

CUT & GLUE SORT



AREA OF A SECTOR

Same Area? Sectors That Look Alike

Twelve pairs of sectors · same area, different areas, or cannot tell · three rows

FREE

1 Sector 1 is 60° on a circle of radius 6 cm. Sector 2 is 120° on a circle of radius 3 cm.	2 Sector 1 is 40° on a circle with diameter 12 cm. Sector 2 is 160° on a circle of radius 3 cm.	3 Sector 1 has an arc 4π cm long. Sector 2 has an arc 4π cm long.	4 Sector 1 has an area of 6π cm ² . Sector 2 is 60° on a circle of radius 6 cm.
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Row 1. The two sectors have the same area.

Row 2. The two sectors have different areas.

Row 3. Cannot tell. The words do not show if the two areas are equal.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Each piece gives two sectors. Find each area as a fraction of $\pi \times \text{radius} \times \text{radius}$, and leave π in. Same area, row 1. Different, row 2. If the words do not show which, row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows a 90° sector and the 270° sector that is the rest of the same circle.

Answers page 5.

Each piece describes two sectors in words. The area of a sector is its angle $\div 360^\circ$, times $\pi \times \text{radius} \times \text{radius}$. It is also half its arc length times the radius. This sheet uses this definition: a *sector* is the part of a circle between two radii and one of their arcs. Some textbooks use other definitions, so tell the class which ones this sheet uses.

The main point is that an area does not grow like a length. A 60° sector of radius 6 cm and a 120° sector of radius 3 cm have arcs of the same length, 2π cm. But their areas are $6\pi \text{ cm}^2$ and $3\pi \text{ cm}^2$, because the radius is multiplied by itself. A 30° sector of radius 6 cm and a 120° sector of radius 3 cm have the same area, $3\pi \text{ cm}^2$. Many students think equal arcs mean equal areas. The piece with two arcs of 4π cm and no radius tests exactly this: it goes in row 3. But two 100° sectors on the same circle have the same area, even with no radius given, so that piece goes in row 1.

Two pieces give a diameter. A 40° sector of a circle with diameter 12 cm has radius 6 cm, so its area is $4\pi \text{ cm}^2$, the same as a 160° sector of radius 3 cm. Guessing does not work: a simple guessing rule, such as “the same angle twice means the same area,” places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — same area or not?

Area of a sector

Name _____

Period _____

Date _____

Each piece on the next page tells you about two sectors. **Do the two sectors have the same area? Glue the piece in its row.** Leave π in your work. No calculator is needed. If the words do not show whether the two areas are equal, the piece goes in row 3.

Row 1. The two sectors have the same area.

Row 2. The two sectors have different areas.

Row 3. Cannot tell. The words do not show if the two areas are equal.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. The box below shows a sector and the other sector of the same circle.



a 90° sector



the other sector, 270°

A **sector** is the part of a circle between two radii and one of their arcs.

The whole circle is 360°. Its area is $\pi \times \text{radius} \times \text{radius}$. We use this today. Why it is true comes later in the course.

A sector is a fraction of the circle. Its fraction is its angle $\div 360^\circ$.

An area is in square units, such as cm^2 . A length is in cm.

The diameter is twice the radius.

1 Sector 1 is 60° on a circle of radius 6 cm. Sector 2 is 120° on a circle of radius 3 cm.	2 Sector 1 is 40° on a circle with diameter 12 cm. Sector 2 is 160° on a circle of radius 3 cm.	3 Sector 1 has an arc 4π cm long. Sector 2 has an arc 4π cm long.	4 Sector 1 has an area of $6\pi \text{ cm}^2$. Sector 2 is 60° on a circle of radius 6 cm.
5 Sector 1 is 90° on a circle of radius 4 cm. Sector 2 is 45° on another circle. Its radius is not given.	6 Sector 1 is 100° on a circle. Sector 2 is 260° on the same circle.	7 Sector 1 is 60°. Sector 2 is 60°.	8 Sector 1 is 90° on a circle with diameter 8 cm. Sector 2 is 90° on a circle of radius 8 cm.
9 Sector 1 is 30° on a circle of radius 6 cm. Sector 2 is 120° on a circle of radius 3 cm.	10 Sector 1 is 40° on a circle of radius 9 cm. Sector 2 is 90° on a circle of radius 4 cm.	11 Sector 1 has an area of $9\pi \text{ cm}^2$. Sector 2 is 90° on another circle. Its radius is not given.	12 Sector 1 is 100° on a circle. Sector 2 is 100° on the same circle.



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Every piece is listed by the number printed on its corner, with the reason. Each row holds four pieces. The area of a sector is its angle $\div 360^\circ$, times $\pi \times \text{radius} \times \text{radius}$.

1 **The two sectors have the same area.**

2. Sector 1 is 40° on a circle with diameter 12 cm. Sector 2 is 160° on a circle of radius 3 cm. *Why:* The radius is half the diameter. Sector 1 is $40/360 \times \pi \times 6 \times 6 = 4\pi \text{ cm}^2$, and sector 2 is $160/360 \times \pi \times 3 \times 3 = 4\pi \text{ cm}^2$. They are equal. The arcs are different ($4\pi/3 \text{ cm}$ and $8\pi/3 \text{ cm}$), but the areas are equal.

4. Sector 1 has an area of $6\pi \text{ cm}^2$. Sector 2 is 60° on a circle of radius 6 cm. *Why:* Sector 1 has area $6\pi \text{ cm}^2$, and sector 2 is $60/360 \times \pi \times 6 \times 6 = 6\pi \text{ cm}^2$. They are equal.

9. Sector 1 is 30° on a circle of radius 6 cm. Sector 2 is 120° on a circle of radius 3 cm. *Why:* Sector 1 is $30/360 \times \pi \times 6 \times 6 = 3\pi \text{ cm}^2$, and sector 2 is $120/360 \times \pi \times 3 \times 3 = 3\pi \text{ cm}^2$. They are equal. The arcs are different ($\pi \text{ cm}$ and $2\pi \text{ cm}$), but the areas are equal.

12. Sector 1 is 100° on a circle. Sector 2 is 100° on the same circle. *Why:* Both sectors are 100° on one circle, so each is the same fraction of the same circle. The areas are equal, whatever the radius.

2 **The two sectors have different areas.**

1. Sector 1 is 60° on a circle of radius 6 cm. Sector 2 is 120° on a circle of radius 3 cm. *Why:* Sector 1 is $60/360 \times \pi \times 6 \times 6 = 6\pi \text{ cm}^2$, and sector 2 is $120/360 \times \pi \times 3 \times 3 = 3\pi \text{ cm}^2$. They are not equal. The two arcs are the same length, $2\pi \text{ cm}$, but the areas are not.

6. Sector 1 is 100° on a circle. Sector 2 is 260° on the same circle. *Why:* On one circle, 100° is $100/360$ of the circle and 260° is $260/360$ of it, so the areas are not equal, whatever the radius.

8. Sector 1 is 90° on a circle with diameter 8 cm. Sector 2 is 90° on a circle of radius 8 cm. *Why:* The radius is half the diameter. Sector 1 is $90/360 \times \pi \times 4 \times 4 = 4\pi \text{ cm}^2$, and sector 2 is $90/360 \times \pi \times 8 \times 8 = 16\pi \text{ cm}^2$. They are not equal.

10. Sector 1 is 40° on a circle of radius 9 cm. Sector 2 is 90° on a circle of radius 4 cm. *Why:* Sector 1 is $40/360 \times \pi \times 9 \times 9 = 9\pi \text{ cm}^2$, and sector 2 is $90/360 \times \pi \times 4 \times 4 = 4\pi \text{ cm}^2$. They are not equal. The two arcs are the same length, $2\pi \text{ cm}$, but the areas are not.

3 **Cannot tell. The words do not show if the two areas are equal.**

3. Sector 1 has an arc $4\pi \text{ cm}$ long. Sector 2 has an arc $4\pi \text{ cm}$ long. *Why:* The area of a sector is half its arc times its radius. No radius is given, so equal arcs can belong to sectors of different areas.

5. Sector 1 is 90° on a circle of radius 4 cm. Sector 2 is 45° on another circle. Its radius is not given. *Why:* No radius is given for sector 2, and the area of a sector depends on its radius. With one radius the two areas are equal, with another they are not.

7. Sector 1 is 60° . Sector 2 is 60° . *Why:* No radius is given for sector 1 or sector 2, and the area of a sector depends on its radius. With one radius the two areas are equal, with another they are not.

11. Sector 1 has an area of $9\pi \text{ cm}^2$. Sector 2 is 90° on another circle. Its radius is not given. *Why:* No radius is given for sector 2, and the area of a sector depends on its radius. With one radius the two areas are equal, with another they are not.

To check a piece with a student, ask: “What is the radius of each sector? Is it given, or is it the same circle? What fraction of its circle is each sector?”