

CUT & GLUE SORT



CLASSIFYING QUADRILATERALS ON THE COORDINATE PLANE

A Square or Not? Quadrilaterals on the Coordinate Plane

Twelve quadrilaterals · corners, slopes, lengths, midpoints, drawings · three rows

FREE

1	2	3
In ABCD, $AB^2 = BC^2 = CD^2 = DA^2 = 10$.	ABCD has corners $A(-2, 1)$, $B(1, 3)$, $C(3, 0)$ and $D(0, -2)$.	In ABCD, $AB^2 = 20$ and $BC^2 = 45$. The slopes of AB, BC, CD and DA are 2, $-1/2$, 2 and $-1/2$.

Row 1. It is a square.

Row 2. It is not a square.

Row 3. Cannot tell. The piece does not give enough to decide.

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Is ABCD a square? A square needs four equal sides and a right angle. Use slopes, lengths and midpoints. If the piece does not give enough, put it in row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows the six names and which tool shows what.
Answers	page 5.

A square needs two things: four equal sides and a right angle. Four equal sides alone make a rhombus. Equal slopes show parallel sides, slopes that multiply to -1 show a right angle, and the squares of the lengths, $(\text{across})^2 + (\text{up})^2$, show equal sides. The diagonals can settle it too: diagonals that share a midpoint, are the same length and are perpendicular belong to a square.

Row 2 holds the pieces that tempt students most. One gives the corners of a rhombus, with four equal sides and no right angle. One is a rectangle whose sides that meet are different lengths. One gives two midpoints that agree in x but not in y . One drawing on squared paper looks square, but one of its sides is not vertical.

Row 3 holds four pieces whose numbers allow a square but do not prove one: four equal sides alone, diagonals that bisect each other at a right angle, two equal sides that meet at a right angle, and equal perpendicular diagonals with no midpoints. The key gives two quadrilaterals for each, one a square and one not, that both fit the piece. Guessing does not work: a simple guessing rule, such as sorting by whether the piece mentions slopes, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — a square or not?

Classifying quadrilaterals

Name _____

Period _____

Date _____

Each piece on the next page tells you about one quadrilateral, ABCD, with its corners in order. **Is it a square? Glue the piece in its row.** Use only what the piece gives. On squared paper, count squares between the dots. The sides of ABCD are AB, BC, CD and DA. The diagonals are AC and BD.

Row 1. It is a square.

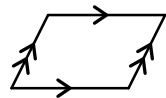
Row 2. It is not a square.

Row 3. Cannot tell. The piece does not give enough to decide.

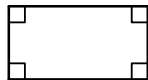
Cut & Glue Sort — the pieces

Cut along the dashed lines

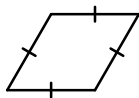
Cut out the twelve pieces. Is the quadrilateral a square? Glue each piece in its row on the other sheet. Use only what the piece gives. On squared paper, count squares between the dots.



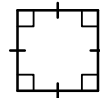
parallelogram
both pairs of opposite
sides parallel



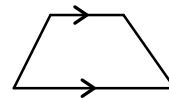
rectangle
a parallelogram with
four right angles



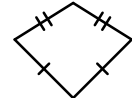
rhombus
four equal sides



square
four equal sides and four
right angles



trapezoid
at least one pair of
parallel sides



kite
two pairs of equal sides
next to each other

Slopes show parallel sides (equal slopes) and right angles (slopes that multiply to -1). Two vertical sides are parallel. A vertical side and a horizontal side are perpendicular.

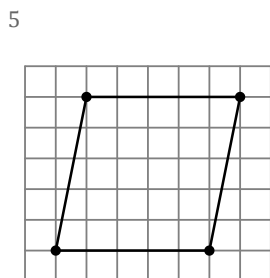
Distances show equal lengths: compare the squares, (across)² + (up)². **Midpoints** show that the diagonals bisect each other: the two midpoints must match in *both* coordinates.

1
In ABCD, $AB^2 = BC^2 =$
 $CD^2 = DA^2 = 10$.

2
ABCD has corners
A(-2, 1), B(1, 3), C(3, 0)
and D(0, -2).

3
In ABCD, $AB^2 = 20$ and
 $BC^2 = 45$. The slopes of
AB, BC, CD and DA are
2, $-1/2$, 2 and $-1/2$.

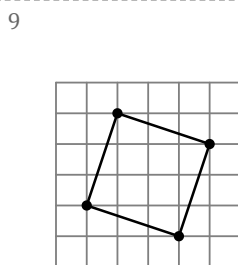
4
In ABCD, $AB^2 = BC^2 =$
 $CD^2 = DA^2 = 13$. The
slopes of AB and BC are
 $2/3$ and $-3/2$.



6
In ABCD, $AC^2 = BD^2 =$
18. The slopes of AC and
BD are 1 and -1 .

7
ABCD has corners
A(-3, -2), B(1, 0),
C(3, 4) and D(-1, 2).

8
In ABCD, the slopes of
AC and BD are 3 and
 $-1/3$. The midpoints of
AC and BD are both
(1.5, 0.5).



10
In ABCD, the midpoint of
AC is (1, 2), and the
midpoint of BD is (1, 3).

11
In ABCD, $AC^2 = BD^2 =$
26. The slopes of AC and
BD are 5 and $-1/5$. The
midpoints of AC and BD
are both (1.5, 2.5).

12
In ABCD, $AB^2 = BC^2 =$
20. The slopes of AB and
BC are 2 and $-1/2$.



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Each row holds four pieces. Every piece is described again here, with the reason. This sheet counts a square as a rectangle and a rhombus, and a rhombus as a kite. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 **It is a square.**

2. ABCD has corners $A(-2, 1)$, $B(1, 3)$, $C(3, 0)$ and $D(0, -2)$. *Why:* The squares of the four sides are all 13. The slope of AB is $2/3$ and the slope of BC is $-3/2$. They multiply to -1 , so the corner at B is a right angle. Four equal sides and a right angle make a square.

4. In ABCD, $AB^2 = BC^2 = CD^2 = DA^2 = 13$. The slopes of AB and BC are $2/3$ and $-3/2$. *Why:* All four sides are the same length, and the slopes of AB and BC multiply to -1 , so the corner at B is a right angle. That makes a square.

9. A quadrilateral on squared paper with corners (counted in squares from the lower left) $(1, 2)$, $(4, 1)$, $(5, 4)$, $(2, 5)$. *Why:* Count squares between the dots. The squares of the four sides are all 10. The slope of AB is $-1/3$ and the slope of BC is 3. They multiply to -1 , so the corner at B is a right angle. Four equal sides and a right angle make a square.

11. In ABCD, $AC^2 = BD^2 = 26$. The slopes of AC and BD are 5 and $-1/5$. The midpoints of AC and BD are both $(1.5, 2.5)$. *Why:* The diagonals share a midpoint, so they bisect each other: a parallelogram. They are the same length, so it is a rectangle. Their slopes multiply to -1 , so they are perpendicular: a rhombus too. A rectangle that is a rhombus is a square.

2 **It is not a square.**

3. In ABCD, $AB^2 = 20$ and $BC^2 = 45$. The slopes of AB, BC, CD and DA are 2, $-1/2$, 2 and $-1/2$. *Why:* Opposite slopes are equal and the slopes that meet multiply to -1 , so it is a rectangle. But AB^2 and BC^2 are different, so two sides that meet are not the same length. One unequal pair is enough to rule out a square.

5. A quadrilateral on squared paper with corners (counted in squares from the lower left) $(1, 1)$, $(6, 1)$, $(7, 6)$, $(2, 6)$. *Why:* Count squares between the dots. The squares of the four sides are 25, 26, 25, 26. AB and BC are not the same length, so it is not a square. The drawing looks close to a square, but AB is horizontal and BC goes 1 across and 5 up, so BC is not vertical and the corner at B is not a right angle.

7. ABCD has corners $A(-3, -2)$, $B(1, 0)$, $C(3, 4)$ and $D(-1, 2)$. *Why:* The squares of the four sides are all 20, so it is a rhombus. The slope of AB is $1/2$ and the slope of BC is 2. They do not multiply to -1 . Four equal sides alone do not make a square.

10. In ABCD, the midpoint of AC is $(1, 2)$, and the midpoint of BD is $(1, 3)$. *Why:* The two midpoints agree in x but not in y, so they are different points. The diagonals do not bisect each other, so it is not a parallelogram, and so not a square.

3 **Cannot tell. The piece does not give enough to decide.**

1. In ABCD, $AB^2 = BC^2 = CD^2 = DA^2 = 10$. *Why:* Four equal sides make a rhombus. Nothing says whether a corner is a right angle. For example, $A(0, 0)$, $B(-3, -1)$, $C(-4, 2)$ and $D(-1, 3)$ is a square and $A(0, 0)$, $B(-3, -1)$, $C(-6, 0)$ and $D(-3, 1)$ is not, and both fit the piece.

6. In ABCD, $AC^2 = BD^2 = 18$. The slopes of AC and BD are 1 and -1 . *Why:* The diagonals are the same length and perpendicular. Nothing says that they bisect each other. For example, $A(0, 0)$, $B(-3, 0)$, $C(-3, -3)$ and $D(0, -3)$ is a square and $A(0, 0)$, $B(-5, 0)$, $C(-3, -3)$ and $D(-2, -3)$ is not, and both fit the piece.

8. In ABCD, the slopes of AC and BD are 3 and $-1/3$. The midpoints of AC and BD are both $(1.5, 0.5)$. *Why:* The diagonals bisect each other and are perpendicular, so it is a rhombus. Nothing says whether the diagonals are the same length. For example, $A(0, -4)$, $B(-3, 2)$, $C(3, 5)$ and $D(6, -1)$ is a square and $A(0, -4)$, $B(0, 1)$, $C(3, 5)$ and $D(3, 0)$ is not, and both fit the piece.

12. In ABCD, $AB^2 = BC^2 = 20$. The slopes of AB and BC are 2 and $-1/2$. *Why:* AB and BC are the same length and meet at a right angle. Nothing is given about C to D or D to A, so D could be anywhere. For example, $A(0, 0)$, $B(-2, -4)$, $C(-6, -2)$ and $D(-4, 2)$ is a square and $A(0, 0)$, $B(-2, -4)$, $C(-6, -2)$ and $D(-8, 0)$ is not, and both fit the piece.

To check a piece with a student, ask: “Do you know all four sides are equal? Do you know there is a right angle?”