

CUT & GLUE SORT



PLACING FIGURES FOR COORDINATE PROOFS

What Does the Placement Force?

Twelve lettered placements · a right angle, equal sides, or neither · three rows

FREE

1	2	3
Corners in order: $(-a, -b), (a, -b),$ $(a, b), (-a, b)$	Corners in order: $(0, 0), (2a, 0), (a, b)$	Corners in order: $(0, 0), (a, 0),$ $(a + b, b), (b, b)$

Row 1. It forces a right angle. Every choice of letters gives a right angle at some corner.

Row 2. It forces two equal sides that meet at a corner, for every choice of letters.

Row 3. It forces neither one. Some choice of letters gives no right angle and no equal sides that meet.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Each piece places a figure on the grid, with letters for the coordinates of its corners. What does it force: a right angle, two equal sides that meet, or neither? Forced means it happens for every choice of letters.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows a good placement.

Answers page 5.

This is the question to ask beside every placement: what does it force? A placement forces a property when every choice of letters gives it. A student checks by picking a few numbers for the letters and looking at the figure. Each row has triangles and quadrilaterals.

Row 3 holds the pieces that tempt students most. A general parallelogram placement can make a rectangle, if one letter is 0, but it does not force one. Piece 3 uses the same letter for both coordinates of a corner, (b, b) . That forces an angle of 45° , but not a right angle or equal sides. The key gives, for each, numbers for the letters that show it.

Other pieces hide their answer. In $(0, 0)$, $(a, 0)$, (a, b) the right angle is at $(a, 0)$, not at the origin. In $(0, 0)$, (a, b) , $(0, 2b)$ one side is upright, but no right angle is forced. That piece forces two equal sides. In $(0, 0)$, (a, b) , $(-2b, 2a)$ no side is upright, but the slopes of the two sides at the origin multiply to -1 . Guessing does not work: a simple guessing rule, such as sorting by the number of letters, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — what does it force?

Placing a general figure

Name _____

Period _____

Date _____

Each piece on the next page gives the corners of a figure, with letters, in order. **What does the placement force? Glue the piece in its row.** Pick a few numbers for the letters and check. A thing is forced only if it happens for every choice of letters.

Row 1. It forces a right angle. Every choice of letters gives a right angle at some corner.

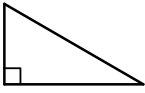
Row 2. It forces two equal sides that meet at a corner, for every choice of letters.

Row 3. It forces neither one. Some choice of letters gives no right angle and no equal sides that meet.

Cut & Glue Sort — the pieces

Cut along the dashed lines

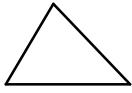
Cut out the twelve pieces. Glue each one in its row on the other sheet. Pick a few numbers for the letters and check. A thing is forced only if it happens for every choice of letters.



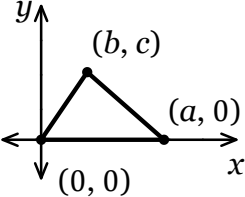
a right angle
at one corner



two equal sides
that meet
at one corner



neither
no right angle, no
equal sides that meet



Forced means it happens for every choice of letters. A general placement forces nothing extra: a general parallelogram can be a rectangle for some letters, but it does not have to be one.
A good placement puts one corner at $(0, 0)$ and one side along the x -axis, with letters where the numbers would be. A letter can be any number, positive, negative or zero, if the figure is real. A doubled letter, like $2a$, is any number too. The corners are listed in order around the figure.

1 Corners in order: $(-a, -b), (a, -b),$ $(a, b), (-a, b)$	2 Corners in order: $(0, 0), (2a, 0), (a, b)$	3 Corners in order: $(0, 0), (a, 0),$ $(a + b, b), (b, b)$	4 Corners in order: $(0, 0), (a, b), (0, 2b)$
5 Corners in order: $(0, 0), (2a, 2b),$ $(2a, 2c)$	6 Corners in order: $(0, 0), (a, 0), (a, b)$	7 Corners in order: $(0, 0), (a, 0),$ $(a + b, c), (b, c)$	8 Corners in order: $(0, 0), (a, b),$ $(-2b, 2a)$
9 Corners in order: $(0, 0), (a, b), (0, c),$ $(-a, b)$	10 Corners in order: $(0, 0), (a, 0),$ $(a, b + c), (0, b)$	11 Corners in order: $(a, 0), (0, b), (-a, 0),$ $(0, -b)$	12 Corners in order: $(-a, 0), (a, 0), (b, c)$



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Each row holds four pieces. Every piece is given again here, with the reason. A letter can be any number, positive, negative or zero, as long as the figure is real. The corners are listed in order. This sheet calls two sides equal when they have the same length. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 It forces a right angle. Every choice of letters gives a right angle at some corner.

1. $(-a, -b), (a, -b), (a, b), (-a, b)$. *Why:* Every choice of letters gives a right angle. Two equal sides that meet are not forced: put $a = 1$ and $b = 2$ and no two sides that meet have the same length $((-1, -2), (1, -2), (1, 2), (-1, 2))$. It is centred on $(0, 0)$, and the minus signs do not matter.
6. $(0, 0), (a, 0), (a, b)$. *Why:* Every choice of letters gives a right angle. Two equal sides that meet are not forced: put $a = 1$ and $b = 2$ and no two sides that meet have the same length $((0, 0), (1, 0), (1, 2))$. The right angle is at $(a, 0)$, not at $(0, 0)$.
8. $(0, 0), (a, b), (-2b, 2a)$. *Why:* Every choice of letters gives a right angle. Two equal sides that meet are not forced: put $a = 1$ and $b = 1$ and no two sides that meet have the same length $((0, 0), (1, 1), (-2, 2))$. The sides from $(0, 0)$ have slopes b/a and $-a/b$, which multiply to -1 , so they are perpendicular. If one of them is upright, the other is flat.
10. $(0, 0), (a, 0), (a, b + c), (0, b)$. *Why:* Every choice of letters gives a right angle. Two equal sides that meet are not forced: put $a = 1, b = 2$ and $c = 1$ and no two sides that meet have the same length $((0, 0), (1, 0), (1, 3), (0, 2))$. The two upright sides can have different lengths, but both meet the base at right angles.

2 It forces two equal sides that meet at a corner, for every choice of letters.

2. $(0, 0), (2a, 0), (a, b)$. *Why:* Every choice of letters gives two equal sides that meet at a corner. A right angle is not forced: put $a = 1$ and $b = 2$ and there is none $((0, 0), (2, 0), (1, 2))$. The top corner stands over the middle of the base.
4. $(0, 0), (a, b), (0, 2b)$. *Why:* Every choice of letters gives two equal sides that meet at a corner. A right angle is not forced: put $a = 1$ and $b = 2$ and there is none $((0, 0), (1, 2), (0, 4))$. The side from $(0, 0)$ to $(0, 2b)$ is upright, but no right angle is forced. The corner (a, b) is level with its middle.
9. $(0, 0), (a, b), (0, c), (-a, b)$. *Why:* Every choice of letters gives two equal sides that meet at a corner. A right angle is not forced: put $a = 1, b = 2$ and $c = 5$ and there is none $((0, 0), (1, 2), (0, 5), (-1, 2))$. It is the same on both sides of the y -axis, so the two sides at $(0, 0)$ have the same length.
11. $(a, 0), (0, b), (-a, 0), (0, -b)$. *Why:* Every choice of letters gives two equal sides that meet at a corner. A right angle is not forced: put $a = 1$ and $b = 2$ and there is none $((1, 0), (0, 2), (-1, 0), (0, -2))$. All four sides have the same length.

3 It forces neither one. Some choice of letters gives no right angle and no equal sides that meet.

3. $(0, 0), (a, 0), (a + b, b), (b, b)$. *Why:* Put $a = 1$ and $b = 1$: the corners are $(0, 0), (1, 0), (2, 1), (1, 1)$, with no right angle and no equal sides that meet. The side from $(0, 0)$ to (b, b) has slope 1, so it makes an angle of 45° with the base. That is not a right angle, and it forces no equal sides.
5. $(0, 0), (2a, 2b), (2a, 2c)$. *Why:* Put $a = 1, b = 1$ and $c = 2$: the corners are $(0, 0), (2, 2), (2, 4)$, with no right angle and no equal sides that meet. The side from $(2a, 2b)$ to $(2a, 2c)$ is upright, but nothing forces a right angle.
7. $(0, 0), (a, 0), (a + b, c), (b, c)$. *Why:* Put $a = 1, b = 1$ and $c = 1$: the corners are $(0, 0), (1, 0), (2, 1), (1, 1)$, with no right angle and no equal sides that meet. With $b = 0$ it is a rectangle, but nothing forces b to be 0.
12. $(-a, 0), (a, 0), (b, c)$. *Why:* Put $a = 1, b = 2$ and $c = 1$: the corners are $(-1, 0), (1, 0), (2, 1)$, with no right angle and no equal sides that meet. The base is centred on $(0, 0)$, but the top corner can stand anywhere.

To check a piece with a student, ask: “Pick numbers for the letters. Does it still happen? Try other numbers.”