

CUT & GLUE SORT



EQUATION OF A CIRCLE: COMPLETING THE SQUARE

Which Circle, If Any? Completing the Square

Twelve equations · two centers · and the ones that are not circles at all

FREE

1	2	3
$x^2 + y^2 - 4x + 6y - 23 = 0$	$x^2 - 6y + y^2 + 4x + 9 = 0$	$2x^2 - 8x + 2y^2 + 12y + 40 = 0$
$r^2 =$ _____	$r^2 =$ _____	$r^2 =$ _____

Row 1. It is a circle with center $(2, -3)$.

Row 2. It is a circle with center $(-2, 3)$.

Row 3. It is not a circle. The number on the right is 0 or less than 0.

Cut & Glue Sort — how to run it

25 minutes, including the cutting

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“Complete both squares on each piece and write the number on the right. If it is 0 or less, the equation is not a circle: row 3. If not, read the center and glue it in row 1 or row 2.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box with the three moves and a worked example.
Answers	page 5.

Every piece needs the whole routine. The two centers, $(2, -3)$ and $(-2, 3)$, have opposite signs, so a student who copies the signs of the brackets into the center puts the piece in the wrong circle row. The terms are in different orders, some stand on the right of the $=$ sign, and two pieces share a common factor.

Row 3 holds the equations that are not circles. After both squares are complete, the number on the right is r^2 . When it is less than 0, no point fits, because two squares never add to a negative number. When it is exactly 0, only the center fits: a single point. Four pieces belong here, one of them a single point. A student who takes the square root anyway puts them in row 1 or row 2.

A student who adds 4 and 9 to the left side only puts four circles in row 3: pieces 2, 6, 9 and 10. On these, the number moved to the right is 0 or less (-9 , -10 , 0 and -4). A student whose rows hold uneven numbers of pieces has made a mistake somewhere. Guessing does not work: a simple rule, such as sorting by the sign in front of x , places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Cut & Glue Sort — which circle, if any?

Completing the square

Name _____

Period _____

Date _____

Each piece on the next page is one equation. **Complete both squares. Write the number on the right, r^2 , on the piece. Then glue the piece in its row.** The three moves are on the next page.

Row 1. It is a circle with center $(2, -3)$.

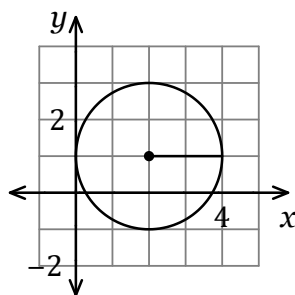
Row 2. It is a circle with center $(-2, 3)$.

Row 3. It is not a circle. The number on the right is 0 or less than 0.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. For each one, complete both squares and write r^2 on it. Is it a circle with center $(2, -3)$, a circle with center $(-2, 3)$, or not a circle? Glue it in its row on the other sheet.



Built form: $(x - h)^2 + (y - k)^2 = r^2$. The center is (h, k) . The radius is r .

1. Group and clear. Put the x terms together and the y terms together. Move the number alone to the right side. A term that crosses the $=$ sign changes its sign. If every term has a common factor, divide every term by it.

2. Complete both squares. Halve the number in front of x . Square it. Add it to **both** sides. Do the same for y .

3. Read. The center comes from the brackets. The number on the right is r^2 . The radius is its square root.

Example: $x^2 + y^2 - 4x - 2y + 1 = 0$

becomes $(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$

so $(x - 2)^2 + (y - 1)^2 = 4$. The dot is the center $(2, 1)$. The radius is 2.

1 $x^2 + y^2 - 4x + 6y - 23 = 0$ $r^2 =$ _____	2 $x^2 - 6y + y^2 + 4x + 9 = 0$ $r^2 =$ _____	3 $2x^2 - 8x + 2y^2 + 12y + 40 = 0$ $r^2 =$ _____	4 $x^2 + y^2 + 4x - 6y = 23$ $r^2 =$ _____
5 $x^2 + y^2 + 4x - 6y = -16$ $r^2 =$ _____	6 $2x^2 + 2y^2 - 8x + 12y + 20 = 0$ $r^2 =$ _____	7 $x^2 + y^2 - 4x + 6y + 13 = 0$ $r^2 =$ _____	8 $x^2 + y^2 - 7 = 4x - 6y$ $r^2 =$ _____
9 $x^2 + y^2 - 6y = -4x$ $r^2 =$ _____	10 $x^2 - 4x + y^2 + 6y + 4 = 0$ $r^2 =$ _____	11 $x^2 + y^2 - 6y - 36 = -4x$ $r^2 =$ _____	12 $x^2 + y^2 + 25 = -4x + 6y$ $r^2 =$ _____



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Each row holds four pieces. Every piece is written again here, with the number on the right and the reason. On this sheet an equation is a circle only when the number on the right is more than 0; 0 gives a single point and less than 0 gives no points. Some textbooks use other definitions, so tell the class which ones this sheet uses.

1 It is a circle with center (2, -3).

1. $x^2 + y^2 - 4x + 6y - 23 = 0$ ($r^2 = 36$) *Why:* First move the number to the right, changing its sign. Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = 36$, since $23 + 4 + 9 = 36$. The center is $(2, -3)$ and the radius is 6 .

6. $2x^2 + 2y^2 - 8x + 12y + 20 = 0$ ($r^2 = 3$) *Why:* First move the number to the right, changing its sign, then divide every term by 2 . Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = 3$, since $-10 + 4 + 9 = 3$. The center is $(2, -3)$ and the radius is $\sqrt{3}$.

8. $x^2 + y^2 - 7 = 4x - 6y$ ($r^2 = 20$) *Why:* First move the x and y terms to the left, changing their signs, then move the number to the right, changing its sign. Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = 20$, since $7 + 4 + 9 = 20$. The center is $(2, -3)$ and the radius is $2\sqrt{5}$.

10. $x^2 - 4x + y^2 + 6y + 4 = 0$ ($r^2 = 9$) *Why:* First move the number to the right, changing its sign. Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = 9$, since $-4 + 4 + 9 = 9$. The center is $(2, -3)$ and the radius is 3 .

2 It is a circle with center (-2, 3).

2. $x^2 - 6y + y^2 + 4x + 9 = 0$ ($r^2 = 4$) *Why:* First move the number to the right, changing its sign. Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = 4$, since $-9 + 4 + 9 = 4$. The center is $(-2, 3)$ and the radius is 2 .

4. $x^2 + y^2 + 4x - 6y = 23$ ($r^2 = 36$) *Why:* Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = 36$, since $23 + 4 + 9 = 36$. The center is $(-2, 3)$ and the radius is 6 .

9. $x^2 + y^2 - 6y = -4x$ ($r^2 = 13$) *Why:* First move the x term to the left, changing its sign, then move the number to the right, changing its sign. Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = 13$, since $0 + 4 + 9 = 13$. The center is $(-2, 3)$ and the radius is $\sqrt{13}$.

11. $x^2 + y^2 - 6y - 36 = -4x$ ($r^2 = 49$) *Why:* First move the x term to the left, changing its sign, then move the number to the right, changing its sign. Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = 49$, since $36 + 4 + 9 = 49$. The center is $(-2, 3)$ and the radius is 7 .

3 It is not a circle. The number on the right is 0 or less than 0.

3. $2x^2 - 8x + 2y^2 + 12y + 40 = 0$ ($r^2 = -7$) *Why:* First move the number to the right, changing its sign, then divide every term by 2 . Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = -7$, since $-20 + 4 + 9 = -7$. The number on the right is less than 0 , and two squares can never add to a negative number, so no point fits.

5. $x^2 + y^2 + 4x - 6y = -16$ ($r^2 = -3$) *Why:* Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = -3$, since $-16 + 4 + 9 = -3$. The number on the right is less than 0 , and two squares can never add to a negative number, so no point fits.

7. $x^2 + y^2 - 4x + 6y + 13 = 0$ ($r^2 = 0$) *Why:* First move the number to the right, changing its sign. Half of -4 is -2 , so the number to add is $(-2)^2 = 4$. Half of 6 is 3 , so the number to add is $3^2 = 9$. Adding 4 and 9 to both sides gives $(x - 2)^2 + (y + 3)^2 = 0$, since $-13 + 4 + 9 = 0$. The number on the right is 0 , so only the point $(2, -3)$ fits: a single point, not a circle.

12. $x^2 + y^2 + 25 = -4x + 6y$ ($r^2 = -12$) *Why:* First move the x and y terms to the left, changing their signs, then move the number to the right, changing its sign. Half of 4 is 2 , so the number to add is $2^2 = 4$. Half of -6 is -3 , so the number to add is $(-3)^2 = 9$. Adding 4 and 9 to both sides gives $(x + 2)^2 + (y - 3)^2 = -12$, since $-25 + 4 + 9 = -12$. The number on the right is less than 0 , and two squares can never add to a negative number, so no point fits.

To check a piece with a student, ask: “What did you add on the left? Is it on the right too? What is the number on the right now?”