

CUT & GLUE SORT



WRITING COORDINATE PROOFS

Does the Proof Work? Judging the Last Sentence

Twelve short proofs · slopes, lengths, midpoints · three rows

FREE

1 Prove: Triangle ABC is isosceles. $AB = \sqrt{13}$ $BC = \sqrt{26}$ $CA = \sqrt{13}$ <i>Last sentence:</i> $AB = CA$, so triangle ABC is isosceles.	2 Prove: ABCD is a parallelogram. $AB = \sqrt{29}$ $BC = \sqrt{10}$ $CD = \sqrt{29}$ $DA = \sqrt{10}$ <i>Last sentence:</i> Both pairs of opposite sides are parallel, so ABCD is a parallelogram.	3 Prove: ABCD is a parallelogram. slope of $AB = 0$ slope of $CD = 0$ $BC = \sqrt{13}$ $DA = \sqrt{13}$ <i>Last sentence:</i> AB is parallel to CD and $BC = DA$, so ABCD is a parallelogram.
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Row 1. Correct: the numbers prove the claim, and the last sentence names the rule they show.

Row 2. Wrong rule: the numbers prove the claim, but the last sentence names a rule they do not show.

Row 3. Not enough: the numbers do not prove the claim.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Every number on the pieces is correct. Do the numbers prove the claim? Does the last sentence name the rule the numbers show? Use the box at the top of the page.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows and a box that shows which tool proves which kind of claim. Page 4 has the pieces.
Answers	page 5.

The last sentence of a coordinate proof is where most marks are lost. It must name the rule the numbers actually show. Distances show congruent segments. Slopes show parallel or perpendicular segments. Midpoints show segments that bisect each other.

Row 2 holds the mistake students find hardest to see. Every statement in the sentence is true, and the figure really is what the sentence says. But the sentence names a rule the page never computed. For example, piece 2 computes four lengths, and its last sentence says the opposite sides are parallel.

Row 3 holds pieces where the numbers are not enough. One pair of parallel sides and one pair of congruent sides can be an isosceles trapezoid. Four congruent sides make a rhombus, not always a square. Two pairs of parallel sides with no right angle make a parallelogram, not a rectangle. And numbers for one rectangle prove nothing about every rectangle. The piece in row 1 that proves the same claim uses letters for the coordinates. Guessing does not work: a simple guessing rule, such as sorting by how many numbers a piece shows, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — does the proof work?

Writing coordinate proofs

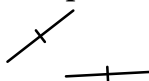
Name _____

Period _____

Date _____

Each piece on the next page is a short proof. Every number on the pieces is correct. **Read the claim, the numbers and the last sentence. Then glue the piece in its row.**

Which tool proves which kind of claim



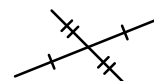
Distance: equal lengths show two segments are **congruent**



Slope: equal slopes show two segments are **parallel**



Slope: slopes that multiply to -1 show **perpendicular** segments



Midpoint: the same midpoint shows two segments **bisect each other**

Numbers for one figure prove a claim about that figure only. A claim about *every* figure needs letters, like p and q , in the coordinates.

Row 1. Correct: the numbers prove the claim, and the last sentence names the rule they show.

Row 2. Wrong rule: the numbers prove the claim, but the last sentence names a rule they do not show.

Row 3. Not enough: the numbers do not prove the claim.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Glue each one in its row on the other sheet. Every number on the pieces is correct. Distance shows congruent segments. Slope shows parallel or perpendicular segments. Midpoint shows segments that bisect each other. Numbers for one figure prove a claim about that figure only.

Parallelogram: both pairs of opposite sides parallel, or both pairs congruent, or diagonals that bisect each other, or one pair of opposite sides parallel and congruent.
Rectangle: a parallelogram with a right angle. **Rhombus:** four congruent sides, or a parallelogram whose diagonals are perpendicular. **Square:** a rhombus with a right angle.
Right triangle: two sides are perpendicular. **Isosceles triangle:** two sides are congruent.

1 Prove: Triangle ABC is isosceles. $AB = \sqrt{13}$ $BC = \sqrt{26}$ $CA = \sqrt{13}$ <i>Last sentence:</i> $AB = CA$, so triangle ABC is isosceles.	2 Prove: ABCD is a parallelogram. $AB = \sqrt{29}$ $BC = \sqrt{10}$ $CD = \sqrt{29}$ $DA = \sqrt{10}$ <i>Last sentence:</i> Both pairs of opposite sides are parallel, so ABCD is a parallelogram.	3 Prove: ABCD is a parallelogram. slope of $AB = 0$ slope of $CD = 0$ $BC = \sqrt{13}$ $DA = \sqrt{13}$ <i>Last sentence:</i> AB is parallel to CD and $BC = DA$, so ABCD is a parallelogram.	4 Prove: ABCD is a parallelogram. midpoint of $AC = (0, 1)$ midpoint of $BD = (0, 1)$ <i>Last sentence:</i> $AB = CD$ and $BC = DA$, so ABCD is a parallelogram.
5 Prove: The diagonals of every rectangle are congruent. Take $A(0, 0)$, $B(6, 0)$, $C(6, 4)$, $D(0, 4)$. $AC = \sqrt{52}$ $BD = \sqrt{52}$ <i>Last sentence:</i> $AC = BD$, so the diagonals of every rectangle are congruent.	6 Prove: ABCD is a parallelogram. midpoint of $AC = (1, 1)$ midpoint of $BD = (1, 1)$ <i>Last sentence:</i> The diagonals bisect each other, so ABCD is a parallelogram.	7 Prove: ABCD is a square. $AB = \sqrt{10}$ $BC = \sqrt{10}$ $CD = \sqrt{10}$ $DA = \sqrt{10}$ <i>Last sentence:</i> All four sides are congruent, so ABCD is a square.	8 Prove: ABCD is a parallelogram. slope of $AB = -1/4$ slope of $BC = 3$ slope of $CD = -1/4$ slope of $DA = 3$ <i>Last sentence:</i> Both pairs of opposite sides are parallel, so ABCD is a parallelogram.
9 Prove: ABCD is a rectangle. slope of $AB = 1/2$ slope of $BC = -2$ slope of $CD = 1/2$ slope of $DA = -2$ <i>Last sentence:</i> Both pairs of opposite sides are congruent and AB is perpendicular to BC, so ABCD is a rectangle.	10 Prove: The diagonals of every rectangle are congruent. Place it at $A(0, 0)$, $B(2p, 0)$, $C(2p, 2q)$, $D(0, 2q)$. $AC = \sqrt{4p^2 + 4q^2}$ $BD = \sqrt{4p^2 + 4q^2}$ <i>Last sentence:</i> $AC = BD$, so the diagonals of every rectangle are congruent.	11 Prove: ABCD is a parallelogram. slope of $AB = -1/4$ slope of $BC = 3/2$ slope of $CD = -1/4$ slope of $DA = 3/2$ <i>Last sentence:</i> The diagonals bisect each other, so ABCD is a parallelogram.	12 Prove: ABCD is a rectangle. slope of $AB = -1/5$ slope of $BC = 3/2$ slope of $CD = -1/5$ slope of $DA = 3/2$ <i>Last sentence:</i> Both pairs of opposite sides are parallel, so ABCD is a rectangle.



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Each row holds four pieces. Every piece is described again here, with the reason. This sheet uses these rules. Parallelogram: both pairs of opposite sides parallel, or both pairs congruent, or diagonals that bisect each other, or one pair of opposite sides parallel and congruent. Rectangle: a parallelogram with a right angle. Rhombus: four congruent sides, or a parallelogram whose diagonals are perpendicular. Square: a rhombus with a right angle. Right triangle: two sides are perpendicular. Isosceles triangle: two sides are congruent. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 **Correct: the numbers prove the claim, and the last sentence names the rule they show.**

1. Prove: Triangle ABC is isosceles. $AB = \sqrt{13}$, $BC = \sqrt{26}$, $CA = \sqrt{13}$. Last sentence: $AB = CA$, so triangle ABC is isosceles. *Why:* The lengths show that $AB = CA$. That is a rule for this claim, and the last sentence names it.

6. Prove: ABCD is a parallelogram. midpoint of $AC = (1, 1)$, midpoint of $BD = (1, 1)$. Last sentence: The diagonals bisect each other, so ABCD is a parallelogram. *Why:* The midpoints show that the diagonals bisect each other. That is a rule for this claim, and the last sentence names it.

8. Prove: ABCD is a parallelogram. slope of $AB = -1/4$, slope of $BC = 3$, slope of $CD = -1/4$, slope of $DA = 3$. Last sentence: Both pairs of opposite sides are parallel, so ABCD is a parallelogram. *Why:* The slopes show that both pairs of opposite sides are parallel. That is a rule for this claim, and the last sentence names it.

10. Prove: The diagonals of every rectangle are congruent. Place it at $A(0, 0)$, $B(2p, 0)$, $C(2p, 2q)$, $D(0, 2q)$. $AC = \sqrt{(4p^2 + 4q^2)}$, $BD = \sqrt{(4p^2 + 4q^2)}$. Last sentence: $AC = BD$, so the diagonals of every rectangle are congruent. *Why:* The coordinates use letters, so the work holds for every rectangle. AC and BD have the same length, so they are congruent, and the sentence says so.

2 **Wrong rule: the numbers prove the claim, but the last sentence names a rule they do not show.**

2. Prove: ABCD is a parallelogram. $AB = \sqrt{29}$, $BC = \sqrt{10}$, $CD = \sqrt{29}$, $DA = \sqrt{10}$. Last sentence: Both pairs of opposite sides are parallel, so ABCD is a parallelogram. *Why:* The lengths show that both pairs of opposite sides are congruent, which is enough for the claim. But the sentence says both pairs of opposite sides are parallel, and lengths cannot show that.

4. Prove: ABCD is a parallelogram. midpoint of $AC = (0, 1)$, midpoint of $BD = (0, 1)$. Last sentence: $AB = CD$ and $BC = DA$, so ABCD is a parallelogram. *Why:* The midpoints show that the diagonals bisect each other, which is enough for the claim. But the sentence says both pairs of opposite sides are congruent, and midpoints cannot show that.

9. Prove: ABCD is a rectangle. slope of $AB = 1/2$, slope of $BC = -2$, slope of $CD = 1/2$, slope of $DA = -2$. Last sentence: Both pairs of opposite sides are congruent and AB is perpendicular to BC, so ABCD is a rectangle. *Why:* The slopes show that both pairs of opposite sides are parallel and AB is perpendicular to BC, which is enough for the claim. But the sentence says both pairs of opposite sides are congruent, and slopes cannot show that.

11. Prove: ABCD is a parallelogram. slope of $AB = -1/4$, slope of $BC = 3/2$, slope of $CD = -1/4$, slope of $DA = 3/2$. Last sentence: The diagonals bisect each other, so ABCD is a parallelogram. *Why:* The slopes show that both pairs of opposite sides are parallel, which is enough for the claim. But the sentence says the diagonals bisect each other, and slopes cannot show that.

3 **Not enough: the numbers do not prove the claim.**

3. Prove: ABCD is a parallelogram. slope of $AB = 0$, slope of $CD = 0$, $BC = \sqrt{13}$, $DA = \sqrt{13}$. Last sentence: AB is parallel to CD and $BC = DA$, so ABCD is a parallelogram. *Why:* The slopes and lengths show only that AB is parallel to CD and $BC = DA$. That is not enough for the claim. For example, the points $A(0, 0)$, $B(6, 0)$, $C(4, 3)$ and $D(2, 3)$ give these numbers, and that figure is not a parallelogram.

5. Prove: The diagonals of every rectangle are congruent. Take $A(0, 0)$, $B(6, 0)$, $C(6, 4)$, $D(0, 4)$. $AC = \sqrt{52}$, $BD = \sqrt{52}$. Last sentence: $AC = BD$, so the diagonals of every rectangle are congruent. *Why:* The numbers are for one rectangle only. A claim about every rectangle needs letters in the coordinates.

7. Prove: ABCD is a square. $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$. Last sentence: All four sides are congruent, so ABCD is a square. *Why:* The lengths show only that all four sides are congruent. That is not enough for the claim. For example, the points $A(0, 0)$, $B(3, 1)$, $C(4, 4)$ and $D(1, 3)$ give these numbers, and that figure is not a square.

12. Prove: ABCD is a rectangle. slope of $AB = -1/5$, slope of $BC = 3/2$, slope of $CD = -1/5$, slope of $DA = 3/2$. Last sentence: Both pairs of opposite sides are parallel, so ABCD is a rectangle. *Why:* The slopes show only that both pairs of opposite sides are parallel. That is not enough for the claim. For example, the points $A(0, 0)$, $B(5, -1)$, $C(7, 2)$ and $D(2, 3)$ give these numbers, and that figure is not a rectangle.

To check a piece with a student, ask: “Which numbers did the page compute? Which rule do they show? Is that rule enough for the claim?”