

MYSTERY PICTURE



CLASSIFYING QUADRILATERALS ON THE COORDINATE PLANE

What Do the Numbers Prove? Classifying Quadrilaterals

25 concluding sentences · slopes, lengths and midpoints · nothing to calculate · a picture appears

FREE

1 The slope of AB is $\frac{3}{4}$ and the slope of CD is $\frac{3}{4}$. So AB is parallel to CD.	2 AB and CD are vertical. So AB is parallel to CD.	3 The slope of AB is 2 and the slope of BC is $-\frac{1}{2}$. So the angle at B is a right angle.	4 $AB^2 = CD^2 = 13$. So AB and CD are the same length.	5 The midpoints of AC and BD are both (1, 4). So the diagonals bisect each other.
6 The slope of AB is $\frac{3}{4}$ and the slope of CD is $\frac{3}{4}$. So AB and CD are the same length.	7 The slope of AB is $\frac{2}{3}$ and the slope of BC is $-\frac{2}{3}$. So the angle at B is a right angle.	8 $AC^2 = BD^2 = 40$. So the diagonals bisect each other.	9 The slope of AB is $\frac{2}{3}$ and the slope of BC is $\frac{3}{2}$. So AB is perpendicular to BC.	10 The midpoints of AC and BD are both (2.5, 1). So ABCD is a parallelogram.
11 $AC^2 = BD^2 = 40$. So the diagonals are the same length.	12 The slope of AC is 3 and the slope of BD is $-\frac{1}{3}$. So the diagonals are perpendicular.	13 The slope of AB is $\frac{1}{2}$ and the slope of CD is $\frac{1}{2}$. So ABCD is a trapezoid.	14 The slopes of AB, BC, CD and DA are $\frac{1}{2}$, $-\frac{3}{2}$, $\frac{1}{2}$ and $-\frac{3}{2}$. So ABCD is a parallelogram.	15 $AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a rhombus.
16 $AB^2 = BC^2 = CD^2 = DA^2 = 5$. The slope of AB is 2 and the slope of BC is $-\frac{1}{2}$. So ABCD is a square.	17 The midpoint of AC is (1, 4) and the midpoint of BD is (1, 2). So the diagonals bisect each other.	18 $AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a square.	19 The slope of AB is 2 and the slope of BC is $-\frac{1}{2}$. So ABCD is a rectangle.	20 The slope of AB is $\frac{1}{2}$ and the slope of CD is $\frac{1}{2}$. So ABCD is a parallelogram.
21 $AB^2 = BC^2 = 8$ and $CD^2 = DA^2 = 13$. So ABCD is a kite.	22 The slopes of AB, BC, CD and DA are 2, $-\frac{1}{2}$, 2 and $-\frac{1}{2}$. So ABCD is a rectangle.	23 $AB^2 = BC^2 = CD^2 = DA^2 = 13$. So ABCD is a kite.	24 $AC^2 = BD^2 = 13$. The midpoints of AC and BD are both (0, 2.5). So ABCD is a rectangle.	25 The slopes of AB, BC, CD and DA are $\frac{1}{3}$, $-\frac{3}{2}$, $\frac{1}{3}$ and $-\frac{3}{2}$. So ABCD is a trapezoid.

Mystery Picture — how to run it

25–35 minutes

1. **Give each student page 3** and a pencil. Nothing is calculated.
2. **Say this once:** *“Every number in a square is correct. Do the numbers prove the sentence after ‘So’? If yes, fill the square with slanted lines. If not, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows the six names, what this sheet means by each, and which tool shows what.
Answers	page 4, for you.

Every square is a finished piece of work with a concluding sentence. The student decides only whether the numbers support the sentence. Slopes show parallel and perpendicular sides. Squared lengths show equal lengths. Midpoints show diagonals that bisect each other. Seventeen squares are filled and eight are white.

The white squares are the mistakes students make in a concluding sentence. One uses slopes to claim equal lengths. One takes equal diagonals to mean diagonals that bisect each other, and one accepts two midpoints that match in only one coordinate. Two take slopes that are only opposites, or only reciprocals, as perpendicular. Three claim a name that needs one more fact, such as “square” from four equal sides. The filled squares include a parallelogram called a trapezoid and a rhombus called a kite: on this sheet both are true.

If part of a student’s picture looks wrong, ask about that square: *“Which tool do these numbers come from, and what can that tool show?”* Guessing does not work. Every simple guessing rule, such as filling every square that names a shape, gets at least six squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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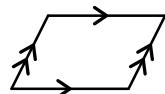
Mystery Picture — what do the numbers prove? *Classifying quadrilaterals*

Name _____

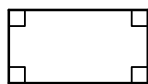
Period _____

Date _____

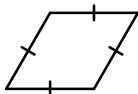
Each square gives some numbers about a quadrilateral ABCD and a sentence. **Every number in a square is correct.** Read the sentence after “So”. If the numbers prove it, fill the square with slanted lines. If they do not prove it, leave the square white, even when the sentence could be true. A picture appears. If part of it is missing, or one square does not fit, check that square again.



parallelogram
both pairs of opposite
sides parallel



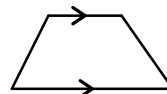
rectangle
a parallelogram with
four right angles



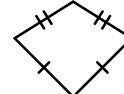
rhombus
four equal sides



square
four equal sides and four
right angles



trapezoid
at least one pair of
parallel sides



kite
two pairs of equal sides
next to each other

A true name counts, even when a more specific name also fits. A square is also a rectangle and a rhombus. A rectangle and a rhombus are also parallelograms. A rhombus is also a kite. A trapezoid has *at least* one pair of parallel sides, so a parallelogram is also a trapezoid.

Slopes show parallel sides (equal slopes) and right angles (slopes that multiply to -1). Two vertical sides are parallel. A vertical side and a horizontal side are perpendicular.

Distances show equal lengths: compare the squares, (across)² + (up)². **Midpoints** show that the diagonals bisect each other: the two midpoints must match in *both* coordinates.

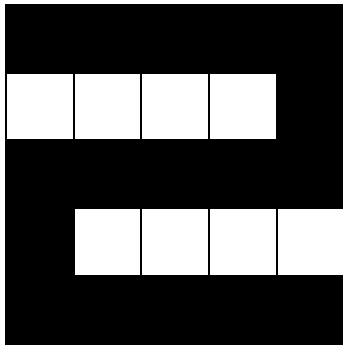
1 The slope of AB is $3/4$ and the slope of CD is $3/4$. So AB is parallel to CD.	2 AB and CD are vertical. So AB is parallel to CD.	3 The slope of AB is 2 and the slope of BC is $-1/2$. So the angle at B is a right angle.	4 $AB^2 = CD^2 = 13$. So AB and CD are the same length.	5 The midpoints of AC and BD are both (1, 4). So the diagonals bisect each other.
6 The slope of AB is $3/4$ and the slope of CD is $3/4$. So AB and CD are the same length.	7 The slope of AB is $2/3$ and the slope of BC is $-2/3$. So the angle at B is a right angle.	8 $AC^2 = BD^2 = 40$. So the diagonals bisect each other.	9 The slope of AB is $2/3$ and the slope of BC is $3/2$. So AB is perpendicular to BC.	10 The midpoints of AC and BD are both (2.5, 1). So ABCD is a parallelogram.
11 $AC^2 = BD^2 = 40$. So the diagonals are the same length.	12 The slope of AC is 3 and the slope of BD is $-1/3$. So the diagonals are perpendicular.	13 The slope of AB is $1/2$ and the slope of CD is $1/2$. So ABCD is a trapezoid.	14 The slopes of AB, BC, CD and DA are $1/2$, -3 , $1/2$ and -3 . So ABCD is a parallelogram.	15 $AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a rhombus.
16 $AB^2 = BC^2 = CD^2 = DA^2 = 5$. The slope of AB is 2 and the slope of BC is $-1/2$. So ABCD is a square.	17 The midpoint of AC is (1, 4) and the midpoint of BD is (1, 2). So the diagonals bisect each other.	18 $AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a square.	19 The slope of AB is 2 and the slope of BC is $-1/2$. So ABCD is a rectangle.	20 The slope of AB is $1/2$ and the slope of CD is $1/2$. So ABCD is a parallelogram.
21 $AB^2 = BC^2 = 8$ and $CD^2 = DA^2 = 13$. So ABCD is a kite.	22 The slopes of AB, BC, CD and DA are 2, $-1/2$, 2 and $-1/2$. So ABCD is a rectangle.	23 $AB^2 = BC^2 = CD^2 = DA^2 = 13$. So ABCD is a kite.	24 $AC^2 = BD^2 = 13$. The midpoints of AC and BD are both (0, 2.5). So ABCD is a rectangle.	25 The slopes of AB, BC, CD and DA are $1/3$, -3 , $1/3$ and -3 . So ABCD is a trapezoid.



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Mystery Picture — answers

For the teacher



The finished picture is **the number 2**. Seventeen squares are filled.

A square is filled when its numbers prove its sentence. This sheet counts a parallelogram as a trapezoid (a trapezoid has at least one pair of parallel sides), a rhombus as a kite, and a square as a rectangle and a rhombus.

Some textbooks use other definitions, so tell the class which ones this sheet uses.

	The square	Answer	Why
1	The slope of AB is $3/4$ and the slope of CD is $3/4$. So AB is parallel to CD.	fill	Equal slopes mean parallel sides.
2	AB and CD are vertical. So AB is parallel to CD.	fill	Two vertical sides are parallel, even though neither has a slope.
3	The slope of AB is 2 and the slope of BC is $-1/2$. So the angle at B is a right angle.	fill	The slopes of AB and BC multiply to -1 .
4	$AB^2 = CD^2 = 13$. So AB and CD are the same length.	fill	Equal squared lengths mean equal lengths.
5	The midpoints of AC and BD are both (1, 4). So the diagonals bisect each other.	fill	The two midpoints match in both coordinates, so the diagonals cut each other in half.
6	The slope of AB is $3/4$ and the slope of CD is $3/4$. So AB and CD are the same length.	white	Slopes show direction, not length. For example, A(0, 0), B(-8, -6), C(-8, -8) and D(-4, -5) fits the numbers, and the sentence is false for it.
7	The slope of AB is $2/3$ and the slope of BC is $-2/3$. So the angle at B is a right angle.	white	The slopes are opposites, but they multiply to $-4/9$, not -1 .
8	$AC^2 = BD^2 = 40$. So the diagonals bisect each other.	white	Equal diagonals are not the same as diagonals that cut each other in half. That needs the midpoints. For example, A(0, 0), B(-8, -1), C(-6, -2) and D(-2, -3) fits the numbers, and the sentence is false for it.
9	The slope of AB is $2/3$ and the slope of BC is $3/2$. So AB is perpendicular to BC.	white	The slopes are reciprocals but not opposites. They multiply to 1, not -1 .
10	The midpoints of AC and BD are both (2.5, 1). So ABCD is a parallelogram.	fill	Diagonals that share a midpoint bisect each other, and that makes a parallelogram.
11	$AC^2 = BD^2 = 40$. So the diagonals are the same length.	fill	Equal squared lengths mean equal diagonals.
12	The slope of AC is 3 and the slope of BD is $-1/3$. So the diagonals are perpendicular.	fill	The slopes of the diagonals multiply to -1 .
13	The slope of AB is $1/2$ and the slope of CD is $1/2$. So ABCD is a trapezoid.	fill	One pair of parallel sides is enough for a trapezoid on this sheet.
14	The slopes of AB, BC, CD and DA are $1/2$, -3 , $1/2$ and -3 . So ABCD is a parallelogram.	fill	Both pairs of opposite sides have equal slopes, so both pairs are parallel.
15	$AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a rhombus.	fill	Four equal sides make a rhombus.
16	$AB^2 = BC^2 = CD^2 = DA^2 = 5$. The slope of AB is 2 and the slope of BC is $-1/2$. So ABCD is a square.	fill	Four equal sides make a rhombus, and the slopes of AB and BC multiply to -1 , so it has a right angle: a square.
17	The midpoint of AC is (1, 4) and the midpoint of BD is (1, 2). So the diagonals bisect each other.	white	The midpoints agree in x but not in y, so they are different points.
18	$AB^2 = BC^2 = CD^2 = DA^2 = 10$. So ABCD is a square.	white	Four equal sides make a rhombus. A square also needs a right angle. For example, A(0, 0), B(-3, -1), C(-6, 0) and D(-3, 1) fits the numbers, and the sentence is false for it.
19	The slope of AB is 2 and the slope of BC is $-1/2$. So ABCD is a rectangle.	white	One right angle is not enough. Nothing shows the other sides are parallel. For example, A(0, 0), B(-4, -8), C(-8, -6) and D(-8, -5) fits the numbers, and the sentence is false for it.
20	The slope of AB is $1/2$ and the slope of CD is $1/2$. So ABCD is a parallelogram.	white	One pair of parallel sides makes a trapezoid. A parallelogram needs both pairs. For example, A(0, 0), B(-8, -4), C(-8, -8) and D(-6, -7) fits the numbers, and the sentence is false for it.
21	$AB^2 = BC^2 = 8$ and $CD^2 = DA^2 = 13$. So ABCD is a kite.	fill	Two pairs of equal sides next to each other make a kite.
22	The slopes of AB, BC, CD and DA are 2, $-1/2$, 2 and $-1/2$. So ABCD is a rectangle.	fill	Opposite sides are parallel and the sides that meet are perpendicular.
23	$AB^2 = BC^2 = CD^2 = DA^2 = 13$. So ABCD is a kite.	fill	Four equal sides make a rhombus, and a rhombus is also a kite.
24	$AC^2 = BD^2 = 13$. The midpoints of AC and BD are both (0, 2.5). So ABCD is a rectangle.	fill	The diagonals bisect each other (a parallelogram) and are the same length, so it is a rectangle.
25	The slopes of AB, BC, CD and DA are $1/3$, -3 , $1/3$ and -3 . So ABCD is a trapezoid.	fill	Both pairs of sides are parallel, so it is a parallelogram, and a parallelogram is also a trapezoid on this sheet.