

ANSWER MAZE



DERIVING THE EQUATION OF A CIRCLE

Write It, Read It: The Equation of a Circle

Center, radius and equation · one page · 10–15 minutes · nothing to prepare

FREE

3

A circle has center $(-2, 5)$ and radius 3.
Write its equation.

$$(x - 2)^2 + (y + 5)^2 = 9 \rightarrow \text{go to box 2}$$

$$(x + 2)^2 + (y - 5)^2 = 9 \rightarrow \text{go to box 6}$$

Hint: Each bracket is x minus the center's x , and y minus the center's y . Subtracting a negative gives a plus.

6

Find the radius of the circle
 $(x - 4)^2 + (y + 1)^2 = 18$. Give it exactly.

$$3\sqrt{2} \rightarrow \text{go to box 1}$$

$$18 \rightarrow \text{go to box 5}$$

Hint: The right side holds the radius times itself. Undo that with a square root, then simplify the root.

1. **Hand out page 3**, one per student, and scrap paper. No calculator is needed.
2. **Say this once:** *“Start at box 3. Each box gives a circle and asks for its equation, its center or its radius. Pick one answer and go to the box it names. Write every box number in the row at the bottom as you go.”*
3. **Check a sheet quickly.** A student with no mistakes writes **3 6 1 8 4 7**: six boxes, then out.

Student sheet	page 3, one per student. A box at the top shows the circle, its right triangle, and the rules for the signs and the radius.
Answers	page 4.

Every box uses $(x - h)^2 + (y - k)^2 = r^2$. The brackets are the two legs of a right triangle from the center (h, k) to a point (x, y) on the circle, and the radius r is its hypotenuse. So each bracket is a difference, and the right side is r^2 , the radius squared. The boxes go both ways: some give a center and a radius and ask for the equation, and some give the equation and ask for the center or the radius. One gives a center and a point on the circle.

Every wrong answer is one of two mistakes. Copying the signs of the center from the page, so that $(x + 6)^2$ reads as $h = 6$, sends the student to box 2. Mixing up the radius with its square, so that a right side of 18 is taken as the radius or a radius of 7 is written as “= 7”, sends the student to box 5. Boxes 2 and 5 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more, and the extra box number tells you which mistake it was. Guessing does not get a student out: every simple guessing rule, such as always taking the larger number, goes wrong within the first two boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Answer Maze — the equation of a circle

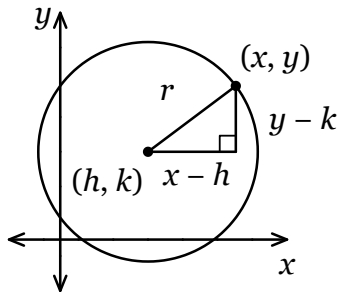
Start at box 3

Name _____

Period _____

Date _____

Start at box 3. **Each box gives a circle and asks for its equation, its center or its radius.** Pick one answer and go to the box it names. Write each box number in the row at the bottom as you go.



A circle is every point at the same distance r from its center (h, k) . The right triangle gives the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

Each bracket is a difference. $(x - 3)^2$ means $h = 3$. $(x + 3)^2$ is $(x - (-3))^2$, so $h = -3$. A lone x^2 means $h = 0$. The same goes for y and k .

The right side is r^2 , the radius times itself. If the right side is 49, then $r = 7$. If the radius is 7, the right side is 49.

Give a radius exactly. If the right side is not a perfect square, write r in simplest radical form: 12 gives $r = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$.

1 Find the center of the circle

$$(x + 6)^2 + y^2 = 10.$$

$(6, 0) \rightarrow$ **go to box 2**

$(-6, 0) \rightarrow$ **go to box 8**

Hint: Rewrite x plus a number as x minus its opposite. A lone y^2 means y minus zero.

2 A circle has center $(4, -3)$ and radius 6. Write its equation.

$$(x - 4)^2 + (y + 3)^2 = 36 \rightarrow$$
 go to box 6

$$(x + 4)^2 + (y - 3)^2 = 36 \rightarrow$$
 go to box 3

Hint: Each bracket subtracts a coordinate of the center. Check the sign in front of each number.

3 A circle has center $(-2, 5)$ and radius 3. Write its equation.

$$(x - 2)^2 + (y + 5)^2 = 9 \rightarrow$$
 go to box 2

$$(x + 2)^2 + (y - 5)^2 = 9 \rightarrow$$
 go to box 6

Hint: Each bracket is x minus the center's x , and y minus the center's y . Subtracting a negative gives a plus.

4 Find the center of the circle

$$(x - 1)^2 + (y + 8)^2 = 50.$$

$(-1, 8) \rightarrow$ **go to box 2**

$(1, -8) \rightarrow$ **go to box 7**

Hint: Read each bracket as a difference: x minus h , and y minus k .

5 Find the radius of the circle

$$(x + 2)^2 + (y - 2)^2 = 36.$$
 Give it exactly.

$36 \rightarrow$ **go to box 6**

$6 \rightarrow$ **go to box 1**

Hint: Read the rules at the top: what does the right side hold? Ask which number times itself gives it.

6 Find the radius of the circle

$$(x - 4)^2 + (y + 1)^2 = 18.$$
 Give it exactly.

$3\sqrt{2} \rightarrow$ **go to box 1**

$18 \rightarrow$ **go to box 5**

Hint: The right side holds the radius times itself. Undo that with a square root, then simplify the root.

7 A circle has center $(2, 3)$ and passes through $(5, 7)$. Write its equation.

$$(x - 2)^2 + (y - 3)^2 = 25 \rightarrow$$
 you are out

$$(x - 2)^2 + (y - 3)^2 = 5 \rightarrow$$
 go to box 5

Hint: Draw the triangle from the center to the point. Square both legs and add them.

8 A circle has center $(3, -1)$ and radius 7. Write its equation.

$$(x - 3)^2 + (y + 1)^2 = 49 \rightarrow$$
 go to box 4

$$(x - 3)^2 + (y + 1)^2 = 7 \rightarrow$$
 go to box 5

Hint: The radius is the hypotenuse of the triangle. Look at how r is written in the equation at the top of the page.

Write every box number here, in order.

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The route, in order. Boxes 2 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table. This sheet writes the equation as $(x - h)^2 + (y - k)^2 = r^2$ with the right side as a whole number, and gives a radius exactly, in simplest radical form when it is not a whole number. Some textbooks use other definitions, so tell the class which ones this sheet uses.

3 → 6 → 1 → 8 → 4 → 7 → out

	The box's question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 3 A circle has center $(-2, 5)$ and radius 3. Write its equation.	$(x + 2)^2 + (y - 5)^2 = 9$. The brackets are $x - (-2)$, which is $x + 2$, and $y - 5$. The right side is $r^2 = 3 \times 3 = 9$.	" $(x - 2)^2 + (y + 5)^2 = 9$ " goes to box 2. A student who copies the signs of the center into the brackets gets this. Each bracket is a difference, so a negative coordinate shows as a plus.
2	Box 6 Find the radius of the circle $(x - 4)^2 + (y + 1)^2 = 18$. Give it exactly.	$3\sqrt{2}$. The right side is $r^2 = 18$, so $r = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$.	"18" goes to box 5. A student who takes the right side as the radius gets this. The right side is r^2 , so r is its square root.
3	Box 1 Find the center of the circle $(x + 6)^2 + y^2 = 10$.	$(-6, 0)$. $(x + 6)^2$ is $(x - (-6))^2$, so $h = -6$. A lone y^2 means $k = 0$.	" $(6, 0)$ " goes to box 2. A student who copies the signs from the page gets this. Each bracket is a difference, so a plus means a negative coordinate.
4	Box 8 A circle has center $(3, -1)$ and radius 7. Write its equation.	$(x - 3)^2 + (y + 1)^2 = 49$. The brackets are $x - 3$ and $y - (-1)$, which is $y + 1$. The right side is $r^2 = 7 \times 7 = 49$.	" $(x - 3)^2 + (y + 1)^2 = 7$ " goes to box 5. A student who writes the radius on the right side gets this. The right side is r^2 , which is 49.
5	Box 4 Find the center of the circle $(x - 1)^2 + (y + 8)^2 = 50$.	$(1, -8)$. $(x - 1)^2$ gives $h = 1$. $(y + 8)^2$ is $(y - (-8))^2$, so $k = -8$.	" $(-1, 8)$ " goes to box 2. A student who copies the signs from the page gets this. Each bracket is a difference, so a plus means a negative coordinate.
6	Box 7 A circle has center $(2, 3)$ and passes through $(5, 7)$. Write its equation.	$(x - 2)^2 + (y - 3)^2 = 25$. The legs from the center to the point are 3 and 4, so $r^2 = 3^2 + 4^2 = 25$. The brackets are $x - 2$ and $y - 3$.	" $(x - 2)^2 + (y - 3)^2 = 5$ " goes to box 5. A student who finds the distance 5 and stops gets this. The right side is the distance squared, 25.
	Box 2 A circle has center $(4, -3)$ and radius 6. Write its equation.	$(x - 4)^2 + (y + 3)^2 = 36$. The brackets are $x - 4$ and $y - (-3)$, which is $y + 3$. The right side is $r^2 = 6 \times 6 = 36$.	" $(x + 4)^2 + (y - 3)^2 = 36$ " goes to box 3. A student who copies the signs of the center into the brackets gets this. Each bracket is a difference, so a negative coordinate shows as a plus.
	Box 5 Find the radius of the circle $(x + 2)^2 + (y - 2)^2 = 36$. Give it exactly.	6. The right side is $r^2 = 36$, and $6 \times 6 = 36$.	"36" goes to box 6. A student who takes the right side as the radius gets this. The right side is r^2 , so r is its square root.

How to check any box. Write each bracket as x minus a number and y minus a number. Those two numbers are the center. The right side is the radius times itself.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.