

SCAVENGER HUNT



DERIVING THE EQUATION OF A CIRCLE

Center and Radius: Reading the Equation of a Circle

$(x - h)^2 + (y - k)^2 = r^2$ · the sign of the center · the radius, exactly · the cards check the answers

FREE

ANSWER

9

A circle has the equation

$$(x + 4)^2 + (y - 1)^2 = 36$$

Find its center.

Both brackets are zero at the center.

N

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1. **Tape the 16 cards around the room**, in any order. Each card has a large letter in its bottom left corner. Start each group at a *different* one of these eight cards: *N A S M Q F B J*.
2. **Say this once:** “Each card gives the equation of a circle. It asks for the center or the radius. Write each bracket as a difference first. The right side is *r squared*, not *r*. Then walk to the card with your answer at the top.”
3. **Each group writes one line per card.** Eight right answers bring the group back to its first card.

16 cards	pages 3–10, one copy. There are two cards on each sheet. Cut the whole stack once.
Record sheet	page 11, one per student. It shows the circle, its right triangle and the rules, with a worked example.
Answers	pages 12–13, for you only. Do not put them up in the room.

The equation $(x - h)^2 + (y - k)^2 = r^2$ is the distance formula. The two brackets are the legs of a right triangle from the center (h, k) to a point (x, y) on the circle, and r is its hypotenuse. So each bracket is a difference, and the right side is the radius squared. The record sheet’s example:

$(x + 3)^2 + (y - 5)^2 = 12$ has center $(-3, 5)$ and radius $\sqrt{12} = 2\sqrt{3}$.

Eight of the sixteen cards make the ring. A group reaches the other eight only by a wrong answer, but all the cards look the same. The ring asks for a center and a radius in turn, so the two most common mistakes take turns. On a center card, some students copy the signs from the page and read $(x + 4)^2$ as $h = 4$. On a radius card, some students give the right side itself as the radius. Each mistake lands on a card off the ring, and that card sends the group back to the card it got wrong.

The check is the length of the group’s list. A clean run is **eight lines**. Each wrong answer adds two. Every card has a line in italics under its question. It tells a group that arrived by a wrong answer what to check. A group that arrived by a right answer skips it. Guessing does not work: no simple rule, such as walking to the next answer in the key, gets a single card right.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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ANSWER

$(-4, 1)$

A circle has the equation

$$(x - 2)^2 + (y + 5)^2 = 20$$

Find its radius. Give it exactly.

Leave a root that is not whole in simplest radical form.

A

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ANSWER

$3\sqrt{3}$

A circle has the equation

$$(x - 2)^2 + (y + 7)^2 = 25$$

Find its center.

A minus sign in front of a negative number makes a plus.

B

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ANSWER

(5, 3)

A circle has the equation

$$(x + 2)^2 + (y - 6)^2 = 64$$

Find its radius. Give it exactly.

Read each bracket as x minus h. Is the sign of h the opposite of the sign shown?

D

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ANSWER

(4, -1)

A circle has the equation

$$x^2 + (y - 5)^2 = 81$$

Find its radius. Give it exactly.

Each bracket is a difference. Check the sign: $x + 4$ means $x - (-4)$.

E

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ANSWER

$(-5, -3)$

A circle has the equation

$$(x - 6)^2 + (y - 2)^2 = 27$$

Find its radius. Give it exactly.

Does the root simplify? Look for a square factor like 4 or 9.

F

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ANSWER

81

A circle has the equation

$$(x - 2)^2 + (y + 7)^2 = 13$$

Find its center.

The right side holds the radius times itself. Find the root.

G

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ANSWER

$(2, -7)$

A circle has the equation

$$(x + 3)^2 + (y - 4)^2 = 81$$

Find its radius. Give it exactly.

The equation comes from the Pythagorean theorem, with r as the hypotenuse.

J

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ANSWER

$(-2, 7)$

A circle has the equation

$$(x - 4)^2 + (y + 2)^2 = 27$$

Find its radius. Give it exactly.

Subtracting a negative number adds. Check the sign of each coordinate.

L

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ANSWER

(3, -6)

A circle has the equation

$$(x + 1)^2 + (y + 2)^2 = 64$$

Find its radius. Give it exactly.

Check: the radius times itself must give the right side.

M

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ANSWER

9

A circle has the equation

$$(x + 4)^2 + (y - 1)^2 = 36$$

Find its center.

Both brackets are zero at the center.

N

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ANSWER

20

A circle has the equation

$$(x + 4)^2 + (y - 1)^2 = 50$$

Find its center.

The right side is r times r. Take its square root to find r.

P

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ANSWER

8

A circle has the equation

$$(x + 5)^2 + (y + 3)^2 = 18$$

Find its center.

Plot the center on scrap paper. Check it against both brackets.

Q

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ANSWER

$2\sqrt{5}$

A circle has the equation

$$(x - 3)^2 + (y + 6)^2 = 49$$

Find its center.

Is each bracket a difference? Read h and k from $x - h$ and $y - k$.

S

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ANSWER

64

A circle has the equation

$$(x - 3)^2 + (y + 6)^2 = 29$$

Find its center.

Is the number on the right r or r^2 ? Its square root is r .

T

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ANSWER

27

A circle has the equation

$$(x + 5)^2 + (y + 3)^2 = 40$$

Find its center.

The radius is the square root of the right side, in simplest form.

U

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ANSWER

(-3, 6)

A circle has the equation

$$(x - 1)^2 + (y + 4)^2 = 20$$

Find its radius. Give it exactly.

A minus in a bracket gives a positive coordinate. A plus gives a negative one.

V

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Scavenger Hunt — where I went

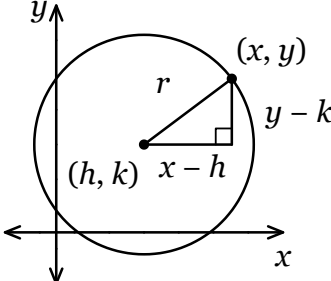
Center and radius

Name _____

Period _____

Date _____

Each card gives the equation of a circle and asks for its center or its radius. Find the card with your answer at the top. A clean run is eight lines. Each card has a line in italics under its question. **If you are on a card because you went wrong, its line says what to check. If you were right, skip it.**



A circle is every point at the same distance r from its center (h, k) . The right triangle gives the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

Each bracket is a difference. $(x - 3)^2$ means $h = 3$. $(x + 3)^2$ is $(x - (-3))^2$, so $h = -3$. A lone x^2 means $h = 0$. The same goes for y and k .

The right side is r^2 , the radius times itself. If the right side is 49, then $r = 7$. If the radius is 7, the right side is 49.

Example: $(x + 3)^2 + (y - 5)^2 = 12$ is $(x - (-3))^2 + (y - 5)^2 = 12$. The center is $(-3, 5)$. The radius is $\sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$.
Give a radius exactly: if the right side is not a perfect square, write r in simplest radical form.

Card letter	The equation, with each bracket written as a difference	Center or radius
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

What does the center of a circle say about the distances to all its points?

Every answer stands for a letter. Write the letter for each answer in the box above it. Read the boxes you filled, left to right. **If they do not spell the answer, one of your answers is wrong.**

$2\sqrt{5} = A$ $3\sqrt{3} = L$ $8 = L$ $9 = E$ $20 = L$ $27 = A$ $64 = L$ $81 = A$ $(-5, -3) = Q$ $(-4, 1) = U$ $(-3, 6) = Q$ $(-2, 7) = L$
 $(2, -7) = A$ $(3, -6) = L$ $(4, -1) = U$ $(5, 3) = E$

$2\sqrt{5}$	$3\sqrt{3}$	8	9	20	27	64	81	$(-5, -3)$	$(-4, 1)$	$(-3, 6)$	$(-2, 7)$	$(2, -7)$	$(3, -6)$	$(4, -1)$	$(5, 3)$



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Scavenger Hunt — answers

For the teacher

The ring in order, with the answer found on each card. A group may begin anywhere on it. Wherever it begins, eight right answers bring it back to that card.

N $(-4, 1) \rightarrow$ **A** $2\sqrt{5} \rightarrow$ **S** $(3, -6) \rightarrow$ **M** $8 \rightarrow$ **Q** $(-5, -3) \rightarrow$ **F** $3\sqrt{3} \rightarrow$ **B** $(2, -7) \rightarrow$ **J** $9 \rightarrow$ **N**

What does the center of a circle say about the distances to all its points? The answer is *ALL EQUAL!* It is read along the boxes of the record sheet, which run in this order: the radii from smallest to largest, then the centers by their first coordinate.

	Card	What the card asks	Answer, and why	The common wrong answer on this card, and why	Letter of its top
1	N	$(x + 4)^2 + (y - 1)^2 = 36$. Find its center.	$(-4, 1)$. $(x + 4)^2$ is $(x - (-4))^2$, so $h = -4$. $(y - 1)^2$ gives $k = 1$. The center is $(-4, 1)$.	$(4, -1)$: the group copies the signs from the page. Each bracket is a difference, so a plus means a negative coordinate.	E
2	A	$(x - 2)^2 + (y + 5)^2 = 20$. Find its radius.	$2\sqrt{5}$. The right side is $r^2 = 20$, so $r = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$.	20: the group takes the right side as the radius. The right side is r^2 , so r is its square root.	U
3	S	$(x - 3)^2 + (y + 6)^2 = 49$. Find its center.	$(3, -6)$. $(x - 3)^2$ gives $h = 3$. $(y + 6)^2$ is $(y - (-6))^2$, so $k = -6$. The center is $(3, -6)$.	$(-3, 6)$: the group copies the signs from the page. Each bracket is a difference, so a plus means a negative coordinate.	A
4	M	$(x + 1)^2 + (y + 2)^2 = 64$. Find its radius.	8 . The right side is $r^2 = 64$, and $8 \times 8 = 64$, so $r = 8$.	64: the group takes the right side as the radius. The right side is r^2 , so r is its square root.	L
5	Q	$(x + 5)^2 + (y + 3)^2 = 18$. Find its center.	$(-5, -3)$. $(x + 5)^2$ is $(x - (-5))^2$, so $h = -5$. $(y + 3)^2$ is $(y - (-3))^2$, so $k = -3$. The center is $(-5, -3)$.	$(5, 3)$: the group copies the signs from the page. Each bracket is a difference, so a plus means a negative coordinate.	L
6	F	$(x - 6)^2 + (y - 2)^2 = 27$. Find its radius.	$3\sqrt{3}$. The right side is $r^2 = 27$, so $r = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$.	27: the group takes the right side as the radius. The right side is r^2 , so r is its square root.	Q
7	B	$(x - 2)^2 + (y + 7)^2 = 25$. Find its center.	$(2, -7)$. $(x - 2)^2$ gives $h = 2$. $(y + 7)^2$ is $(y - (-7))^2$, so $k = -7$. The center is $(2, -7)$.	$(-2, 7)$: the group copies the signs from the page. Each bracket is a difference, so a plus means a negative coordinate.	L
8	J	$(x + 3)^2 + (y - 4)^2 = 81$. Find its radius.	9 . The right side is $r^2 = 81$, and $9 \times 9 = 81$, so $r = 9$.	81: the group takes the right side as the radius. The right side is r^2 , so r is its square root.	A

Scavenger Hunt — the cards off the ring

For the teacher

A group reaches these eight cards only by a wrong answer. The value at the top of each one is the wrong answer from one ring card. Its right answer is the value at the top of that ring card, so the group goes to the card it got wrong and tries it again.

Card	What the card asks	Answer, and why	The common wrong answer on this card, and why	Letter of its top
E	$x^2 + (y - 5)^2 = 81$. Find its radius.	9. The right side is $r^2 = 81$, and $9 \times 9 = 81$, so $r = 9$.	—	U
P	$(x + 4)^2 + (y - 1)^2 = 50$. Find its center.	(-4, 1). $(x + 4)^2$ is $(x - (-4))^2$, so $h = -4$. $(y - 1)^2$ gives $k = 1$. The center is $(-4, 1)$.	—	L
V	$(x - 1)^2 + (y + 4)^2 = 20$. Find its radius.	$2\sqrt{5}$. The right side is $r^2 = 20$, so $r = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$.	—	Q
T	$(x - 3)^2 + (y + 6)^2 = 29$. Find its center.	(3, -6). $(x - 3)^2$ gives $h = 3$. $(y + 6)^2$ is $(y - (-6))^2$, so $k = -6$. The center is $(3, -6)$.	—	L
D	$(x + 2)^2 + (y - 6)^2 = 64$. Find its radius.	8. The right side is $r^2 = 64$, and $8 \times 8 = 64$, so $r = 8$.	—	E
U	$(x + 5)^2 + (y + 3)^2 = 40$. Find its center.	(-5, -3). $(x + 5)^2$ is $(x - (-5))^2$, so $h = -5$. $(y + 3)^2$ is $(y - (-3))^2$, so $k = -3$. The center is $(-5, -3)$.	—	A
L	$(x - 4)^2 + (y + 2)^2 = 27$. Find its radius.	$3\sqrt{3}$. The right side is $r^2 = 27$, so $r = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$.	—	L
G	$(x - 2)^2 + (y + 7)^2 = 13$. Find its center.	(2, -7). $(x - 2)^2$ gives $h = 2$. $(y + 7)^2$ is $(y - (-7))^2$, so $k = -7$. The center is $(2, -7)$.	—	A

Other mistakes lead to no card. A group might copy only one of the two signs from the page, swap the x and y coordinates of the center, or halve the right side to get the radius. Its answer is then not at the top of any card, and the group knows to check its work. A radius that is not a whole number is written in simplest radical form, like $2\sqrt{5}$, never as a decimal. Some textbooks use other definitions, so tell the class which ones this sheet uses.