

MYSTERY PICTURE



PLACING FIGURES FOR COORDINATE PROOFS

General or Not? Placing a Figure on the Axes

25 lettered placements · is each one general? · nothing to calculate · a picture appears

FREE

1 Rectangle (0, 0), (a, 0), (a, a), (0, a)	2 Rhombus (0, 0), (a, 0), (a + b, c), (b, c)	3 Right triangle (0, 0), (2a, 0), (a, b)	4 Triangle (0, 0), (a, 0), (0, b)	5 Rectangle (0, 0), (a, 0), (a, b), (0, c)
6 Triangle (-a, 0), (a, 0), (b, c)	7 Isosceles triangle (0, 0), (a, 0), (0, b)	8 Parallelogram (0, 0), (a, 0), (a, c), (0, c)	9 Square (0, 0), (a, 0), (a, b), (0, b)	10 Parallelogram (0, 0), (2a, 0), (a + 2b, 2c), (2b, 2c)
11 Parallelogram (0, 0), (a, 0), (a + b, c), (b, c)	12 Isosceles triangle (0, 0), (2a, 0), (a, b)	13 Rhombus (a, 0), (0, a), (-a, 0), (0, -a)	14 Rectangle (0, 0), (a, 0), (a + b, b), (b, b)	15 Parallelogram (0, 0), (a, 0), (d, e), (b, c)
16 Rectangle (-a, -b), (a, -b), (a, b), (-a, b)	17 Right triangle (0, 0), (a, 0), (a, b)	18 Square (0, 0), (a, 0), (a, a), (0, a)	19 Isosceles triangle (0, 0), (2a, 0), (a, a)	20 Square (0, 0), (a, 0), (a, 2a), (0, 2a)
21 Rhombus (a, 0), (0, b), (-a, 0), (0, -b)	22 Triangle (0, 0), (2a, 0), (2b, 2c)	23 Isosceles triangle (-a, 0), (a, 0), (0, b)	24 Parallelogram (0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c)	25 Parallelogram (0, 0), (a, 0), (a, b), (0, c)

1. **Give each student page 3** and a pencil. Nothing is calculated.
2. **Say this once:** *“Each square gives a figure and a placement with letters. Is the placement general? It must give that figure for every choice of letters, and force nothing extra. If yes, fill the square with slanted lines. If no, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows the figure names, a good placement and the letter counts.
Answers	page 4, for you.

A placement is general when it does two things. Every choice of letters gives the figure named. And it forces nothing extra, so the letters can make every figure of that kind. Ten squares are general and are filled. The other fifteen are white.

The white squares are the mistakes students make. Five force too much, such as a rectangle placed with one letter, which makes it a square. Four force too little, such as a rhombus placed like any parallelogram. Six give the wrong figure, such as an isosceles triangle placed with a right angle at $(0, 0)$. Six white squares use exactly the right number of letters, so counting letters alone is not enough.

Some filled squares look wrong and are not. A letter with a minus sign, like $(-a, 0)$, can be any number. A figure centred on $(0, 0)$ is fine. Doubled letters, like $2a$, cost nothing.

If part of a student’s picture looks wrong, ask about that square: *“Pick numbers for the letters. Is it always the figure? Could it be any figure of that kind?”* Guessing does not work. Every simple guessing rule, such as filling the squares with the right number of letters, gets at least six squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Mystery Picture — general or not?

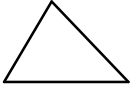
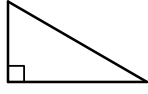
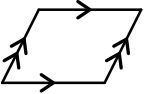
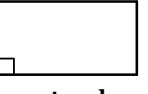
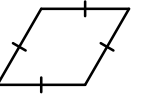
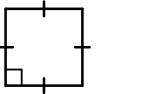
Placing a general figure

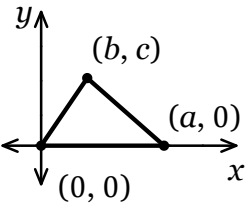
Name _____

Period _____

Date _____

Each square names a figure and gives a placement of it: its corners, with letters, in order. **Is the placement general? If yes, fill the square with slanted lines. If no, leave it white.** Check two things. Does every choice of letters give that figure? Does it force nothing extra? A picture appears. If part of it is missing, or one square does not fit, check that square again.

 triangle three corners, not in a line	 right triangle one right angle	 isosceles triangle at least two sides the same length	 parallelogram opposite sides parallel. It need not be a rectangle.	 rectangle four right angles	 rhombus four sides the same length	 square four right angles and four sides the same length
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General means the placement makes the named figure for every choice of letters, and it forces nothing extra. A general parallelogram does not have to be a rectangle.

A good placement puts one corner at $(0, 0)$ and one side along the x -axis, with letters where the numbers would be. A letter can be any number, positive, negative or zero, if the figure is real. A doubled letter, like $2a$, is any number too. The corners are listed in order around the figure.

Letters a general placement needs: triangle 3, parallelogram 3, right triangle 2, isosceles triangle 2, rectangle 2, rhombus 2, square 1. The right count is needed, but it is not enough.

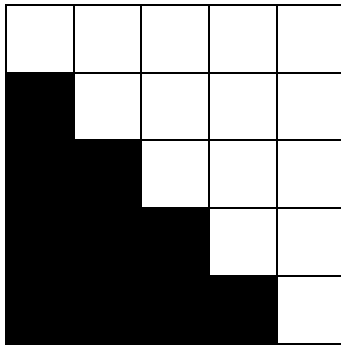
1 Rectangle $(0, 0), (a, 0), (a, a), (0, a)$	2 Rhombus $(0, 0), (a, 0), (a + b, c), (b, c)$	3 Right triangle $(0, 0), (2a, 0), (a, b)$	4 Triangle $(0, 0), (a, 0), (0, b)$	5 Rectangle $(0, 0), (a, 0), (a, b), (0, c)$
6 Triangle $(-a, 0), (a, 0), (b, c)$	7 Isosceles triangle $(0, 0), (a, 0), (0, b)$	8 Parallelogram $(0, 0), (a, 0), (a, c), (0, c)$	9 Square $(0, 0), (a, 0), (a, b), (0, b)$	10 Parallelogram $(0, 0), (2a, 0), (a + 2b, 2c), (2b, 2c)$
11 Parallelogram $(0, 0), (a, 0), (a + b, c), (b, c)$	12 Isosceles triangle $(0, 0), (2a, 0), (a, b)$	13 Rhombus $(a, 0), (0, a), (-a, 0), (0, -a)$	14 Rectangle $(0, 0), (a, 0), (a + b, b), (b, b)$	15 Parallelogram $(0, 0), (a, 0), (d, e), (b, c)$
16 Rectangle $(-a, -b), (a, -b), (a, b), (-a, b)$	17 Right triangle $(0, 0), (a, 0), (a, b)$	18 Square $(0, 0), (a, 0), (a, a), (0, a)$	19 Isosceles triangle $(0, 0), (2a, 0), (a, a)$	20 Square $(0, 0), (a, 0), (a, 2a), (0, 2a)$
21 Rhombus $(a, 0), (0, b), (-a, 0), (0, -b)$	22 Triangle $(0, 0), (2a, 0), (2b, 2c)$	23 Isosceles triangle $(-a, 0), (a, 0), (0, b)$	24 Parallelogram $(0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c)$	25 Parallelogram $(0, 0), (a, 0), (a, b), (0, c)$



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Mystery Picture — answers

For the teacher



The finished picture is a **staircase**. Ten squares are filled.

A square is filled when the placement always gives the figure named and forces nothing extra. A letter can be any number, positive, negative or zero, as long as the figure is real. This sheet calls a triangle isosceles when at least two sides are the same length, and a square is a rectangle and a rhombus. Some textbooks use other definitions, so tell the class which ones this sheet uses.

	The square	Answer	Why
1	Rectangle (0, 0), (a, 0), (a, a), (0, a)	white	It forces too much: every figure it gives is a square, and a rectangle need not be one.
2	Rhombus (0, 0), (a, 0), (a + b, c), (b, c)	white	It forces too little. Put a = 1, b = 1 and c = 1: the corners are (0, 0), (1, 0), (2, 1), (1, 1), which is a parallelogram, not a rhombus.
3	Right triangle (0, 0), (2a, 0), (a, b)	white	It is not always a right triangle. Put a = 1 and b = 2: the corners are (0, 0), (2, 0), (1, 2), which is an isosceles triangle, not a right triangle.
4	Triangle (0, 0), (a, 0), (0, b)	white	It forces too much: every figure it gives is a right triangle, and a triangle need not be one.
5	Rectangle (0, 0), (a, 0), (a, b), (0, c)	white	It forces too little. Put a = 1, b = 1 and c = 2: the corners are (0, 0), (1, 0), (1, 1), (0, 2), which is not a rectangle. The top side has slope -1 and the bottom side has slope 0, so they are not parallel.
6	Triangle (-a, 0), (a, 0), (b, c)	fill	Every choice of letters gives a triangle, and the letters can make any triangle. The minus sign is fine: a can be any number.
7	Isosceles triangle (0, 0), (a, 0), (0, b)	white	It is not always an isosceles triangle. Put a = 1 and b = 2: the corners are (0, 0), (1, 0), (0, 2), which is a right triangle, not an isosceles triangle.
8	Parallelogram (0, 0), (a, 0), (a, c), (0, c)	white	It forces too much: every figure it gives is a rectangle, and a parallelogram need not be one.
9	Square (0, 0), (a, 0), (a, b), (0, b)	white	It forces too little. Put a = 1 and b = 2: the corners are (0, 0), (1, 0), (1, 2), (0, 2), which is a rectangle, not a square.
10	Parallelogram (0, 0), (2a, 0), (a + 2b, 2c), (2b, 2c)	white	It is not always a parallelogram. Put a = 1, b = 1 and c = 1: the corners are (0, 0), (2, 0), (3, 2), (2, 2), which is not a parallelogram. The bottom side is 2 long and the top side is 1 long.
11	Parallelogram (0, 0), (a, 0), (a + b, c), (b, c)	fill	Every choice of letters gives a parallelogram, and the letters can make any parallelogram. The fourth corner is the sum of the two moves out of (0, 0).
12	Isosceles triangle (0, 0), (2a, 0), (a, b)	fill	Every choice of letters gives an isosceles triangle, and the letters can make any isosceles triangle. The top corner stands above the middle of the base.
13	Rhombus (a, 0), (0, a), (-a, 0), (0, -a)	white	It forces too much: every figure it gives is a square, and a rhombus need not be one.
14	Rectangle (0, 0), (a, 0), (a + b, b), (b, b)	white	It is not always a rectangle. Put a = 1 and b = 1: the corners are (0, 0), (1, 0), (2, 1), (1, 1), which is a parallelogram, not a rectangle.
15	Parallelogram (0, 0), (a, 0), (d, e), (b, c)	white	It forces too little. Put a = 1, b = 1, c = 1, d = 2 and e = 3: the corners are (0, 0), (1, 0), (2, 3), (1, 1), which is not a parallelogram. The side from (1, 0) to (2, 3) has slope 3 and the side from (1, 1) to (0, 0) has slope 1, so they are not parallel.
16	Rectangle (-a, -b), (a, -b), (a, b), (-a, b)	fill	Every choice of letters gives a rectangle, and the letters can make any rectangle. It is centred on (0, 0). The minus signs are fine.
17	Right triangle (0, 0), (a, 0), (a, b)	fill	Every choice of letters gives a right triangle, and the letters can make any right triangle. The right angle is at (a, 0), not at (0, 0). That is allowed.
18	Square (0, 0), (a, 0), (a, a), (0, a)	fill	Every choice of letters gives a square, and the letters can make any square. For a square, one letter is exactly right.
19	Isosceles triangle (0, 0), (2a, 0), (a, a)	white	It forces too much: every figure it gives is an isosceles right triangle, and an isosceles triangle need not be one.
20	Square (0, 0), (a, 0), (a, 2a), (0, 2a)	white	It is not always a square. Put a = 1: the corners are (0, 0), (1, 0), (1, 2), (0, 2), which is a rectangle, not a square.
21	Rhombus (a, 0), (0, b), (-a, 0), (0, -b)	fill	Every choice of letters gives a rhombus, and the letters can make any rhombus. Each side joins a corner on one axis to a corner on the other, and every side is the same length.
22	Triangle (0, 0), (2a, 0), (2b, 2c)	fill	Every choice of letters gives a triangle, and the letters can make any triangle. Doubling every letter costs nothing: 2a is any number too.
23	Isosceles triangle (-a, 0), (a, 0), (0, b)	fill	Every choice of letters gives an isosceles triangle, and the letters can make any isosceles triangle. The minus sign is fine: a can be any number.
24	Parallelogram (0, 0), (2a, 0), (2a + 2b, 2c), (2b, 2c)	fill	Every choice of letters gives a parallelogram, and the letters can make any parallelogram. Every letter is doubled, so it is still general.
25	Parallelogram (0, 0), (a, 0), (a, b), (0, c)	white	It is not always a parallelogram. Put a = 1, b = 1 and c = 2: the corners are (0, 0), (1, 0), (1, 1), (0, 2), which is not a parallelogram. The top side has slope -1 and the bottom side has slope 0, so they are not parallel.