

CUT & GLUE SORT



DERIVING THE EQUATION OF A CIRCLE

On the Circle or Not? Testing a Point with the Equation

Twelve circles and points · equations, centers, radii, drawings · three rows

FREE

1 A circle has center $(4, -1)$ and radius 13. Is $P(9, 11)$ on it?	2 A circle has center $(2, -1)$ and radius 5. Is $P(3, 1)$ on it?	3 A circle has radius 10. Is $P(6, 8)$ on it?
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Row 1. Yes. P is on the circle.

Row 2. No. P is not on the circle.

Row 3. Cannot tell. The piece does not give one circle.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Is P on the circle? Put P’s numbers into the left side of the equation. If you get r squared, P is on the circle. If the piece does not give one circle, put it in row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows the circle, its right triangle and the rules.
Answers	page 5.

A point is on the circle exactly when its numbers make the equation true. Put x and y into $(x - h)^2 + (y - k)^2$ and compare the result with r^2 . The left side is the square of P’s distance from the center, so equal means P is exactly one radius away.

Two pieces are drawn, and they use the same words. On piece 8, P is on the circle. On piece 4, P misses by about 2% of the radius, which nobody can see. Only the numbers decide. Two more pieces catch the usual mistakes: on piece 9, a student who copies the signs of the center from the page finds P on the wrong circle, and on piece 2, a student who compares with r instead of r^2 says yes.

Row 3 holds the pieces that do not give one circle. The equation needs both the center and the radius. A center alone, a radius alone, two points, or an equation with r missing are not enough. Many students assume a circle with only a radius has its center at $(0, 0)$. The key gives two circles for each row 3 piece: one with P on it and one without, and both fit the piece. Guessing does not work: a simple guessing rule, such as sorting by whether the piece prints an equation, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — on the circle or not?

The equation of a circle

Name _____

Period _____

Date _____

Each piece on the next page gives a circle and a point P. **Is P on the circle? Glue the piece in its row.** Use only what the piece gives. Decide from the numbers. Do not measure the drawings.

Row 1. Yes. P is on the circle.

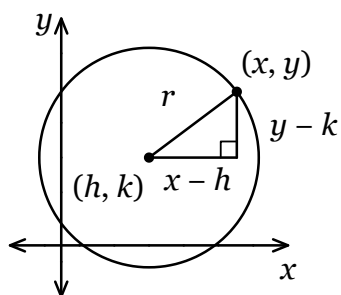
Row 2. No. P is not on the circle.

Row 3. Cannot tell. The piece does not give one circle.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Is P on the circle? Glue each piece in its row on the other sheet. Use only what the piece gives. Decide from the numbers. Do not measure the drawings.



A circle is every point at the same distance r from its center (h, k) . The right triangle gives the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

Each bracket is a difference. $(x - 3)^2$ means $h = 3$. $(x + 3)^2$ is $(x - (-3))^2$, so $h = -3$. A lone x^2 means $h = 0$. The same goes for y and k .

The right side is r^2 , the radius times itself. If the right side is 49, then $r = 7$. If the radius is 7, the right side is 49.

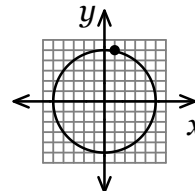
Is a point on the circle? Put its x and y into the left side. Equal to r^2 : on the circle. Not equal: not on it. Decide from the numbers, never by looking.

1 A circle has center $(4, -1)$ and radius 13. Is $P(9, 11)$ on it?

2 A circle has center $(2, -1)$ and radius 5. Is $P(3, 1)$ on it?

3 A circle has radius 10. Is $P(6, 8)$ on it?

4 The dot is $P(1, 5)$. It looks like it might be on the circle $x^2 + y^2 = 25$. Is it?

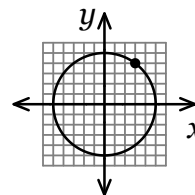


5 A circle has the equation $(x - 3)^2 + (y + 1)^2 = r^2$. The radius r is not given. Is $P(6, 3)$ on it?

6 A circle has the equation $(x + 2)^2 + (y - 1)^2 = 100$. Is $P(4, 9)$ on it?

7 A circle has center $(5, 2)$. Is $P(8, 6)$ on it?

8 The dot is $P(3, 4)$. It looks like it might be on the circle $x^2 + y^2 = 25$. Is it?



9 A circle has the equation $(x + 3)^2 + (y + 1)^2 = 25$. Is $P(6, 5)$ on it?

10 A circle has center $(1, 1)$ and radius $2\sqrt{5}$. Is $P(5, 3)$ on it?

11 A circle passes through $(3, 4)$ and $(-3, 4)$. Is $P(0, 5)$ on it?

12 A circle has center $(-1, 4)$ and passes through $(2, 8)$. Is $P(-5, 8)$ on it?



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Each row holds four pieces. Every piece is described again here, with the reason. This sheet writes the equation as $(x - h)^2 + (y - k)^2 = r^2$. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 **Yes. P is on the circle.**

1. A circle has center $(4, -1)$ and radius 13. Is $P(9, 11)$ on it? *Why:* The equation is $(x - 4)^2 + (y + 1)^2 = 169$. At P : $5^2 + 12^2 = 25 + 144 = 169$, equal to $r^2 = 169$, so P is on the circle.

6. A circle has the equation $(x + 2)^2 + (y - 1)^2 = 100$. Is $P(4, 9)$ on it? *Why:* At P : $6^2 + 8^2 = 36 + 64 = 100$, equal to $r^2 = 100$, so P is on the circle.

8. The dot is $P(3, 4)$. It looks like it might be on the circle $x^2 + y^2 = 25$. Is it? *Why:* At P : $3^2 + 4^2 = 9 + 16 = 25$, equal to $r^2 = 25$, so P is on the circle.

10. A circle has center $(1, 1)$ and radius $2\sqrt{5}$. Is $P(5, 3)$ on it? *Why:* The equation is $(x - 1)^2 + (y - 1)^2 = 20$. At P : $4^2 + 2^2 = 16 + 4 = 20$, equal to $r^2 = 20$, so P is on the circle.

2 **No. P is not on the circle.**

2. A circle has center $(2, -1)$ and radius 5. Is $P(3, 1)$ on it? *Why:* The equation is $(x - 2)^2 + (y + 1)^2 = 25$. At P : $1^2 + 2^2 = 1 + 4 = 5$, not 25, so P is not on the circle. A student who compares with the radius 5 instead of $r^2 = 25$ says yes.

4. The dot is $P(1, 5)$. It looks like it might be on the circle $x^2 + y^2 = 25$. Is it? *Why:* At P : $1^2 + 5^2 = 1 + 25 = 26$, not 25, so P is not on the circle. P misses by about 2% of the radius, too little to see.

9. A circle has the equation $(x + 3)^2 + (y + 1)^2 = 25$. Is $P(6, 5)$ on it? *Why:* At P : $9^2 + 6^2 = 81 + 36 = 117$, not 25, so P is not on the circle. A student who reads the center as $(3, 1)$ gets $9 + 16 = 25$ and says yes.

12. A circle has center $(-1, 4)$ and passes through $(2, 8)$. Is $P(-5, 8)$ on it? *Why:* The equation is $(x + 1)^2 + (y - 4)^2 = 25$. At P : $(-4)^2 + 4^2 = 16 + 16 = 32$, not 25, so P is not on the circle.

3 **Cannot tell. The piece does not give one circle.**

3. A circle has radius 10. Is $P(6, 8)$ on it? *Why:* A radius alone does not give the center. The center does not have to be $(0, 0)$. For example, P is on $x^2 + y^2 = 100$ but not on $(x + 1)^2 + y^2 = 100$, and both fit the piece.

5. A circle has the equation $(x - 3)^2 + (y + 1)^2 = r^2$. The radius r is not given. Is $P(6, 3)$ on it? *Why:* Without r , the center alone is known. For example, P is on $(x - 3)^2 + (y + 1)^2 = 25$ but not on $(x - 3)^2 + (y + 1)^2 = 26$, and both fit the piece.

7. A circle has center $(5, 2)$. Is $P(8, 6)$ on it? *Why:* A center alone does not give the radius. For example, P is on $(x - 5)^2 + (y - 2)^2 = 25$ but not on $(x - 5)^2 + (y - 2)^2 = 26$, and both fit the piece.

11. A circle passes through $(3, 4)$ and $(-3, 4)$. Is $P(0, 5)$ on it? *Why:* Many circles pass through two points. For example, P is on $x^2 + y^2 = 25$ but not on $x^2 + (y + 1)^2 = 34$, and both fit the piece.

To check a piece with a student, ask: “Do you know the center? Do you know the radius? What do P ’s numbers give on the left side?”