

ANSWER MAZE



EQUATION OF A CIRCLE: COMPLETING THE SQUARE

Center and Radius: Complete the Square

Equations of circles in any layout · one page · 10–15 minutes · nothing to prepare

FREE

3 $x^2 - 6x + y^2 + 4y - 3 = 0$

center $(3, -2)$, radius 16 → **go to box 2**

center $(3, -2)$, radius 4 → **go to box 6**

Hint: Check what the number on the right stands for.

6 $x^2 + y^2 + 8x - 2y = 8$

center $(4, -1)$, radius 5 → **go to box 5**

center $(-4, 1)$, radius 5 → **go to box 1**

Hint: Look at the sign inside each bracket.

1. **Hand out page 3**, one per student, and scrap paper. No calculator is needed.
2. **Say this once:** *“Start at box 3. Each box has the equation of a circle. Complete both squares and find the center and the radius. Pick the answer you get and go to the box it names. Write every box number in the row at the bottom.”*
3. **Check a sheet quickly.** A student with no mistakes writes **3 6 1 8 4 7**: six boxes, then out.

Student sheet	page 3, one per student. A box at the top shows the three moves, a worked example with its circle, and when an equation is not a circle.
Answers	page 4.

Every box is the same job in a different layout. Students put the x terms together and the y terms together, move the number to the right, complete both squares, and add each new number to both sides. The center comes from the brackets, and the number on the right is r^2 . If that number is 0 or less, the equation is not a circle. Box 1 is such an equation.

Each wrong answer is a real mistake. A mistake in the radius sends the student to box 2: writing r^2 as the radius, adding the new numbers to the left side only, or taking a negative number as r^2 . A mistake in the center sends the student to box 5: copying the signs of the brackets, squaring without halving, or moving a term across the $=$ sign without changing its sign. Boxes 2 and 5 ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more, and the extra box number tells you whether the center or the radius went wrong. Guessing does not get a student out: every simple guessing rule, such as always taking the smaller radius, goes wrong within the first three boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Answer Maze — center and radius

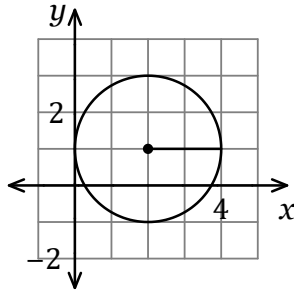
Start at box 3

Name _____

Period _____

Date _____

Start at box 3. **Each box has the equation of a circle. Find the center and the radius.** Pick the answer you get and go to the box it names. Write each box number in the row at the bottom as you go.



Built form: $(x - h)^2 + (y - k)^2 = r^2$. The center is (h, k) . The radius is r .

1. Group and clear. Put the x terms together and the y terms together. Move the number alone to the right side. A term that crosses the $=$ sign changes its sign. If every term has a common factor, divide every term by it.

2. Complete both squares. Halve the number in front of x . Square it. Add it to **both** sides. Do the same for y .

3. Read. The center comes from the brackets. The number on the right is r^2 . The radius is its square root.

Example: $x^2 + y^2 - 4x - 2y + 1 = 0$
 becomes $(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$
 so $(x - 2)^2 + (y - 1)^2 = 4$. The dot is the center $(2, 1)$. The radius is 2.

Leave the radius exact: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$. Do not use a decimal. Write the center as a pair, like $(2, 1)$.

Not a circle: if the number on the right is 0 or less than 0, the equation is not a circle.

1 $x^2 + y^2 - 4x - 6y + 16 = 0$

not a circle → **go to box 8**

center $(2, 3)$, radius $\sqrt{3}$ → **go to box 2**

Hint: Look at the number on the right after both squares.

2 $2x^2 + 2y^2 + 4x - 8y = 40$

center $(-1, 2)$, radius 5 → **go to box 6**

center $(-1, 2)$, radius $2\sqrt{5}$ → **go to box 3**

Hint: Divide every term by the factor. Then check both sides.

3 $x^2 - 6x + y^2 + 4y - 3 = 0$

center $(3, -2)$, radius 16 → **go to box 2**

center $(3, -2)$, radius 4 → **go to box 6**

Hint: Check what the number on the right stands for.

4 $x^2 - 8y + y^2 + 6x + 9 = 0$

not a circle → **go to box 2**

center $(-3, 4)$, radius 4 → **go to box 7**

Hint: Check that each new number is on both sides.

5 $x^2 + y^2 - 12x + 2y + 1 = 0$

center $(-6, 1)$, radius 6 → **go to box 6**

center $(6, -1)$, radius 6 → **go to box 1**

Hint: Each bracket gives one coordinate. Look at its sign.

6 $x^2 + y^2 + 8x - 2y = 8$

center $(4, -1)$, radius 5 → **go to box 5**

center $(-4, 1)$, radius 5 → **go to box 1**

Hint: Look at the sign inside each bracket.

7 $x^2 + y^2 - 4y + 9 = 10x$

center $(5, 2)$, radius $2\sqrt{5}$ → **you are out**

center $(-5, 2)$, radius $2\sqrt{5}$ → **go to box 3**

Hint: When the x term moves to the left, what happens to its sign?

8 $x^2 - 2x + y^2 + 6y = 8$

center $(1, -3)$, radius $3\sqrt{2}$ → **go to box 4**

center $(2, -6)$, radius $4\sqrt{3}$ → **go to box 5**

Hint: Check which number goes inside each bracket.

Write every box number here, in order.

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The route, in order. Boxes 2 and 5 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table. On this sheet the radius is exact, in simplest radical form, and an equation whose number on the right is 0 or less is not a circle (0 gives a single point, less than 0 gives no points). Some textbooks use other definitions, so tell the class which ones this sheet uses.

3 → 6 → 1 → 8 → 4 → 7 → out

	The box's question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 3 $x^2 - 6x + y^2 + 4y - 3 = 0$. Find the center and the radius.	center (3, -2), radius 4. Half of -6 is -3, so the number to add is $(-3)^2 = 9$. Half of 4 is 2, so the number to add is $2^2 = 4$. Move -3 to the right and add 9 and 4 to both sides: $(x - 3)^2 + (y + 2)^2 = 3 + 9 + 4 = 16$. The center is (3, -2) and the radius is $\sqrt{16} = 4$.	"center (3, -2), radius 16" goes to box 2. The student stops at $r^2 = 16$ and writes 16 as the radius.
2	Box 6 $x^2 + y^2 + 8x - 2y = 8$. Find the center and the radius.	center (-4, 1), radius 5. Half of 8 is 4, so the number to add is $4^2 = 16$. Half of -2 is -1, so the number to add is $(-1)^2 = 1$. Add 16 and 1 to both sides: $(x + 4)^2 + (y - 1)^2 = 8 + 16 + 1 = 25$. The center is (-4, 1) and the radius is $\sqrt{25} = 5$.	"center (4, -1), radius 5" goes to box 5. The student copies the signs in the brackets of $(x + 4)^2 + (y - 1)^2 = 25$ into the center.
3	Box 1 $x^2 + y^2 - 4x - 6y + 16 = 0$. Find the center and the radius.	not a circle. Half of -4 is -2, so the number to add is $(-2)^2 = 4$. Half of -6 is -3, so the number to add is $(-3)^2 = 9$. Move 16 to the right and add 4 and 9 to both sides: $(x - 2)^2 + (y - 3)^2 = -16 + 4 + 9 = -3$. The number on the right is -3, which is less than 0, so no point fits: it is not a circle.	"center (2, 3), radius $\sqrt{3}$ " goes to box 2. The student takes the square root of 3 and drops the minus sign. A square cannot be negative, so nothing fits.
4	Box 8 $x^2 - 2x + y^2 + 6y = 8$. Find the center and the radius.	center (1, -3), radius $3\sqrt{2}$. Half of -2 is -1, so the number to add is $(-1)^2 = 1$. Half of 6 is 3, so the number to add is $3^2 = 9$. Add 1 and 9 to both sides: $(x - 1)^2 + (y + 3)^2 = 8 + 1 + 9 = 18$. The center is (1, -3) and the radius is $\sqrt{18} = 3\sqrt{2}$.	"center (2, -6), radius $4\sqrt{3}$ " goes to box 5. The student squares 2 and 6 without halving, adds 4 and 36, and writes $(x - 2)^2 + (y + 6)^2 = 48$.
5	Box 4 $x^2 - 8y + y^2 + 6x + 9 = 0$. Find the center and the radius.	center (-3, 4), radius 4. Half of 6 is 3, so the number to add is $3^2 = 9$. Half of -8 is -4, so the number to add is $(-4)^2 = 16$. Move 9 to the right and add 9 and 16 to both sides: $(x + 3)^2 + (y - 4)^2 = -9 + 9 + 16 = 16$. The center is (-3, 4) and the radius is $\sqrt{16} = 4$.	"not a circle" goes to box 2. The student adds 9 and 16 on the left only, so the right side stays -9, which is less than 0.
6	Box 7 $x^2 + y^2 - 4y + 9 = 10x$. Find the center and the radius.	center (5, 2), radius $2\sqrt{5}$. Move the x term to the left, changing its sign. Half of -10 is -5, so the number to add is $(-5)^2 = 25$. Half of -4 is -2, so the number to add is $(-2)^2 = 4$. Move 9 to the right and add 25 and 4 to both sides: $(x - 5)^2 + (y - 2)^2 = -9 + 25 + 4 = 20$. The center is (5, 2) and the radius is $\sqrt{20} = 2\sqrt{5}$.	"center (-5, 2), radius $2\sqrt{5}$ " goes to box 5. The student moves the x term to the left without changing its sign, so the x of the center flips.
	Box 2 $2x^2 + 2y^2 + 4x - 8y = 40$. Find the center and the radius.	center (-1, 2), radius 5. Divide every term by 2. Half of 2 is 1, so the number to add is $1^2 = 1$. Half of -4 is -2, so the number to add is $(-2)^2 = 4$. Add 1 and 4 to both sides: $(x + 1)^2 + (y - 2)^2 = 20 + 1 + 4 = 25$. The center is (-1, 2) and the radius is $\sqrt{25} = 5$.	"center (-1, 2), radius $2\sqrt{5}$ " goes to box 3. The student adds 1 and 4 on the left only, so the right side stays 20.
	Box 5 $x^2 + y^2 - 12x + 2y + 1 = 0$. Find the center and the radius.	center (6, -1), radius 6. Half of -12 is -6, so the number to add is $(-6)^2 = 36$. Half of 2 is 1, so the number to add is $1^2 = 1$. Move 1 to the right and add 36 and 1 to both sides: $(x - 6)^2 + (y + 1)^2 = -1 + 36 + 1 = 36$. The center is (6, -1) and the radius is $\sqrt{36} = 6$.	"center (-6, 1), radius 6" goes to box 6. The student copies the signs in the brackets of $(x - 6)^2 + (y + 1)^2 = 36$ into the center.

How to check any box. Expand the answer back: $(x - h)^2 + (y - k)^2 = r^2$, multiplied out and moved around, must give the box's equation term for term. Or step one radius to the right of the center and check that the point fits the box's equation.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.