

MYSTERY PICTURE



WRITING COORDINATE PROOFS

Do the Numbers Prove It? Coordinate Proofs

25 lines of proofs · which numbers prove which claim · nothing to calculate · a picture appears

FREE

1 slope of $AB = 1/2$, slope of $CD = 1/2$, so AB is parallel to CD .	2 $AB = \sqrt{13}$, $AC = \sqrt{13}$, so triangle ABC is isosceles.	3 midpoint of $AC = (2, 0)$, midpoint of $BD = (2, 0)$, so AC and BD bisect each other.	4 slope of $AB = 2$, slope of $BC = -1/2$, so AB is perpendicular to BC .	5 $AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB is parallel to CD .
6 slope of $AB = 1/2$, slope of $CD = 1/2$, so AB and CD are congruent.	7 slope of $AB = 2$, slope of $CD = 2$, so $ABCD$ is a parallelogram.	8 $AM = 5$, $MB = 5$, so M is the midpoint of AB .	9 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a rhombus.	10 $AB = \sqrt{26}$, $BC = \sqrt{10}$, $CD = \sqrt{26}$, $DA = \sqrt{10}$, so $ABCD$ is a parallelogram.
11 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a square.	12 midpoint of $AB = (1, 4)$, $M = (1, 4)$, so M is the midpoint of AB .	13 slope of $AB = 1/3$, slope of $BC = -3$, so triangle ABC is a right triangle.	14 For $A(0, 0)$, $B(2p, 0)$, $C(2p, 2q)$ and $D(0, 2q)$: $AC = \sqrt{4p^2 + 4q^2}$, $BD = \sqrt{4p^2 + 4q^2}$, so the diagonals of every rectangle are congruent.	15 midpoint of $AC = (2, 1)$, midpoint of $BD = (2, 1)$, so $ABCD$ is a rectangle.
16 For $A(0, 0)$, $B(6, 0)$, $C(6, 4)$ and $D(0, 4)$: $AC = \sqrt{52}$, $BD = \sqrt{52}$, so the diagonals of every rectangle are congruent.	17 slope of $AB = 3$, slope of $AC = -3$, so triangle ABC is isosceles.	18 slope of $AB = 1/2$, slope of $BC = -2$, so $ABCD$ is a rectangle.	19 slope of $AB = -1/2$, slope of $BC = 2$, slope of $CD = -1/2$, slope of $DA = 2$, so $ABCD$ is a rectangle.	20 $AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB and CD are congruent.
21 midpoint of $AC = (1, 1)$, midpoint of $BD = (1, 1)$, so $ABCD$ is a parallelogram.	22 slope of $AB = 2/5$, slope of $CD = 2/5$, $AB = \sqrt{29}$, $CD = \sqrt{29}$, so $ABCD$ is a parallelogram.	23 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, slope of $AB = -1/3$, slope of $BC = 3$, so $ABCD$ is a square.	24 slope of $AB = -1/4$, slope of $BC = 3/2$, slope of $CD = -1/4$, slope of $DA = 3/2$, so $ABCD$ is a parallelogram.	25 $AC = \sqrt{34}$, $BD = \sqrt{34}$, so $ABCD$ is a rectangle.

Mystery Picture — how to run it

25–35 minutes

1. **Give each student page 3** and a pencil. Nothing is calculated.
2. **Say this once:** *“Each square is one line of a proof. Every number is correct. Do the numbers prove what comes after ‘so’? If yes, fill the square with slanted lines. If no, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows which tool proves which kind of claim, and the rules for each figure.
Answers	page 4, for you.

The question in every square is the check a proof writer should make on the last sentence: point at the numbers that prove it. Fifteen squares are filled and ten are white, and the finished picture is the number 3.

The white squares are the mistakes students make. Four use the wrong tool, such as two equal lengths “proving” two sides parallel. Five name a rule that is not enough, such as four congruent sides for a square. One uses numbers for one rectangle to prove a claim about every rectangle; a filled square proves the same claim with letters.

If part of a student’s picture looks wrong, ask about that square: *“Which tool made these numbers? What can that tool show?”* Guessing does not work. Every simple guessing rule, such as filling every square that says “parallel”, gets at least ten squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture — do the numbers prove it?

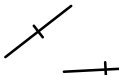
Writing coordinate proofs

Name _____

Period _____

Date _____

Each square is one line of a proof about quadrilateral ABCD or triangle ABC. Every number is correct. A true name counts even when a more specific name also fits. **Do the numbers prove what comes after “so”?** If yes, fill the square with slanted lines. If no, leave it white. A picture appears. If part of it is missing, or one square does not fit, check that square again.

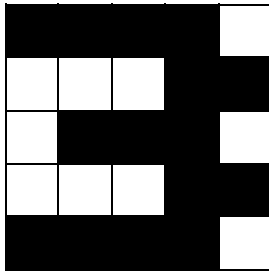
Which tool proves which kind of claim							
	Distance: equal lengths → congruent		Slope: equal slopes → parallel		Slope: slopes multiply to -1 → perpendicular		Midpoint: same midpoint → bisect each other
Parallelogram: both pairs of opposite sides parallel, or both pairs congruent, or diagonals that bisect each other, or one pair of opposite sides parallel and congruent. Rectangle: a parallelogram with a right angle. Rhombus: four congruent sides, or a parallelogram whose diagonals are perpendicular. Square: a rhombus with a right angle. Right triangle: two sides are perpendicular. Isosceles triangle: two sides are congruent. Numbers for one figure prove a claim about that figure only. A claim about <i>every</i> figure needs letters, like p and q , in the coordinates.							
1 slope of $AB = 1/2$, slope of $CD = 1/2$, so AB is parallel to CD .	2 $AB = \sqrt{13}$, $AC = \sqrt{13}$, so triangle ABC is isosceles.	3 midpoint of $AC = (2, 0)$, midpoint of $BD = (2, 0)$, so AC and BD bisect each other.	4 slope of $AB = 2$, slope of $BC = -1/2$, so AB is perpendicular to BC .	5 $AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB is parallel to CD .			
6 slope of $AB = 1/2$, slope of $CD = 1/2$, so AB and CD are congruent.	7 slope of $AB = 2$, slope of $CD = 2$, so $ABCD$ is a parallelogram.	8 $AM = 5$, $MB = 5$, so M is the midpoint of AB .	9 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a rhombus.	10 $AB = \sqrt{26}$, $BC = \sqrt{10}$, $CD = \sqrt{26}$, $DA = \sqrt{10}$, so $ABCD$ is a parallelogram.			
11 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a square.	12 midpoint of $AB = (1, 4)$, $M = (1, 4)$, so M is the midpoint of AB .	13 slope of $AB = 1/3$, slope of $BC = -3$, so triangle ABC is a right triangle.	14 For $A(0, 0)$, $B(2p, 0)$, $C(2p, 2q)$ and $D(0, 2q)$: $AC = \sqrt{4p^2 + 4q^2}$, $BD = \sqrt{4p^2 + 4q^2}$, so the diagonals of every rectangle are congruent.	15 midpoint of $AC = (2, 1)$, midpoint of $BD = (2, 1)$, so $ABCD$ is a rectangle.			
16 For $A(0, 0)$, $B(6, 0)$, $C(6, 4)$ and $D(0, 4)$: $AC = \sqrt{52}$, $BD = \sqrt{52}$, so the diagonals of every rectangle are congruent.	17 slope of $AB = 3$, slope of $AC = -3$, so triangle ABC is isosceles.	18 slope of $AB = 1/2$, slope of $BC = -2$, so $ABCD$ is a rectangle.	19 slope of $AB = -1/2$, slope of $BC = 2$, slope of $CD = -1/2$, slope of $DA = 2$, so $ABCD$ is a rectangle.	20 $AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB and CD are congruent.			
21 midpoint of $AC = (1, 1)$, midpoint of $BD = (1, 1)$, so $ABCD$ is a parallelogram.	22 slope of $AB = 2/5$, slope of $CD = 2/5$, $AB = \sqrt{29}$, $CD = \sqrt{29}$, so $ABCD$ is a parallelogram.	23 $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, slope of $AB = -1/3$, slope of $BC = 3$, so $ABCD$ is a square.	24 slope of $AB = -1/4$, slope of $BC = 3/2$, slope of $CD = -1/4$, slope of $DA = 3/2$, so $ABCD$ is a parallelogram.	25 $AC = \sqrt{34}$, $BD = \sqrt{34}$, so $ABCD$ is a rectangle.			



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Mystery Picture — answers

For the teacher



The finished picture is **the number 3**. Fifteen squares are filled.

A square is filled when its numbers prove what comes after “so”. This sheet uses these rules. Parallelogram: both pairs of opposite sides parallel, or both pairs congruent, or diagonals that bisect each other, or one pair of opposite sides parallel and congruent. Rectangle: a parallelogram with a right angle. Rhombus: four congruent sides, or a parallelogram whose diagonals are perpendicular. Square: a rhombus with a right angle. Right triangle: two sides are perpendicular. Isosceles triangle: two sides are congruent. Some textbooks use other definitions, so tell the class which ones this sheet uses.

	The line	Answer	Why
1	slope of $AB = 1/2$, slope of $CD = 1/2$, so AB is parallel to CD .	fill	The slopes show that AB is parallel to CD . That is exactly what the claim needs.
2	$AB = \sqrt{13}$, $AC = \sqrt{13}$, so triangle ABC is isosceles.	fill	The lengths show that $AB = AC$. That is exactly what the claim needs. The figure is also a right triangle; a true name still counts.
3	midpoint of $AC = (2, 0)$, midpoint of $BD = (2, 0)$, so AC and BD bisect each other.	fill	The midpoints show that the diagonals bisect each other. That is exactly what the claim needs.
4	slope of $AB = 2$, slope of $BC = -1/2$, so AB is perpendicular to BC .	fill	The slopes show that AB is perpendicular to BC . That is exactly what the claim needs.
5	$AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB is parallel to CD .	white	The lengths show only that $AB = CD$. That is not what the claim is about, so lengths cannot prove it.
6	slope of $AB = 1/2$, slope of $CD = 1/2$, so AB and CD are congruent.	white	The slopes show only that AB is parallel to CD . That is not what the claim is about, so slopes cannot prove it.
7	slope of $AB = 2$, slope of $CD = 2$, so $ABCD$ is a parallelogram.	white	The slopes show that AB is parallel to CD . That is only part of what the claim needs.
8	$AM = 5$, $MB = 5$, so M is the midpoint of AB .	white	$AM = MB$ shows only that M is as far from A as from B . M need not be on AB : every point on the perpendicular bisector of AB has $AM = MB$. So this does not prove that M is the midpoint.
9	$AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a rhombus.	fill	The lengths show that all four sides are congruent. That is exactly what the claim needs.
10	$AB = \sqrt{26}$, $BC = \sqrt{10}$, $CD = \sqrt{26}$, $DA = \sqrt{10}$, so $ABCD$ is a parallelogram.	fill	The lengths show that both pairs of opposite sides are congruent. That is exactly what the claim needs.
11	$AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, so $ABCD$ is a square.	white	The lengths show that all four sides are congruent. That is only part of what the claim needs.
12	midpoint of $AB = (1, 4)$, $M = (1, 4)$, so M is the midpoint of AB .	fill	The midpoints show that M is the midpoint of AB . That is exactly what the claim needs.
13	slope of $AB = 1/3$, slope of $BC = -3$, so triangle ABC is a right triangle.	fill	The slopes show that AB is perpendicular to BC . That is exactly what the claim needs. The figure is also isosceles; a true name still counts.
14	For $A(0, 0)$, $B(2p, 0)$, $C(2p, 2q)$ and $D(0, 2q)$: $AC = \sqrt{(4p^2 + 4q^2)}$, $BD = \sqrt{(4p^2 + 4q^2)}$, so the diagonals of every rectangle are congruent.	fill	The coordinates use letters, so the lengths hold for every rectangle. Equal lengths show congruent diagonals.
15	midpoint of $AC = (2, 1)$, midpoint of $BD = (2, 1)$, so $ABCD$ is a rectangle.	white	The midpoints show that the diagonals bisect each other. That is only part of what the claim needs.
16	For $A(0, 0)$, $B(6, 0)$, $C(6, 4)$ and $D(0, 4)$: $AC = \sqrt{52}$, $BD = \sqrt{52}$, so the diagonals of every rectangle are congruent.	white	These numbers are for one rectangle. They prove nothing about every rectangle.
17	slope of $AB = 3$, slope of $AC = -3$, so triangle ABC is isosceles.	white	Slopes give directions, not lengths. For example, $A(0, 0)$, $B(1, 3)$ and $C(2, -6)$ have these slopes, but $AB^2 = 10$ and $AC^2 = 40$, so that triangle is not isosceles.
18	slope of $AB = 1/2$, slope of $BC = -2$, so $ABCD$ is a rectangle.	white	The slopes show that AB is perpendicular to BC . That is only part of what the claim needs.
19	slope of $AB = -1/2$, slope of $BC = 2$, slope of $CD = -1/2$, slope of $DA = 2$, so $ABCD$ is a rectangle.	fill	The slopes show that both pairs of opposite sides are parallel and AB is perpendicular to BC . That is exactly what the claim needs.
20	$AB = \sqrt{20}$, $CD = \sqrt{20}$, so AB and CD are congruent.	fill	The lengths show that $AB = CD$. That is exactly what the claim needs.
21	midpoint of $AC = (1, 1)$, midpoint of $BD = (1, 1)$, so $ABCD$ is a parallelogram.	fill	The midpoints show that the diagonals bisect each other. That is exactly what the claim needs.
22	slope of $AB = 2/5$, slope of $CD = 2/5$, $AB = \sqrt{29}$, $CD = \sqrt{29}$, so $ABCD$ is a parallelogram.	fill	The slopes and lengths show that AB is parallel to CD and $AB = CD$. That is exactly what the claim needs.
23	$AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$, $DA = \sqrt{10}$, slope of $AB = -1/3$, slope of $BC = 3$, so $ABCD$ is a square.	fill	The lengths and slopes show that all four sides are congruent and AB is perpendicular to BC . That is exactly what the claim needs.
24	slope of $AB = -1/4$, slope of $BC = 3/2$, slope of $CD = -1/4$, slope of $DA = 3/2$, so $ABCD$ is a parallelogram.	fill	The slopes show that both pairs of opposite sides are parallel. That is exactly what the claim needs.
25	$AC = \sqrt{34}$, $BD = \sqrt{34}$, so $ABCD$ is a rectangle.	white	The lengths show that $AC = BD$. That is only part of what the claim needs.