

MYSTERY PICTURE



AREA OF A SECTOR

Area or Length? Sectors and Arcs

25 workings · area or arc length · no calculator · a picture appears

FREE

1 The radius is 9 cm. $60/360 \times 2 \times \pi \times 9$	2 The radius is 6 cm. $60/360 \times \pi \times 6 \times 6$	3 The radius is 6 cm. $30/360 \times 36\pi$	4 The radius is 6 cm. The arc is 4π cm long. $\frac{1}{2} \times 4\pi \times 6$	5 The radius is 9 cm. $120/360 \times \pi \times 9^2$
6 The radius is 8 cm. $90/360 \times 16\pi$	7 The radius is 7 cm. $\frac{1}{2} \times 2 \times \pi \times 7$	8 The diameter is 12 cm. $135/360 \times \pi \times 12$	9 The diameter is 10 cm. $72/360 \times \pi \times 5^2$	10 The radius is 3 cm. $200/360 \times 9\pi$
11 The radius is 9 cm. The angle is $\pi/3$ radians. $\pi/3 \times 9$	12 The radius is 9 cm. $40/360 \times 18\pi$	13 The radius is 12 cm. $\pi \times 12 \times 12 \div 4$	14 The radius is 6 cm. The angle is $\pi/2$ radians. $\pi/2 \times 6$	15 The radius is 9 cm. $80/360 \times 81\pi$
16 The radius is 8 cm. $2 \times \pi \times 8 \div 4$	17 The radius is 6 cm. The angle is $\pi/3$ radians. $\frac{1}{2} \times 6 \times 6 \times \pi/3$	18 The radius is 5 cm. $300/360 \times 10\pi$	19 The radius is 4 cm. $150/360 \times 2 \times \pi \times 4$	20 The radius is 9 cm. The arc is 3π cm long. $\frac{1}{2} \times 3\pi \times 9$
21 The radius is 4 cm. $270/360 \times 16\pi$	22 The radius is 6 cm. $120/360 \times 2 \times \pi \times 6$	23 The diameter is 10 cm. $\frac{1}{2} \times \pi \times 10$	24 The diameter is 14 cm. $100/360 \times \pi \times 14$	25 The radius is 10 cm. $45/360 \times 20\pi$

1. **Give each student page 3** and a pencil. No calculator is needed.
2. **Say this once:** *“Each square shows someone’s working for a sector. It finds the area of the sector, or the length of its arc. An area multiplies a length by a length, like radius $\times$ radius. If the working finds the area, fill the square with slanted lines. If it finds a length, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows a 90° sector and the 270° sector that is the rest of the same circle.
Answers	pages 4–5, for you.

Each square shows a circle with a sector filled with slanted lines, the radius or the diameter, and one line of working. Every working is right, but some find the area of the sector and some find the length of its arc. An area is in square units, and its working multiplies a length by a length: the radius by itself, or half the arc by the radius. A length uses the radius only once, as in $2 \times \pi \times \text{radius}$. Three squares give the angle in radians. Then the arc is $\text{angle} \times \text{radius}$, a length, and the area is $\frac{1}{2} \times \text{radius} \times \text{radius} \times \text{angle}$. This sheet uses this definition: a *sector* is the part of a circle between two radii and one of their arcs. Some textbooks use other definitions, so tell the class which ones this sheet uses. Eleven squares are filled.

The hardest squares show one number times π . In eight squares the working is a fraction times a number such as 36π . The student must check that number against the radius: with a radius of 6 cm, 36π is $\pi \times 6 \times 6$, the area of the circle, and 12π is $2 \times \pi \times 6$, the distance around. Students need to multiply two small numbers in their heads, but they never work out an answer.

If part of a student’s picture looks wrong, the mistake is in that square. Ask: *“Does this working multiply a length by a length?”* Guessing does not work. Every simple guessing rule, such as filling every square that shows a 2 , gets at least six squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture — area or length?

Area of a sector

Name _____

Period _____

Date _____

Each square shows someone's working for the striped sector. The working finds the area of the sector, or the length of its arc. **Does the working find the area? If yes, fill the square with slanted lines. If it finds a length, leave it white.** No calculator is needed. A picture appears. If part of it is missing, check that square again.

 a 90° sector	 the other sector, 270°	A sector is the part of a circle between two radii and one of their arcs. The whole circle is 360°. Its area is $\pi \times \text{radius} \times \text{radius}$. We use this today. Why it is true comes later in the course. An area is in square units, such as cm^2. A length is in cm. An area's working multiplies a length by a length, like $\text{radius} \times \text{radius}$. An angle can be in radians. $\pi \text{ radians} = 180^\circ$.
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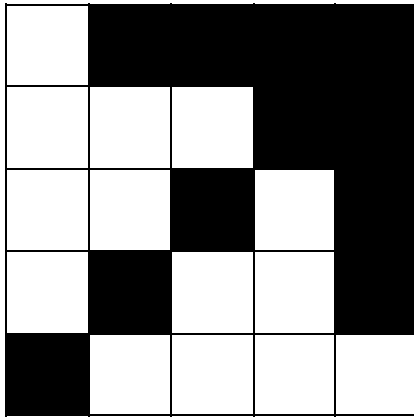
1  The radius is 9 cm. 60/360 × 2 × π × 9	2  The radius is 6 cm. 60/360 × π × 6 × 6	3  The radius is 6 cm. 30/360 × 36π	4  The radius is 6 cm. The arc is 4π cm long. 1/2 × 4π × 6	5  The radius is 9 cm. 120/360 × π × 9²
6  The radius is 8 cm. 90/360 × 16π	7  The radius is 7 cm. 1/2 × 2 × π × 7	8  The diameter is 12 cm. 135/360 × π × 12	9  The diameter is 10 cm. 72/360 × π × 5²	10  The radius is 3 cm. 200/360 × 9π
11  The radius is 9 cm. The angle is π/3 radians. π/3 × 9	12  The radius is 9 cm. 40/360 × 18π	13  The radius is 12 cm. π × 12 × 12 ÷ 4	14  The radius is 6 cm. The angle is π/2 radians. π/2 × 6	15  The radius is 9 cm. 80/360 × 81π
16  The radius is 8 cm. 2 × π × 8 ÷ 4	17  The radius is 6 cm. The angle is π/3 radians. 1/2 × 6 × 6 × π/3	18  The radius is 5 cm. 300/360 × 10π	19  The radius is 4 cm. 150/360 × 2 × π × 4	20  The radius is 9 cm. The arc is 3π cm long. 1/2 × 3π × 9
21  The radius is 4 cm. 270/360 × 16π	22  The radius is 6 cm. 120/360 × 2 × π × 6	23  The diameter is 10 cm. 1/2 × π × 10	24  The diameter is 14 cm. 100/360 × π × 14	25  The radius is 10 cm. 45/360 × 20π



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Mystery Picture — answers

For the teacher



The finished picture is **an up-right arrow**. Eleven squares are filled.

A square is filled when its working finds the area of the sector: it multiplies a length by a length. A white square's working finds the length of the arc. The table gives the reason for every square. It goes on over the page.

	The square says	Answer	Why
1	The radius is 9 cm. Working: $60/360 \times 2 \times \pi \times 9$	white	$2 \times \pi \times 9$ is the distance around the circle, a length. The working finds the arc's length, 3π cm.
2	The radius is 6 cm. Working: $60/360 \times \pi \times 6 \times 6$	fill	$\pi \times 6 \times 6$ is the area of the whole circle, $\pi \times \text{radius} \times \text{radius}$. The working takes $60/360$ of it: the area, 6π cm ² .
3	The radius is 6 cm. Working: $30/360 \times 36\pi$	fill	36π is $\pi \times 6 \times 6$, the area of the whole circle. The working takes $30/360$ of it: the area, 3π cm ² .
4	The radius is 6 cm. The arc is 4π cm long. Working: $\frac{1}{2} \times 4\pi \times 6$	fill	Half the arc (4π cm) times the radius (6 cm) is a length times a length, so it is an area: 12π cm ² , which is $120/360$ of the circle.
5	The radius is 9 cm. Working: $120/360 \times \pi \times 9^2$	fill	$\pi \times 9^2$ is the area of the whole circle. The working takes $120/360$ of it: the area, 27π cm ² .
6	The radius is 8 cm. Working: $90/360 \times 16\pi$	white	16π is $2 \times \pi \times 8$, the distance around the circle, a length. The working finds the arc's length, 4π cm. (The area would use $\pi \times 8 \times 8 = 64\pi$.)
7	The radius is 7 cm. Working: $\frac{1}{2} \times 2 \times \pi \times 7$	white	$2 \times \pi \times 7$ is the distance around the circle, and half of it is the arc of a half circle: a length, 7π cm.
8	The diameter is 12 cm. Working: $135/360 \times \pi \times 12$	white	$\pi \times 12$ ($\pi \times \text{diameter}$) is the distance around the circle, a length. The working finds the arc's length, $9\pi/2$ cm.
9	The diameter is 10 cm. Working: $72/360 \times \pi \times 5^2$	fill	The radius is half the diameter, 5 cm, and $\pi \times 5^2$ is the area of the whole circle. The working takes $72/360$ of it: the area, 5π cm ² .
10	The radius is 3 cm. Working: $200/360 \times 9\pi$	fill	9π is $\pi \times 3 \times 3$, the area of the whole circle. The working takes $200/360$ of it: the area, 5π cm ² .
11	The radius is 9 cm. The angle is $\pi/3$ radians. Working: $\pi/3 \times 9$	white	Angle $\times$ radius, with the angle in radians, is the length of the arc: 3π cm. Nothing multiplies the radius by itself.
12	The radius is 9 cm. Working: $40/360 \times 18\pi$	white	18π is $2 \times \pi \times 9$, the distance around the circle, a length. The working finds the arc's length, 2π cm. (The area would use $\pi \times 9 \times 9 = 81\pi$.)

Mystery Picture — answers, squares 13 to 25

For the teacher

	The square says	Answer	Why
13	The radius is 12 cm. Working: $\pi \times 12 \times 12 \div 4$	fill	$\pi \times 12 \times 12$ is the area of the whole circle, and a 90° sector is a quarter of it: the area, $36\pi \text{ cm}^2$.
14	The radius is 6 cm. The angle is $\pi/2$ radians. Working: $\pi/2 \times 6$	white	Angle $\times$ radius, with the angle in radians, is the length of the arc: $3\pi \text{ cm}$. Nothing multiplies the radius by itself.
15	The radius is 9 cm. Working: $80/360 \times 81\pi$	fill	81π is $\pi \times 9 \times 9$, the area of the whole circle. The working takes $80/360$ of it: the area, $18\pi \text{ cm}^2$.
16	The radius is 8 cm. Working: $2 \times \pi \times 8 \div 4$	white	$2 \times \pi \times 8$ is the distance around the circle, a length. The working finds the arc's length, $4\pi \text{ cm}$.
17	The radius is 6 cm. The angle is $\pi/3$ radians. Working: $\frac{1}{2} \times 6 \times 6 \times \pi/3$	fill	$\frac{1}{2} \times \text{radius} \times \text{radius} \times \text{angle}$, with the angle in radians, is the area of a sector: $6\pi \text{ cm}^2$. The radius is multiplied by itself.
18	The radius is 5 cm. Working: $300/360 \times 10\pi$	white	10π is $2 \times \pi \times 5$, the distance around the circle, a length. The working finds the arc's length, $25\pi/3 \text{ cm}$. (The area would use $\pi \times 5 \times 5 = 25\pi$.)
19	The radius is 4 cm. Working: $150/360 \times 2 \times \pi \times 4$	white	$2 \times \pi \times 4$ is the distance around the circle, a length. The working finds the arc's length, $10\pi/3 \text{ cm}$.
20	The radius is 9 cm. The arc is $3\pi \text{ cm}$ long. Working: $\frac{1}{2} \times 3\pi \times 9$	fill	Half the arc ($3\pi \text{ cm}$) times the radius (9 cm) is a length times a length, so it is an area: $27\pi/2 \text{ cm}^2$, which is $60/360$ of the circle.
21	The radius is 4 cm. Working: $270/360 \times 16\pi$	fill	16π is $\pi \times 4 \times 4$, the area of the whole circle. The working takes $270/360$ of it: the area, $12\pi \text{ cm}^2$.
22	The radius is 6 cm. Working: $120/360 \times 2 \times \pi \times 6$	white	$2 \times \pi \times 6$ is the distance around the circle, a length. The working finds the arc's length, $4\pi \text{ cm}$.
23	The diameter is 10 cm. Working: $\frac{1}{2} \times \pi \times 10$	white	$\pi \times 10$ is the distance around the circle, and half of it is the arc of a half circle: a length, $5\pi \text{ cm}$.
24	The diameter is 14 cm. Working: $100/360 \times \pi \times 14$	white	$\pi \times 14$ ($\pi \times \text{diameter}$) is the distance around the circle, a length. The working finds the arc's length, $35\pi/9 \text{ cm}$.
25	The radius is 10 cm. Working: $45/360 \times 20\pi$	white	20π is $2 \times \pi \times 10$, the distance around the circle, a length. The working finds the arc's length, $5\pi/2 \text{ cm}$. (The area would use $\pi \times 10 \times 10 = 100\pi$.)