 25$. The 6 and the x are on different chords.	E
G	From P outside the circle, a line with outside piece 6 and inside piece 18, and a tangent x.	12. The tangent times itself equals outside piece $\times$ whole length: $x \times x = 6 \times (6 + 18) = 6 \times 24 = 144$. So $x = \sqrt{144} = 12$.	144. 144 is $x \times x$, not x. The square root is left out. A tangent of 144 would be longer than the whole line from P, which is 24.	U

Other mistakes lead to no card. A group might give the whole length from P in place of the inside piece, add the pieces in place of multiplying them, use the outside piece times the inside piece for a tangent, or mix the chords the other way. Its answer is then not the number at the top of any card.