

ANSWER MAZE



PLACING FIGURES FOR COORDINATE PROOFS

Which Placement Is General?

Placing a figure on the axes with letters · one page · 10–15 minutes · nothing to prepare

FREE

4 Rectangle

$(0, 0), (a, 0), (a, a), (0, a) \rightarrow$ **go to box 2**

$(0, 0), (a, 0), (a, b), (0, b) \rightarrow$ **go to box 7**

Hint: Ask whether the width and the height are free to differ.

7 Parallelogram

$(0, 0), (a, 0), (a + b, c), (b, c) \rightarrow$ **go to box 1**

$(0, 0), (a, 0), (d, e), (b, c) \rightarrow$ **go to box 5**

Hint: The fourth corner is not free. Look at how it is built.

1. **Hand out page 3**, one per student. No calculator is needed.
2. **Say this once:** *“Start at box 4. Each box names a figure and gives two placements. Pick the general one: it always makes that figure and forces nothing extra. Write every box number in the row at the bottom as you go.”*
3. **Check a sheet quickly.** A student with no mistakes writes **4 7 1 8 3 6**: six boxes, then out.

Student sheet page 3, one per student. A box at the top shows the figure names and a good placement.
Answers page 4.

Every box names a figure and gives two placements of it with letters. One is general: every choice of letters gives that figure, and the letters can make any figure of that kind. The other is one of two mistakes.

The two mistakes send students to two different boxes. A placement that forces too much makes the figure special, such as a rectangle placed with one letter, which is always a square. It sends the student to box 2. A placement that forces too little does not make the figure at all, such as a parallelogram whose fourth corner has letters of its own. It sends the student to box 5. Boxes 2 and 5 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more, and the extra box number tells you which mistake it was. Guessing does not get a student out: every simple guessing rule, such as always taking the placement with more letters, goes wrong within the first three boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

chooseyourlesson.com — 135 free geometry lessons.

Answer Maze — which placement is general?

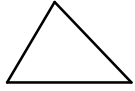
Start at box 4

Name _____

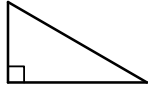
Period _____

Date _____

Start at box 4. **Each box names a figure and gives two placements. Pick the general one** and go to the box it names. Write each box number in the row at the bottom.



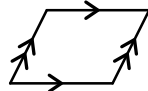
triangle
three corners, not in a line



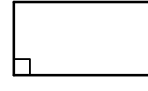
right triangle
one right angle



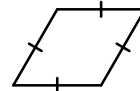
isosceles triangle
at least two sides the same length



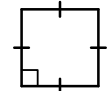
parallelogram
opposite sides parallel. It need not be a rectangle.



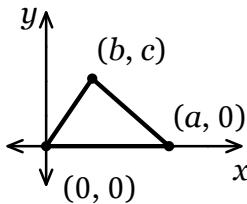
rectangle
four right angles



rhombus
four sides the same length



square
four right angles and four sides the same length



General means the placement makes the named figure for every choice of letters, and it forces nothing extra. A general parallelogram does not have to be a rectangle.

A good placement puts one corner at $(0, 0)$ and one side along the x -axis, with letters where the numbers would be. A letter can be any number, positive, negative or zero, if the figure is real. A doubled letter, like $2a$, is any number too. The corners are listed in order around the figure.

1 Triangle

$(0, 0), (2a, 0), (a, b) \rightarrow$ **go to box 2**

$(0, 0), (2a, 0), (2b, 2c) \rightarrow$ **go to box 8**

Hint: Ask whether the top corner must stand over the middle of the base.

2 Right triangle

$(0, 0), (a, 0), (0, b) \rightarrow$ **go to box 7**

$(0, 0), (a, 0), (0, a) \rightarrow$ **go to box 4**

Hint: Ask whether the two short sides are forced to match.

3 Parallelogram

$(0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c) \rightarrow$ **go to box 6**

$(0, 0), (2a, 0), (2a, 2c), (0, 2c) \rightarrow$ **go to box 2**

Hint: Ask whether the slanted sides are forced to stand upright.

4 Rectangle

$(0, 0), (a, 0), (a, a), (0, a) \rightarrow$ **go to box 2**

$(0, 0), (a, 0), (a, b), (0, b) \rightarrow$ **go to box 7**

Hint: Ask whether the width and the height are free to differ.

5 Square

$(0, 0), (a, 0), (a, b), (0, b) \rightarrow$ **go to box 7**

$(0, 0), (a, 0), (a, a), (0, a) \rightarrow$ **go to box 1**

Hint: Check whether the width and the height can come out unequal.

6 Isosceles triangle

$(0, 0), (a, 0), (b, c) \rightarrow$ **go to box 5**

$(-a, 0), (a, 0), (0, b) \rightarrow$ **you are out**

Hint: Check that two sides match for every choice of letters.

7 Parallelogram

$(0, 0), (a, 0), (a + b, c), (b, c) \rightarrow$ **go to box 1**

$(0, 0), (a, 0), (d, e), (b, c) \rightarrow$ **go to box 5**

Hint: The fourth corner is not free. Look at how it is built.

8 Rhombus

$(a, 0), (0, b), (-a, 0), (0, -b) \rightarrow$ **go to box 3**

$(0, 0), (a, 0), (a + b, c), (b, c) \rightarrow$ **go to box 5**

Hint: Check that all four sides match for every choice of letters.

Write every box number here, in order.

--	--	--	--	--	--	--	--	--	--



chooseyourlesson.com
135 free geometry lessons

The route, in order. Boxes 2 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table. A letter can be any number, positive, negative or zero, as long as the figure is real. This sheet calls a triangle isosceles when at least two sides are the same length, and a square is a rectangle and a rhombus. Some textbooks use other definitions, so tell the class which ones this sheet uses.

4 → 7 → 1 → 8 → 3 → 6 → out

	The box's question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 4. Which placement of a rectangle is general? (0, 0), (a, 0), (a, b), (0, b) or (0, 0), (a, 0), (a, a), (0, a)	(0, 0), (a, 0), (a, b), (0, b). Every choice of letters gives a rectangle, and the letters can make any rectangle.	(0, 0), (a, 0), (a, a), (0, a) goes to box 2. It forces too much: every figure it gives is a square, and a rectangle need not be one.
2	Box 7. Which placement of a parallelogram is general? (0, 0), (a, 0), (a + b, c), (b, c) or (0, 0), (a, 0), (d, e), (b, c)	(0, 0), (a, 0), (a + b, c), (b, c). Every choice of letters gives a parallelogram, and the letters can make any parallelogram.	(0, 0), (a, 0), (d, e), (b, c) goes to box 5. It forces too little. Put a = 1, b = 1, c = 1, d = 2 and e = 3: the corners are (0, 0), (1, 0), (2, 3), (1, 1), which is not a parallelogram. The side from (1, 0) to (2, 3) has slope 3 and the side from (1, 1) to (0, 0) has slope 1, so they are not parallel.
3	Box 1. Which placement of a triangle is general? (0, 0), (2a, 0), (2b, 2c) or (0, 0), (2a, 0), (a, b)	(0, 0), (2a, 0), (2b, 2c). Every choice of letters gives a triangle, and the letters can make any triangle.	(0, 0), (2a, 0), (a, b) goes to box 2. It forces too much: every figure it gives is an isosceles triangle, and a triangle need not be one.
4	Box 8. Which placement of a rhombus is general? (a, 0), (0, b), (−a, 0), (0, −b) or (0, 0), (a, 0), (a + b, c), (b, c)	(a, 0), (0, b), (−a, 0), (0, −b). Every choice of letters gives a rhombus, and the letters can make any rhombus.	(0, 0), (a, 0), (a + b, c), (b, c) goes to box 5. It forces too little. Put a = 1, b = 1 and c = 1: the corners are (0, 0), (1, 0), (2, 1), (1, 1), which is a parallelogram, not a rhombus.
5	Box 3. Which placement of a parallelogram is general? (0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c) or (0, 0), (2a, 0), (2a, 2c), (0, 2c)	(0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c). Every choice of letters gives a parallelogram, and the letters can make any parallelogram.	(0, 0), (2a, 0), (2a, 2c), (0, 2c) goes to box 2. It forces too much: every figure it gives is a rectangle, and a parallelogram need not be one.
6	Box 6. Which placement of an isosceles triangle is general? (−a, 0), (a, 0), (0, b) or (0, 0), (a, 0), (b, c)	(−a, 0), (a, 0), (0, b). Every choice of letters gives an isosceles triangle, and the letters can make any isosceles triangle.	(0, 0), (a, 0), (b, c) goes to box 5. It forces too little. Put a = 1, b = 1 and c = 2: the corners are (0, 0), (1, 0), (1, 2), which is a right triangle, not an isosceles triangle.
	Box 2. Which placement of a right triangle is general? (0, 0), (a, 0), (0, b) or (0, 0), (a, 0), (0, a)	(0, 0), (a, 0), (0, b). Every choice of letters gives a right triangle, and the letters can make any right triangle.	(0, 0), (a, 0), (0, a) goes to box 4. It forces too much: every figure it gives is an isosceles right triangle, and a right triangle need not be one.
	Box 5. Which placement of a square is general? (0, 0), (a, 0), (a, a), (0, a) or (0, 0), (a, 0), (a, b), (0, b)	(0, 0), (a, 0), (a, a), (0, a). Every choice of letters gives a square, and the letters can make any square.	(0, 0), (a, 0), (a, b), (0, b) goes to box 7. It forces too little. Put a = 1 and b = 2: the corners are (0, 0), (1, 0), (1, 2), (0, 2), which is a rectangle, not a square.

How to check any box. Pick small numbers for the letters and plot the corners. Is it always the figure named? Then ask whether the placement forces anything the name does not say.

A student can find the last four boxes of the route by following the arrows backward from “you are out.” So ask a student with a clean row to explain one box.