

MYSTERY PICTURE



PARTITIONING A DIRECTED LINE SEGMENT

Partitioning a Segment: Which Rule Finds P?

25 rules · the ratio turned into steps · no calculator · a picture appears

FREE

¹ $AP : PB = 1 : 3$ $P = A + \frac{1}{4}(B - A)$	² $AP : PB = 4 : 3$ $P = A + \frac{4}{7}(B - A)$	³ $AP : PB = 5 : 4$ $P = B + \frac{4}{9}(A - B)$	⁴ $BP : PA = 5 : 2$ $P = A + \frac{2}{7}(B - A)$	⁵ $AP : PB = 3 : 5$ $P = \frac{3}{8} \cdot B + \frac{5}{8} \cdot A$
⁶ $AP : PB = 1 : 4$ $P = A + \frac{4}{5}(B - A)$	⁷ $BP : PA = 2 : 5$ $P = B + \frac{2}{5}(A - B)$	⁸ $AP : PB = 4 : 1$ $P = B + \frac{4}{5}(A - B)$	⁹ $AP : PB = 5 : 4$ $P = \frac{4}{9} \cdot A + \frac{5}{9} \cdot B$	¹⁰ $AP : PB = 4 : 5$ $P = A + \frac{4}{9}(B - A)$
¹¹ $BP : PA = 3 : 5$ $P = \frac{5}{8} \cdot A + \frac{3}{8} \cdot B$	¹² $AP : PB = 5 : 1$ $P = B + \frac{5}{6}(A - B)$	¹³ $BP : PA = 2 : 3$ $P = \frac{2}{5} \cdot B + \frac{3}{5} \cdot A$	¹⁴ $BP : PA = 1 : 3$ $P = A + \frac{3}{4}(B - A)$	¹⁵ $AP : PB = 3 : 2$ $P = A + \frac{3}{5}(B - A)$
¹⁶ $AP : PB = 3 : 1$ $P = B + \frac{3}{4}(A - B)$	¹⁷ $BP : PA = 1 : 4$ $P = A + \frac{1}{5}(B - A)$	¹⁸ $AP : PB = 3 : 4$ $P = \frac{4}{7} \cdot A + \frac{3}{7} \cdot B$	¹⁹ $AP : PB = 1 : 4$ $P = A + \frac{1}{5}(B - A)$	²⁰ $AP : PB = 1 : 3$ $P = A + \frac{1}{3}(B - A)$
²¹ $AP : PB = 5 : 3$ $P = \frac{3}{8} \cdot B + \frac{5}{8} \cdot A$	²² $BP : PA = 5 : 2$ $P = \frac{5}{7} \cdot A + \frac{2}{7} \cdot B$	²³ $AP : PB = 2 : 5$ $P = B + \frac{5}{7}(A - B)$	²⁴ $BP : PA = 4 : 3$ $P = A + \frac{4}{7}(B - A)$	²⁵ $AP : PB = 2 : 3$ $P = A + \frac{3}{5}(B - A)$

1. **Give each student page 3** and a pencil. No calculator is needed.
2. **Say this once:** *“Each square gives a ratio and a rule for P. The first letter of the ratio is the start. If the rule is correct, fill the square with slanted lines. If not, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet page 3, one per student. A box at the top shows a segment cut into equal steps.
Answers pages 4–5, for you.

Each square gives a ratio and one rule for the point P, written with the letters only. On this sheet $AP : PB = 2 : 1$ means the trip starts at A and P is $2/3$ of the way from A to B. Some textbooks use other definitions, so tell the class which ones this sheet uses. Eight squares start at B, such as $BP : PA = 1 : 3$.

Thirteen squares are filled, and the rule comes in three correct forms. From the start: $P = A + 1/4(B - A)$ for $AP : PB = 1 : 3$. From the other end: $P = B + 3/4(A - B)$. As weights: $P = 3/4 \cdot A + 1/4 \cdot B$. The point is nearer A, so A gets the larger weight. Students find the weights hard, because they are the ratio turned around.

The finished picture is the number 7. Each of the twelve white squares has one common mistake. In two the ratio is read as a fraction, such as $1 : 4$ read as $1/4$ instead of $1/5$. In two the start gets the wrong fraction. In five the rule starts at the wrong end. In three the weights are the wrong way round.

If part of a student's picture looks wrong, ask: *“Where does the trip start? How many equal steps are there? Which end is P nearer?”* Guessing does not work. Every simple guessing rule, such as filling every square whose fraction has the two ratio numbers added at the bottom, gets at least nine squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture — is the rule for P right?

Partitioning a segment

Name _____

Period _____

Date _____

Each square gives a ratio and a rule for the point P . The first letter of the ratio is the start, so $BP : PA$ starts at B . **If the rule is correct, fill the square with slanted lines. If not, leave it white.** A picture appears. If part of it is missing, or one square does not fit, check that square again.

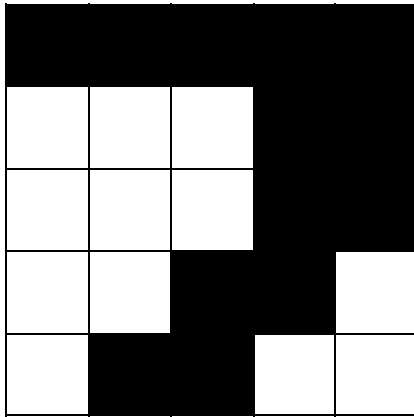
 $AP : PB = 2 : 1$ means: start at A. Cut the trip from A to B into $2 + 1 = 3$ equal steps. P is 2 steps from A, so P is $\frac{2}{3}$ of the way from A to B. So $P = A + \frac{2}{3}(B - A)$. From the other end, P is 1 step of 3 from B, so $P = B + \frac{1}{3}(A - B)$. As weights, $P = \frac{1}{3} \cdot A + \frac{2}{3} \cdot B$. P is nearer B, so B gets the larger weight. The two weights add to 1.				
1 $AP : PB = 1 : 3$ $P = A + \frac{1}{4}(B - A)$	2 $AP : PB = 4 : 3$ $P = A + \frac{4}{7}(B - A)$	3 $AP : PB = 5 : 4$ $P = B + \frac{4}{9}(A - B)$	4 $BP : PA = 5 : 2$ $P = A + \frac{2}{7}(B - A)$	5 $AP : PB = 3 : 5$ $P = \frac{3}{8} \cdot B + \frac{5}{8} \cdot A$
6 $AP : PB = 1 : 4$ $P = A + \frac{4}{5}(B - A)$	7 $BP : PA = 2 : 5$ $P = B + \frac{2}{5}(A - B)$	8 $AP : PB = 4 : 1$ $P = B + \frac{4}{5}(A - B)$	9 $AP : PB = 5 : 4$ $P = \frac{4}{9} \cdot A + \frac{5}{9} \cdot B$	10 $AP : PB = 4 : 5$ $P = A + \frac{4}{9}(B - A)$
11 $BP : PA = 3 : 5$ $P = \frac{5}{8} \cdot A + \frac{3}{8} \cdot B$	12 $AP : PB = 5 : 1$ $P = B + \frac{5}{6}(A - B)$	13 $BP : PA = 2 : 3$ $P = \frac{2}{5} \cdot B + \frac{3}{5} \cdot A$	14 $BP : PA = 1 : 3$ $P = A + \frac{3}{4}(B - A)$	15 $AP : PB = 3 : 2$ $P = A + \frac{3}{5}(B - A)$
16 $AP : PB = 3 : 1$ $P = B + \frac{3}{4}(A - B)$	17 $BP : PA = 1 : 4$ $P = A + \frac{1}{5}(B - A)$	18 $AP : PB = 3 : 4$ $P = \frac{4}{7} \cdot A + \frac{3}{7} \cdot B$	19 $AP : PB = 1 : 4$ $P = A + \frac{1}{5}(B - A)$	20 $AP : PB = 1 : 3$ $P = A + \frac{1}{3}(B - A)$
21 $AP : PB = 5 : 3$ $P = \frac{3}{8} \cdot B + \frac{5}{8} \cdot A$	22 $BP : PA = 5 : 2$ $P = \frac{5}{7} \cdot A + \frac{2}{7} \cdot B$	23 $AP : PB = 2 : 5$ $P = B + \frac{5}{7}(A - B)$	24 $BP : PA = 4 : 3$ $P = A + \frac{4}{7}(B - A)$	25 $AP : PB = 2 : 3$ $P = A + \frac{3}{5}(B - A)$



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Mystery Picture — answers

For the teacher



The finished picture is **the number 7**. Thirteen squares are filled.

The first letter of the ratio is the start. A rule is correct when the start's side gets the right share: the trip has as many equal steps as the two ratio numbers added. The table gives the reason for every square. It goes on over the page.

	The square	Answer	Why
1	AP : PB = 1 : 3. P = A + $\frac{1}{4}(B - A)$.	fill	The trip starts at A and has $1 + 3 = 4$ equal steps. P is 1 of them from A, so $\frac{1}{4}$ of the change from A to B is added to A.
2	AP : PB = 4 : 3. P = A + $\frac{4}{7}(B - A)$.	fill	The trip starts at A and has $4 + 3 = 7$ equal steps. P is 4 of them from A, so $\frac{4}{7}$ of the change from A to B is added to A.
3	AP : PB = 5 : 4. P = B + $\frac{4}{9}(A - B)$.	fill	P is 4 steps of 9 from the end B. So going from B back toward A by $\frac{4}{9}$ of the change is correct.
4	BP : PA = 5 : 2. P = A + $\frac{2}{7}(B - A)$.	fill	P is 2 steps of 7 from the end A. So going from A back toward B by $\frac{2}{7}$ of the change is correct.
5	AP : PB = 3 : 5. P = $\frac{3}{8} \cdot B + \frac{5}{8} \cdot A$.	fill	P is nearer A, so A gets the larger weight. A gets $\frac{5}{8}$ and B gets $\frac{3}{8}$. The weights add to 1.
6	AP : PB = 1 : 4. P = A + $\frac{4}{5}(B - A)$.	white	It goes $\frac{4}{5}$ of the way from A. P is only 1 step from A, so it should be $\frac{1}{5}$.
7	BP : PA = 2 : 5. P = B + $\frac{2}{5}(A - B)$.	white	The ratio $2 : 5$ is read as the fraction $\frac{2}{5}$. The trip has 7 steps, so the fraction is $\frac{2}{7}$.
8	AP : PB = 4 : 1. P = B + $\frac{4}{5}(A - B)$.	white	It starts from B, the end of the trip, but uses $\frac{4}{5}$, which is P's share counted from A. That point is 4 steps from B, not from A.
9	AP : PB = 5 : 4. P = $\frac{4}{9} \cdot A + \frac{5}{9} \cdot B$.	fill	P is nearer B, so B gets the larger weight. A gets $\frac{4}{9}$ and B gets $\frac{5}{9}$. The weights add to 1.
10	AP : PB = 4 : 5. P = A + $\frac{4}{9}(B - A)$.	fill	The trip starts at A and has $4 + 5 = 9$ equal steps. P is 4 of them from A, so $\frac{4}{9}$ of the change from A to B is added to A.
11	BP : PA = 3 : 5. P = $\frac{5}{8} \cdot A + \frac{3}{8} \cdot B$.	white	The weights are the wrong way round. B should get $\frac{5}{8}$ and A should get $\frac{3}{8}$, because P is nearer B.
12	AP : PB = 5 : 1. P = B + $\frac{5}{6}(A - B)$.	white	It starts from B, the end of the trip, but uses $\frac{5}{6}$, which is P's share counted from A. That point is 5 steps from B, not from A.

Mystery Picture — answers, squares 13 to 25

For the teacher

	The square	Answer	Why
13	$BP : PA = 2 : 3$. $P = \frac{2}{5} \cdot B + \frac{3}{5} \cdot A$.	white	The weights are the wrong way round. B should get $\frac{3}{5}$ and A should get $\frac{2}{5}$, because P is nearer B.
14	$BP : PA = 1 : 3$. $P = A + \frac{3}{4}(B - A)$.	fill	P is 3 steps of 4 from the end A. So going from A back toward B by $\frac{3}{4}$ of the change is correct.
15	$AP : PB = 3 : 2$. $P = A + \frac{3}{5}(B - A)$.	fill	The trip starts at A and has $3 + 2 = 5$ equal steps. P is 3 of them from A, so $\frac{3}{5}$ of the change from A to B is added to A.
16	$AP : PB = 3 : 1$. $P = B + \frac{3}{4}(A - B)$.	white	It starts from B, the end of the trip, but uses $\frac{3}{4}$, which is P's share counted from A. That point is 3 steps from B, not from A.
17	$BP : PA = 1 : 4$. $P = A + \frac{1}{5}(B - A)$.	white	It starts from A, the end of the trip, but uses $\frac{1}{5}$, which is P's share counted from B. That point is 1 step from A, not from B.
18	$AP : PB = 3 : 4$. $P = \frac{4}{7} \cdot A + \frac{3}{7} \cdot B$.	fill	P is nearer A, so A gets the larger weight. A gets $\frac{4}{7}$ and B gets $\frac{3}{7}$. The weights add to 1.
19	$AP : PB = 1 : 4$. $P = A + \frac{1}{5}(B - A)$.	fill	The trip starts at A and has $1 + 4 = 5$ equal steps. P is 1 of them from A, so $\frac{1}{5}$ of the change from A to B is added to A.
20	$AP : PB = 1 : 3$. $P = A + \frac{1}{3}(B - A)$.	white	The ratio $1 : 3$ is read as the fraction $\frac{1}{3}$. The trip has 4 steps, so the fraction is $\frac{1}{4}$.
21	$AP : PB = 5 : 3$. $P = \frac{3}{8} \cdot B + \frac{5}{8} \cdot A$.	white	The weights are the wrong way round. A should get $\frac{3}{8}$ and B should get $\frac{5}{8}$, because P is nearer B.
22	$BP : PA = 5 : 2$. $P = \frac{5}{7} \cdot A + \frac{2}{7} \cdot B$.	fill	P is nearer A, so A gets the larger weight. B gets $\frac{2}{7}$ and A gets $\frac{5}{7}$. The weights add to 1.
23	$AP : PB = 2 : 5$. $P = B + \frac{5}{7}(A - B)$.	fill	P is 5 steps of 7 from the end B. So going from B back toward A by $\frac{5}{7}$ of the change is correct.
24	$BP : PA = 4 : 3$. $P = A + \frac{4}{7}(B - A)$.	white	It starts from A, the end of the trip, but uses $\frac{4}{7}$, which is P's share counted from B. That point is 4 steps from A, not from B.
25	$AP : PB = 2 : 3$. $P = A + \frac{3}{5}(B - A)$.	white	It goes $\frac{3}{5}$ of the way from A. P is only 2 steps from A, so it should be $\frac{2}{5}$.