

CUT & GLUE SORT



CLASSIFYING TRIANGLES ON THE COORDINATE PLANE

Isosceles or Not? Triangles on the Coordinate Plane

Twelve triangles · corners, lengths, slopes, drawings · three rows

FREE

1	2	3
In triangle ABC, the slope of AB is 2, and the slope of AC is -2.	A triangle has corners A(-3, 1), B(3, 3) and C(1, -3).	

Row 1. Isosceles. At least two sides are the same length.

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Row 2. Not isosceles. All three sides have different lengths.

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Row 3. Cannot tell. The piece does not give enough to decide.

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1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Is the triangle isosceles? Use the corners, the side lengths, or count squares on the grid. Slopes do not show lengths. Do not measure. If the piece does not give enough, put it in row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows the six triangle names.

Answers page 5.

A triangle is isosceles when two sides have the same length. From corners, the student finds the square of each side, $(\text{across})^2 + (\text{up})^2$, and compares the three squares. On squared paper the student counts squares. Two lengths written in different radical forms, like $\sqrt{40}$ and $2\sqrt{10}$, are the same number.

Row 2 holds the pieces that tempt students most. One gives two slopes that are opposites, but its sides have different lengths: slopes give directions, never lengths. One drawing on squared paper looks isosceles, but its squares are 25, 26, 41. One gives three lengths that are close but all different.

Row 3 holds four pieces that do not fix the three side lengths: two opposite slopes alone, a right angle alone, two of the three squares, and a drawing on plain paper with no numbers. The key gives two triangles for each, one isosceles and one not, that both fit the piece. Guessing does not work: a simple guessing rule, such as sorting by whether the piece mentions slopes, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — isosceles or not?

Classifying triangles

Name _____

Period _____

Date _____

Each piece on the next page tells you about one triangle. **Is the triangle isosceles? Glue the piece in its row.** Use only what the piece gives. On squared paper, count squares between the dots. Do not measure a drawing on plain paper.

Row 1. Isosceles. At least two sides are the same length.

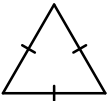
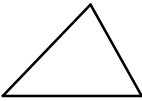
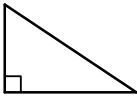
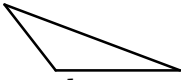
Row 2. Not isosceles. All three sides have different lengths.

Row 3. Cannot tell. The piece does not give enough to decide.

Cut & Glue Sort — the pieces

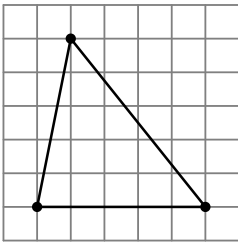
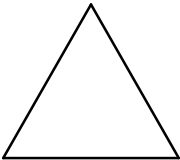
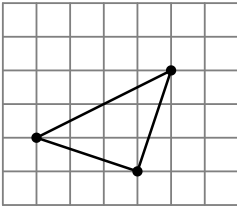
Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. Use only what the piece gives. On squared paper, count squares between the dots. Do not measure a drawing on plain paper.

					
scalene no two sides the same length	isosceles at least two sides the same length	equilateral all three sides the same length	acute $c^2 < a^2 + b^2$	right $c^2 = a^2 + b^2$	obtuse $c^2 > a^2 + b^2$

Isosceles: at least two sides are the same length. Find the square of each side, (across)² + (up)², and compare the three squares.

Slopes give the direction of a side, not its length.

1 In triangle ABC, the slope of AB is 2, and the slope of AC is -2.	2 A triangle has corners A(-3, 1), B(3, 3) and C(1, -3).	3 	4 The three sides of a triangle are $\sqrt{40}$, $2\sqrt{10}$ and $4\sqrt{2}$ long.
5 A triangle has corners X(-3, 2), Y(2, 7) and Z(1, -4).	6 In triangle ABC, $AB^2 = 20$ and $AC^2 = 25$.	7 A triangle has corners A(0, 4), B(-1, 1) and C(2, -2). The slope of AB is 3, and the slope of AC is -3.	8 
9 	10 In triangle ABC, the angle at A is a right angle.	11 In triangle ABC, $AB^2 = 26$ and $AC^2 = 26$. The slope of AB is 5, and the slope of AC is $1/5$.	12 The three sides of a triangle are 5, $\sqrt{26}$ and $\sqrt{29}$ long.



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Each row holds four pieces. Every piece is described again here, with the reason. This sheet calls a triangle isosceles when at least two sides are the same length. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 Isosceles. At least two sides are the same length.

2. A triangle has corners $A(-3, 1)$, $B(3, 3)$ and $C(1, -3)$. *Why:* $AB^2 = 40$, $BC^2 = 40$, $CA^2 = 32$. Two squares match, so two sides are the same length.

4. The three sides of a triangle are $\sqrt{40}$, $2\sqrt{10}$ and $4\sqrt{2}$ long. *Why:* Square each length: 40, 40, 32. Two squares match. A radical and its simplified form are the same number.

9. A triangle on squared paper, its corners on grid crossings. *Why:* One side goes 3 across and 1 up: $9 + 1 = 10$. One goes 1 across and 3 up: $1 + 9 = 10$. One goes 4 across and 2 up: $16 + 4 = 20$. Two squares match, so two sides are the same length. It is also a right triangle: a right triangle can be isosceles.

11. In triangle ABC, $AB^2 = 26$ and $AC^2 = 26$. The slope of AB is 5, and the slope of AC is $1/5$. *Why:* AB^2 and AC^2 are equal, so $AB = AC$. The slopes are extra information. They say nothing about lengths.

2 Not isosceles. All three sides have different lengths.

3. A triangle on squared paper, its corners on grid crossings. *Why:* One side goes 5 across and 0 up, so its square is 25. One goes 4 across and 5 up: $16 + 25 = 41$. One goes 1 across and 5 up: $1 + 25 = 26$. The three squares are all different, so no two sides match. The drawing looks isosceles, but counting shows it is not.

5. A triangle has corners $X(-3, 2)$, $Y(2, 7)$ and $Z(1, -4)$. *Why:* $XY^2 = 50$, $YZ^2 = 122$, $ZX^2 = 52$. The three squares are all different, so no two sides match.

7. A triangle has corners $A(0, 4)$, $B(-1, 1)$ and $C(2, -2)$. The slope of AB is 3, and the slope of AC is -3 . *Why:* $AB^2 = 10$, $BC^2 = 18$, $CA^2 = 40$. The three squares are all different, so no two sides match. The two slopes are opposites, but slopes give directions, not lengths.

12. The three sides of a triangle are 5, $\sqrt{26}$ and $\sqrt{29}$ long. *Why:* Square each length: 25, 26, 29. They are close, but all different, so no two sides match.

3 Cannot tell. The piece does not give enough to decide.

1. In triangle ABC, the slope of AB is 2, and the slope of AC is -2 . *Why:* Opposite slopes give directions, not lengths. For example, $A(0, 0)$, $B(-3, -6)$ and $C(-3, 6)$ is isosceles and $A(0, 0)$, $B(-3, -6)$ and $C(-2, 4)$ is not, and both fit the piece.

6. In triangle ABC, $AB^2 = 20$ and $AC^2 = 25$. *Why:* The third side is not given. It could match one of these two, or not. For example, $A(0, 0)$, $B(-4, -2)$ and $C(-4, 3)$ is isosceles and $A(0, 0)$, $B(-4, -2)$ and $C(-5, 0)$ is not, and both fit the piece.

8. A triangle on plain paper, with no grid, no numbers and no marks. *Why:* The triangle looks isosceles, but there are no numbers and no marks, and a drawing is not measured. For example, $A(0, 0)$, $B(-6, -6)$ and $C(-6, 0)$ is isosceles and $A(0, 0)$, $B(-6, -6)$ and $C(-6, -5)$ is not, and both fit the piece.

10. In triangle ABC, the angle at A is a right angle. *Why:* A right angle says nothing about the side lengths. For example, $A(0, 0)$, $B(-6, -6)$ and $C(-6, 6)$ is isosceles and $A(0, 0)$, $B(-6, -6)$ and $C(-5, 5)$ is not, and both fit the piece.

To check a piece with a student, ask: “Which numbers give the length of each side? Do you have all three?”