

MYSTERY PICTURE



EQUATION OF A CIRCLE: COMPLETING THE SQUARE

Is the Next Line Right? Completing the Square

25 lines of work · the new numbers on both sides · fill the right ones · a picture appears

FREE

¹ $x^2 + y^2 - 4x + 6y - 3 = 0$ $(x^2 - 4x + 4)$ $+ (y^2 + 6y + 9) = 3$	² $x^2 - 10x + y^2 - 6y = -25$ $(x^2 - 10x - 25)$ $+ (y^2 - 6y - 9) = -25 - 25 - 9$	³ $x^2 + y^2 - 6x + 4y - 12 = 0$ $(x^2 - 6x + 9)$ $+ (y^2 + 4y + 4) = 12 + 9 + 4$	⁴ $x^2 + y^2 - 4x - 10y = 7$ $(x^2 - 4x + 4)$ $+ (y^2 - 10y + 25) = 7 + 4 + 25$	⁵ $x^2 + y^2 - 6 = -6x - 2y$ $(x^2 + 6x + 9)$ $+ (y^2 + 2y + 1) = 6 + 9 + 1$
⁶ $x^2 + 8x + y^2 - 2y = 8$ $(x^2 + 8x + 16)$ $+ (y^2 - 2y + 1) = 8 + 16$	⁷ $2x^2 + 2y^2 - 16x - 4y - 6 = 0$ $(x^2 - 8x + 16)$ $+ (y^2 - 2y + 1) = 3 + 16 + 1$	⁸ $x^2 + y^2 + 8x - 5 = 6y$ $(x^2 + 8x + 16)$ $+ (y^2 + 6y + 9) = 5 + 16 + 9$	⁹ $x^2 + y^2 + 10x + 4y = -4$ $(x^2 + 10x + 25)$ $+ (y^2 + 4y + 4) = -4 + 25 + 4$	¹⁰ $x^2 + y^2 - 10x + 4y - 7 = 0$ $(x^2 - 10x + 25)$ $+ (y^2 + 4y + 4) = 7 + 25 + 4$
¹¹ $x^2 + y^2 + 2x + 2y - 9 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 + 2y + 1) = -9 + 1 + 1$	¹² $x^2 + y^2 + 10y - 23 = 2x$ $(x^2 - 2x + 1)$ $+ (y^2 + 10y + 25) = 23 + 1 + 25$	¹³ $x^2 - 8x + y^2 + 6y - 15 = 0$ $(x^2 - 8x + 4)$ $+ (y^2 + 6y + 9) = 15 + 4 + 9$	¹⁴ $x^2 + y^2 + 4x + 6y - 7 = 0$ $(x^2 + 4x + 4)$ $+ (y^2 + 6y + 9) = 7$	¹⁵ $x^2 + y^2 + 8y + 15 = -4x$ $(x^2 - 4x + 4)$ $+ (y^2 + 8y + 16) = -15 + 4 + 16$
¹⁶ $x^2 + y^2 - 2x - 4y = 4$ $(x^2 - 2x + 1)$ $+ (y^2 - 4y + 4) = 4 + 1 + 4$	¹⁷ $y^2 + 2y + x^2 - 4x - 9 = 0$ $(x^2 - 4x + 4)$ $+ (y^2 + 2y + 1) = 9 + 4 + 1$	¹⁸ $x^2 - 4y + y^2 + 4x + 4 = 0$ $(x^2 + 4x + 4)$ $+ (y^2 - 4y + 4) = 4 + 4 + 4$	¹⁹ $x^2 + y^2 + 2y - 5 = 6x$ $(x^2 + 6x + 9)$ $+ (y^2 + 2y + 1) = 5 + 9 + 1$	²⁰ $2x^2 - 8x + 2y^2 - 16y = 0$ $(x^2 - 4x + 16)$ $+ (y^2 - 8y + 64) = 16 + 64$
²¹ $2x^2 + 2y^2 + 4x - 16y - 8 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 - 8y + 16) = 8 + 1 + 16$	²² $x^2 + 2x + y^2 - 8y + 8 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 - 8y + 16) = -8 + 1 + 16$	²³ $3x^2 - 6x + 3y^2 - 18y = -12$ $(x^2 - 2x + 1)$ $+ (y^2 - 6y + 9) = -4 + 1 + 9$	²⁴ $x^2 + y^2 = 6x + 8y$ $(x^2 - 6x + 9)$ $+ (y^2 - 8y + 16) = 9 + 16$	²⁵ $x^2 - 10y + y^2 + 6x + 32 = 0$ $(x^2 + 6x + 9)$ $+ (y^2 - 10y + 25) = -32 + 9 + 25$

1. **Give each student page 3** and a pencil. No calculator is needed.
2. **Say this once:** *“Each square shows an equation and a student’s next line. Is the line right? Check the signs, check each number added, and check that the right side gets the same numbers. If it is right, fill the square with slanted lines. If not, leave it white.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows the three moves and a worked example with its circle.
Answers	page 4, for you.

Every square is the first half of completing the square. The student checks one line: are the terms on the correct sides with the correct signs, is each number added the square of half the number in front of x or y , and does the right side get the same new numbers? Nine lines are right and sixteen are wrong.

Each white square has one of the mistakes students make most. Two add the new numbers on the left side only. Two write the square of a negative half as negative. One adds the new y number on the left side only. Three move an x or y term across the $=$ sign without changing its sign. Three square the numbers without halving them. Two move the number across the $=$ sign without changing its sign. Two add the half instead of its square. One divides only the x and y terms by the common factor.

If part of a student’s picture looks wrong, ask about that square: *“What did you add on the left, and is it on the right too?”* Guessing does not work. Every simple guessing rule, such as filling every square whose right side has three numbers, gets at least nine squares wrong.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
chooseyourlesson.com — 135 free geometry lessons.

Mystery Picture — is the next line right?

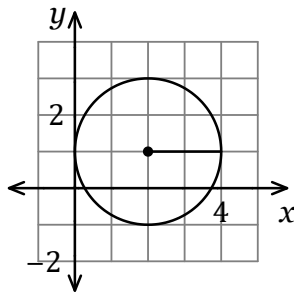
Completing the square

Name _____

Period _____

Date _____

Each square has an equation and, under it, a student's next line. In that line both squares are complete, and the new numbers are written on both sides. **Is the line right?** If yes, fill the square with slanted lines. If no, leave it white. A picture appears. If part of it is missing, or one square does not fit, check that square again.



Built form: $(x - h)^2 + (y - k)^2 = r^2$. The center is (h, k) . The radius is r .

1. Group and clear. Put the x terms together and the y terms together. Move the number alone to the right side. A term that crosses the $=$ sign changes its sign. If every term has a common factor, divide every term by it.

2. Complete both squares. Halve the number in front of x . Square it. Add it to **both** sides. Do the same for y .

Example: $x^2 + y^2 - 4x - 2y + 1 = 0$

becomes $(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$

so $(x - 2)^2 + (y - 1)^2 = 4$. The dot is the center $(2, 1)$. The radius is 2.

A line is right when all four are true. Each term that moves across the $=$ sign changes its sign. A common factor comes out of every term, the number too. Each bracket adds (half the number in front of x or y)². The right side gets the same new numbers.

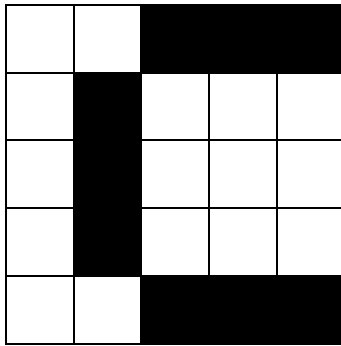
1 $x^2 + y^2 - 4x + 6y - 3 = 0$ $(x^2 - 4x + 4)$ $+ (y^2 + 6y + 9) = 3$	2 $x^2 - 10x + y^2 - 6y = -25$ $(x^2 - 10x - 25)$ $+ (y^2 - 6y - 9)$ $= -25 - 25 - 9$	3 $x^2 + y^2 - 6x + 4y - 12 = 0$ $(x^2 - 6x + 9)$ $+ (y^2 + 4y + 4)$ $= 12 + 9 + 4$	4 $x^2 + y^2 - 4x - 10y = 7$ $(x^2 - 4x + 4)$ $+ (y^2 - 10y + 25)$ $= 7 + 4 + 25$	5 $x^2 + y^2 - 6 = -6x - 2y$ $(x^2 + 6x + 9)$ $+ (y^2 + 2y + 1) = 6 + 9 + 1$
6 $x^2 + 8x + y^2 - 2y = 8$ $(x^2 + 8x + 16)$ $+ (y^2 - 2y + 1) = 8 + 16$	7 $2x^2 + 2y^2 - 16x - 4y - 6 = 0$ $(x^2 - 8x + 16)$ $+ (y^2 - 2y + 1)$ $= 3 + 16 + 1$	8 $x^2 + y^2 + 8x - 5 = 6y$ $(x^2 + 8x + 16)$ $+ (y^2 + 6y + 9)$ $= 5 + 16 + 9$	9 $x^2 + y^2 + 10x + 4y = -4$ $(x^2 + 10x + 25)$ $+ (y^2 + 4y + 4)$ $= -4 + 25 + 4$	10 $x^2 + y^2 - 10x + 4y - 7 = 0$ $(x^2 - 10x - 25)$ $+ (y^2 + 4y + 4)$ $= 7 - 25 + 4$
11 $x^2 + y^2 + 2x + 2y - 9 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 + 2y + 1)$ $= -9 + 1 + 1$	12 $x^2 + y^2 + 10y - 23 = 2x$ $(x^2 - 2x + 1)$ $+ (y^2 + 10y + 25)$ $= 23 + 1 + 25$	13 $x^2 - 8x + y^2 + 6y - 15 = 0$ $(x^2 - 8x + 4)$ $+ (y^2 + 6y + 9)$ $= 15 + 4 + 9$	14 $x^2 + y^2 + 4x + 6y - 7 = 0$ $(x^2 + 4x + 4)$ $+ (y^2 + 6y + 9) = 7$	15 $x^2 + y^2 + 8y + 15 = -4x$ $(x^2 - 4x + 4)$ $+ (y^2 + 8y + 16)$ $= -15 + 4 + 16$
16 $x^2 + y^2 - 2x - 4y = 4$ $(x^2 - 2x + 1)$ $+ (y^2 - 4y + 4)$ $= 4 + 1 + 4$	17 $y^2 + 2y + x^2 - 4x - 9 = 0$ $(x^2 - 4x + 4)$ $+ (y^2 + 2y + 1) = 9 + 4 + 1$	18 $x^2 - 4y + y^2 + 4x + 4 = 0$ $(x^2 + 4x + 4)$ $+ (y^2 - 4y + 4) = 4 + 4 + 4$	19 $x^2 + y^2 + 2y - 5 = 6x$ $(x^2 + 6x + 9)$ $+ (y^2 + 2y + 1) = 5 + 9 + 1$	20 $2x^2 - 8x + 2y^2 - 16y = 0$ $(x^2 - 4x + 16)$ $+ (y^2 - 8y + 64) = 16 + 64$
21 $2x^2 + 2y^2 + 4x - 16y - 8 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 - 8y + 16)$ $= 8 + 1 + 16$	22 $x^2 + 2x + y^2 - 8y + 8 = 0$ $(x^2 + 2x + 1)$ $+ (y^2 - 8y + 4)$ $= -8 + 1 + 4$	23 $3x^2 - 6x + 3y^2 - 18y = -12$ $(x^2 - 2x + 1)$ $+ (y^2 - 6y + 9)$ $= -4 + 1 + 9$	24 $x^2 + y^2 = 6x + 8y$ $(x^2 - 6x + 9)$ $+ (y^2 - 8y + 16) = 9 + 16$	25 $x^2 - 10y + y^2 + 6x + 32 = 0$ $(x^2 + 6x + 9)$ $+ (y^2 - 10y + 25)$ $= -32 + 9 + 25$



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Mystery Picture — answers

For the teacher



The finished picture is **the letter C**. Nine squares are filled.

A square is filled when its line is right: each bracket is a perfect square, like $x^2 - 6x + 9 = (x - 3)^2$, and the line is the same equation as the one above it, so every new number is on both sides. Some textbooks use other definitions, so tell the class which ones this sheet uses.

The square	Answer	Why
1 $x^2 + y^2 - 4x + 6y - 3 = 0$ $(x^2 - 4x + 4) + (y^2 + 6y + 9) = 3$	white	The new numbers 4 and 9 are added on the left only. The right side needs them too.
2 $x^2 - 10x + y^2 - 6y = -25$ $(x^2 - 10x - 25) + (y^2 - 6y - 9) = -25 - 25 - 9$	white	Half of -10 is -5 , and $(-5)^2 = 25$ is positive. The line adds -25 , so the x bracket is not a perfect square.
3 $x^2 + y^2 - 6x + 4y - 12 = 0$ $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$	fill	The number changes sign as it moves right. $(6 \div 2)^2 = 9$ and $(4 \div 2)^2 = 4$ are added to both sides.
4 $x^2 + y^2 - 4x - 10y = 7$ $(x^2 - 4x + 4) + (y^2 - 10y + 25) = 7 + 4 + 25$	fill	$(4 \div 2)^2 = 4$ and $(10 \div 2)^2 = 25$ are added to both sides.
5 $x^2 + y^2 - 6 = -6x - 2y$ $(x^2 + 6x + 9) + (y^2 + 2y + 1) = 6 + 9 + 1$	fill	The x and y terms change sign as they move left, the number changes sign as it moves right. $(6 \div 2)^2 = 9$ and $(2 \div 2)^2 = 1$ are added to both sides.
6 $x^2 + 8x + y^2 - 2y = 8$ $(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8 + 16$	white	The new y number, 1, is added on the left but not on the right.
7 $2x^2 + 2y^2 - 16x - 4y - 6 = 0$ $(x^2 - 8x + 16) + (y^2 - 2y + 1) = 3 + 16 + 1$	fill	Every term is divided by 2, the number changes sign as it moves right. $(8 \div 2)^2 = 16$ and $(2 \div 2)^2 = 1$ are added to both sides.
8 $x^2 + y^2 + 8x - 5 = 6y$ $(x^2 + 8x + 16) + (y^2 + 6y + 9) = 5 + 16 + 9$	white	The term $6y$ moves to the left side, so it becomes $-6y$. The line keeps $6y$.
9 $x^2 + y^2 + 10x + 4y = -4$ $(x^2 + 10x + 25) + (y^2 + 4y + 16) = -4 + 25 + 16$	white	The numbers 10 and 4 are squared without halving. Half of 10 is 5, so add 25. Half of 4 is 2, so add 4.
10 $x^2 + y^2 - 10x + 4y - 7 = 0$ $(x^2 - 10x - 25) + (y^2 + 4y + 4) = 7 - 25 + 4$	white	Half of -10 is -5 , and $(-5)^2 = 25$ is positive. The line adds -25 , so the x bracket is not a perfect square.
11 $x^2 + y^2 + 2x + 2y - 9 = 0$ $(x^2 + 2x + 1) + (y^2 + 2y + 1) = -9 + 1 + 1$	white	The number -9 moves to the right side, so it becomes 9. The line keeps -9 .
12 $x^2 + y^2 + 10y - 23 = 2x$ $(x^2 - 2x + 1) + (y^2 + 10y + 25) = 23 + 1 + 25$	fill	The x term changes sign as it moves left, the number changes sign as it moves right. $(2 \div 2)^2 = 1$ and $(10 \div 2)^2 = 25$ are added to both sides.
13 $x^2 - 8x + y^2 + 6y - 15 = 0$ $(x^2 - 8x + 4) + (y^2 + 6y + 3) = 15 + 4 + 3$	white	Half of 8 is 4, and the number to add is $4^2 = 16$, not 4. The x bracket is not a perfect square.
14 $x^2 + y^2 + 4x + 6y - 7 = 0$ $(x^2 + 4x + 4) + (y^2 + 6y + 9) = 7$	white	The new numbers 4 and 9 are added on the left only. The right side needs them too.
15 $x^2 + y^2 + 8y + 15 = -4x$ $(x^2 - 4x + 4) + (y^2 + 8y + 16) = -15 + 4 + 16$	white	The term $-4x$ moves to the left side, so it becomes $4x$. The line keeps $-4x$.
16 $x^2 + y^2 - 2x - 4y = 4$ $(x^2 - 2x + 4) + (y^2 - 4y + 16) = 4 + 4 + 16$	white	The numbers 2 and 4 are squared without halving. Half of 2 is 1, so add 1. Half of 4 is 2, so add 4.
17 $y^2 + 2y + x^2 - 4x - 9 = 0$ $(x^2 - 4x + 4) + (y^2 + 2y + 1) = 9 + 4 + 1$	fill	The number changes sign as it moves right. $(4 \div 2)^2 = 4$ and $(2 \div 2)^2 = 1$ are added to both sides.
18 $x^2 - 4y + y^2 + 4x + 4 = 0$ $(x^2 + 4x + 4) + (y^2 - 4y + 4) = 4 + 4 + 4$	white	The number 4 moves to the right side, so it becomes -4 . The line keeps 4.
19 $x^2 + y^2 + 2y - 5 = 6x$ $(x^2 + 6x + 9) + (y^2 + 2y + 1) = 5 + 9 + 1$	white	The term $6x$ moves to the left side, so it becomes $-6x$. The line keeps $6x$.
20 $2x^2 - 8x + 2y^2 - 16y = 0$ $(x^2 - 4x + 16) + (y^2 - 8y + 64) = 16 + 64$	white	The numbers 4 and 8 are squared without halving. Half of 4 is 2, so add 4. Half of 8 is 4, so add 16.
21 $2x^2 + 2y^2 + 4x - 16y - 8 = 0$ $(x^2 + 2x + 1) + (y^2 - 8y + 16) = 8 + 1 + 16$	white	Every term is divided by 2, the number too, so the number on the right is 4. The line divides the x and y terms but leaves the number 8.
22 $x^2 + 2x + y^2 - 8y + 8 = 0$ $(x^2 + 2x + 1) + (y^2 - 8y + 4) = -8 + 1 + 4$	white	Half of 8 is 4, and the number to add is $4^2 = 16$, not 4. The y bracket is not a perfect square.
23 $3x^2 - 6x + 3y^2 - 18y = -12$ $(x^2 - 2x + 1) + (y^2 - 6y + 9) = -4 + 1 + 9$	fill	Every term is divided by 3. $(2 \div 2)^2 = 1$ and $(6 \div 2)^2 = 9$ are added to both sides.
24 $x^2 + y^2 = 6x + 8y$ $(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$	fill	The x and y terms change sign as they move left, the number changes sign as it moves right. $(6 \div 2)^2 = 9$ and $(8 \div 2)^2 = 16$ are added to both sides.
25 $x^2 - 10y + y^2 + 6x + 32 = 0$ $(x^2 + 6x + 9) + (y^2 - 10y + 25) = -32 + 9 + 25$	fill	The number changes sign as it moves right. $(6 \div 2)^2 = 9$ and $(10 \div 2)^2 = 25$ are added to both sides.