

CUT & GLUE SORT



ARC LENGTH AND RADIAN MEASURE

Same Measure, Same Length?

Twelve pairs of arcs · same length, different lengths, or cannot tell · three rows

FREE

¹ Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm.	² Arc 1 is 180° on a circle of radius 2 cm. Arc 2 is 1 radian on a circle of radius 4 cm.	³ Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 45° on another circle. Its radius is not given.	⁴ Arc 1 is 60° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm.
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Row 1. The two arcs have the same length.

Row 2. The two arcs have different lengths.

Row 3. Cannot tell. The words do not show if the two lengths are equal.

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Each piece gives two arcs. Find each length: the angle $\div 360^\circ \times 2\pi \times$ radius, or radians $\times$ radius. Leave π in. If no radius is given, the arcs can be compared only on the same circle. Same length, row 1. Different, row 2. If the words do not show which, row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box with two arcs as pictures.
Answers	page 5.

Each piece describes two arcs in words. The length of an arc is its angle $\div 360^\circ$, times $2\pi \times$ radius. An angle in radians gives the length at once: radians $\times$ radius. One radian is the angle whose arc is as long as the radius, so 180° is π radians. This sheet uses these words: the *measure* of an arc is its angle, in degrees, and its *length* is a distance, in cm. Some textbooks use other definitions, so tell the class which ones this sheet uses.

The main point is that the measure alone does not give the length. Two 60° arcs on circles of radius 4 cm and 6 cm have the same measure and different lengths. A 90° arc of radius 4 cm and a 60° arc of radius 6 cm have different measures and the same length, 2π cm. Many students call two arcs with the same measure “the same”. The piece with two 60° arcs and no radius tests exactly this: it goes in row 3. But two 50° arcs on the same circle are the same length, even with no radius given, so that piece goes in row 1.

Two pieces give a diameter. A 60° arc on a circle with diameter 12 cm has radius 6 cm. It matches a 60° arc of radius 6 cm, and not one of radius 12 cm. Guessing does not work: a simple guessing rule, such as “the same angle twice means the same length,” places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — same length or not?

Arc length

Name _____

Period _____

Date _____

Each piece on the next page tells you about two arcs. **Do the two arcs have the same length? Glue the piece in its row.** Leave π in your work. No calculator is needed. If the words do not show whether the two lengths are equal, the piece goes in row 3.

Row 1. The two arcs have the same length.

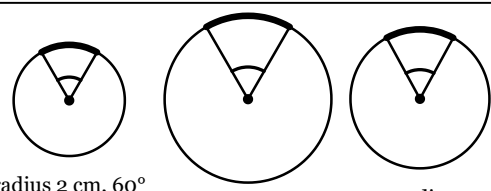
Row 2. The two arcs have different lengths.

Row 3. Cannot tell. The words do not show if the two lengths are equal.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. The box below shows two arcs with the same measure and different lengths.



radius 2 cm, 60°

radius 3 cm, 60°

1 radian

Arc measure is in degrees. **Arc length** is a distance, in cm.
 The whole circle is 360°. The whole way around is $2\pi \times \text{radius}$. We use this today. Why it is true comes later in the course.
 An arc is a fraction of the circle. Its fraction is its angle $\div 360^\circ$.
Radian measure of an angle = arc length $\div$ radius. It has no unit. π radians = 180°.
 The diameter is twice the radius.

1 Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm.	2 Arc 1 is 180° on a circle of radius 2 cm. Arc 2 is 1 radian on a circle of radius 4 cm.	3 Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 45° on another circle. Its radius is not given.	4 Arc 1 is 60° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm.
5 Arc 1 is 2 radians on a circle of radius 3 cm. Arc 2 is 3 radians on a circle of radius 2 cm.	6 Arc 1 is 60°. Arc 2 is 60°.	7 Arc 1 is 60° on a circle with diameter 12 cm. Arc 2 is 60° on a circle of radius 6 cm.	8 Arc 1 is 5 cm long, on a circle of radius 2 cm. Arc 2 is 2.5 radians on another circle. Its radius is not given.
9 Arc 1 is 40° on a circle. Arc 2 is 80° on the same circle.	10 Arc 1 is 6 cm long. Arc 2 is 1 radian.	11 Arc 1 is 50° on a circle. Arc 2 is 50° on the same circle.	12 Arc 1 is 60° on a circle with diameter 12 cm. Arc 2 is 60° on a circle of radius 12 cm.



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Every piece is listed by the number printed on its corner, with the reason. Each row holds four pieces. The length of an arc is its angle $\div 360^\circ$, times $2\pi \times$ radius, or its angle in radians times the radius.

1 The two arcs have the same length.

1. Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm. *Why:* Arc 1 is 2π cm long and arc 2 is 2π cm long. They are equal.

5. Arc 1 is 2 radians on a circle of radius 3 cm. Arc 2 is 3 radians on a circle of radius 2 cm. *Why:* Arc 1 is 6 cm long (2×3) and arc 2 is 6 cm long (3×2). They are equal.

7. Arc 1 is 60° on a circle with diameter 12 cm. Arc 2 is 60° on a circle of radius 6 cm. *Why:* The radius is half the diameter. Arc 1 is 2π cm long and arc 2 is 2π cm long. They are equal.

11. Arc 1 is 50° on a circle. Arc 2 is 50° on the same circle. *Why:* Both arcs are 50° on one circle, so each is the same fraction of the same circle. The lengths are equal, whatever the radius.

2 The two arcs have different lengths.

2. Arc 1 is 180° on a circle of radius 2 cm. Arc 2 is 1 radian on a circle of radius 4 cm. *Why:* Arc 1 is 2π cm long and arc 2 is 4 cm long (1×4), and 2π is about 6.28 cm. They are not equal.

4. Arc 1 is 60° on a circle of radius 4 cm. Arc 2 is 60° on a circle of radius 6 cm. *Why:* Arc 1 is $4\pi/3$ cm long and arc 2 is 2π cm long. They are not equal.

9. Arc 1 is 40° on a circle. Arc 2 is 80° on the same circle. *Why:* On one circle, 80° is twice 40° , so arc 2 is twice as long as arc 1, whatever the radius.

12. Arc 1 is 60° on a circle with diameter 12 cm. Arc 2 is 60° on a circle of radius 12 cm. *Why:* The radius is half the diameter. Arc 1 is 2π cm long and arc 2 is 4π cm long. They are not equal.

3 Cannot tell. The words do not show if the two lengths are equal.

3. Arc 1 is 90° on a circle of radius 4 cm. Arc 2 is 45° on another circle. Its radius is not given. *Why:* No radius is given for arc 2, and the length of an arc depends on its radius. With one radius the two lengths are equal, with another they are not.

6. Arc 1 is 60° . Arc 2 is 60° . *Why:* No radius is given for arc 1 or arc 2, and the length of an arc depends on its radius. With one radius the two lengths are equal, with another they are not.

8. Arc 1 is 5 cm long, on a circle of radius 2 cm. Arc 2 is 2.5 radians on another circle. Its radius is not given. *Why:* No radius is given for arc 2, and the length of an arc depends on its radius. With one radius the two lengths are equal, with another they are not.

10. Arc 1 is 6 cm long. Arc 2 is 1 radian. *Why:* No radius is given for arc 2, and the length of an arc depends on its radius. With one radius the two lengths are equal, with another they are not.

To check a piece with a student, ask: “What is the radius of each arc? Is it given, or is it the same circle? What fraction of its circle is each arc?”