

SCAVENGER HUNT



EQUATION OF A CIRCLE: COMPLETING THE SQUARE

Untidy Equations: Find the Center and the Radius

Completing the square in both letters · terms in any order · the cards check the answers

FREE

ANSWER

**center $(-6, -1)$
radius 7**

Complete the square. Find the center and the radius.

$$2x^2 + 4x + 2y^2 + 8y = 62$$

Move every x term and every y term to the left side first.

F

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1. **Tape the 16 cards around the room**, in any order. Each card has a large letter in its bottom left corner. Start each group at a *different* one of these eight cards: *F M T D L E C K*.
2. **Say this once:** “Each card has the equation of a circle. Put the x terms together and the y terms together, with the number on the right. Complete both squares, and add each new number to both sides. Read the center and the radius. Then walk to the card with your answer at the top.”
3. **Each group writes one line per card.** Eight right answers bring the group back to its first card.

16 cards	pages 3–10, one copy. There are two cards on each sheet. Cut the whole stack once.
Record sheet	page 11, one per student. It shows the three moves and a worked example.
Answers	pages 12–13, for you only. Do not put them up in the room.

Every card is the same job in a different layout. The terms may be out of order, the number may stand on the left, an x or y term may stand on the right, and every term may share a factor. Students first put the equation in order, then complete both squares. For example, $x^2 + y^2 - 4x - 2y + 1 = 0$ becomes $(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$, so $(x - 2)^2 + (y - 1)^2 = 4$: the center is $(2, 1)$ and the radius is 2.

Eight of the sixteen cards make the ring. A group reaches the other eight only by a wrong answer, but all the cards look the same. Four mistakes take turns around the ring: adding the new numbers to the left side only, copying the signs of the brackets into the center, squaring the number in front of x or y without halving it, and writing r^2 as the radius. Each mistake lands on a card off the ring. That card has the same circle as an earlier card in a new layout, and its answer sends the group back to the card it got wrong.

The check is the length of the group’s list. A clean run is **eight lines**. Each wrong answer adds two. Every card has a line in italics under its question. It tells a group that arrived by a wrong answer what to check. A group that arrived by a right answer skips it. Guessing does not work: no simple rule, such as walking to the nearest center, gets more than five of the cards.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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ANSWER

center (2, 2)
radius $2\sqrt{6}$

Complete the square. Find the center and the radius.

$$x^2 + y^2 + 8 = -12x + 2y$$

Which number goes in the bracket? Half of the number in the x or y term.

A

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ANSWER

center (-2, 6)
radius $5\sqrt{2}$

Complete the square. Find the center and the radius.

$$x^2 + y^2 - 10x - 2y = 23$$

Halve the number in front of x first. Then square the half.

B

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ANSWER

center $(-6, 1)$
radius $\sqrt{29}$

Complete the square. Find the center and the radius.

$$x^2 - 2x + y^2 - 2y = 16$$

Is the number on the right more than 0? Only then is the equation a circle.

C

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ANSWER

center $(-1, 3)$
radius $2\sqrt{5}$

Complete the square. Find the center and the radius.

$$x^2 - 12y + y^2 + 10x + 45 = 0$$

Put the x terms next to each other and the y terms next to each other.

D

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ANSWER

center (6, -3)
radius 9

Complete the square. Find the center and the radius.

$$x^2 + 12x + y^2 - 2y + 8 = 0$$

Check one point. Go one radius to the right of the center. Does it fit the equation?

E

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ANSWER

center (-6, -1)
radius 7

Complete the square. Find the center and the radius.

$$2x^2 + 4x + 2y^2 + 8y = 62$$

Move every x term and every y term to the left side first.

F

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ANSWER

center $(-5, 6)$
radius 16

Complete the square. Find the center and the radius.

$$x^2 + 2x + y^2 - 6y = 10$$

The number on the right is r^2 . Take its square root for the radius.

G

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ANSWER

center $(-1, -2)$
radius $\sqrt{31}$

Complete the square. Find the center and the radius.

$$2x^2 + 2y^2 - 24 = -24x - 4y$$

Add each new number to both sides of the equation, not just the left.

H

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ANSWER

**center $(-6, -1)$
radius 49**

Complete the square. Find the center and the radius.

$$x^2 + y^2 - 16 = 2x + 2y$$

Is the radius r or r^2 ? The radius is the square root of the number.

J

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ANSWER

**center $(1, 1)$
radius $3\sqrt{2}$**

Complete the square. Find the center and the radius.

$$x^2 + y^2 + 12x - 12 = -2y$$

Write each new number in the same line as the squares it makes.

K

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ANSWER

**center $(-5, 6)$
radius 4**

Complete the square. Find the center and the radius.

$$6y + x^2 - 12x + y^2 = 36$$

A term that crosses the equals sign changes its sign.

L

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ANSWER

**center $(-1, -2)$
radius 6**

Complete the square. Find the center and the radius.

$$x^2 + y^2 - 10x - 2y - 23 = 0$$

Does every term share a factor? Divide every term by it, the number too.

M

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ANSWER

center (6, -3)
radius 6

Complete the square. Find the center and the radius.

$$x^2 + y^2 + 10x + 45 = 12y$$

The right side gets every number the left side gets. Check the right side.

N

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ANSWER

center (6, -1)
radius $\sqrt{29}$

Complete the square. Find the center and the radius.

$$x^2 + y^2 - 12x + 6y - 36 = 0$$

Is each coordinate of the center the opposite sign of the number in its bracket?

P

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ANSWER

**center $(-5, -1)$
radius 7**

Complete the square. Find the center and the radius.

$$x^2 + 4y + y^2 + 2x - 31 = 0$$

In $(x + 6)^2$, the center's x is -6 . Check the sign of each coordinate.

Q

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ANSWER

**center $(5, 1)$
radius 7**

Complete the square. Find the center and the radius.

$$x^2 + y^2 + 2x - 10 = 6y$$

A radius can be a square root, like $3\sqrt{5}$. Leave it exact.

T

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Scavenger Hunt — where I went

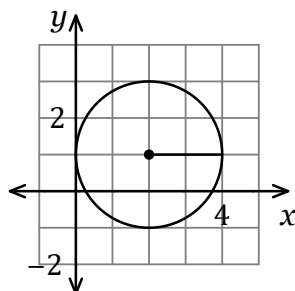
Completing the square

Name _____

Period _____

Date _____

Find the center and the radius. Go to the card with your answer at the top. A clean run is eight lines. Each card has a line in italics under its question. **If you are on a card because you went wrong, its line says what to check. If you were right, skip it.**



Built form: $(x - h)^2 + (y - k)^2 = r^2$. The center is (h, k) . The radius is r .

1. Group and clear. Put the x terms together and the y terms together. Move the number alone to the right side. A term that crosses the $=$ sign changes its sign. If every term has a common factor, divide every term by it.

2. Complete both squares. Halve the number in front of x . Square it. Add it to **both** sides. Do the same for y .

3. Read. The center comes from the brackets. The number on the right is r^2 . The radius is its square root.

Example: $x^2 + y^2 - 4x - 2y + 1 = 0$ becomes $(x^2 - 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$, so $(x - 2)^2 + (y - 1)^2 = 4$. The dot is the center $(2, 1)$. The radius is 2.

Leave the radius exact: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$. Do not use a decimal. Write the center as a pair, like $(2, 1)$.

Card letter	Both squares completed, the new numbers on both sides	Center	Radius
1			
2			
3			
4			
5			
6			
7			
8			
9			

Why is a circle always so calm?

Find each of your answers (center, radius) in the list. Write its letter in the box with the same number. Read the boxes you filled, left to right. **If they do not spell the answer, one of your answers is wrong.**

1. $(-6, -1)$, $7 = C$ 2. $(-6, -1)$, $49 = N$ 3. $(-6, 1)$, $\sqrt{29} = E$ 4. $(-5, -1)$, $7 = T$ 5. $(-5, 6)$, $4 = N$ 6. $(-5, 6)$, $16 = E$
 7. $(-2, 6)$, $5\sqrt{2} = C$ 8. $(-1, -2)$, $\sqrt{31} = E$ 9. $(-1, -2)$, $6 = T$ 10. $(-1, 3)$, $2\sqrt{5} = E$ 11. $(1, 1)$, $3\sqrt{2} = R$ 12. $(2, 2)$, $2\sqrt{6} = D$
 13. $(5, 1)$, $7 = E$ 14. $(6, -3)$, $6 = R$ 15. $(6, -3)$, $9 = D$ 16. $(6, -1)$, $\sqrt{29} = E$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16



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Scavenger Hunt — answers

For the teacher

The ring in order, with the answer found on each card as center and radius. A group may begin anywhere on it. Wherever it begins, eight right answers bring it back to that card.

F (-1, -2), 6 → M (5, 1), 7 → T (-1, 3), $2\sqrt{5}$ → D (-5, 6), 4 → L (6, -3), 9 → E (-6, 1), $\sqrt{29}$ → C (1, 1), $3\sqrt{2}$ → K (-6, -1), 7 → F

Why is a circle always so calm? The answer is *CENTERED*! It is read along the boxes of the record sheet, which run in this order: by the center's x, then its y, then the radius.

Card	The card's equation	Answer, and why	The common wrong answer on this card, and why	Letter of its top
1 F	$2x^2 + 4x + 2y^2 + 8y = 62$	center (-1, -2), radius 6. Divide every term by 2. Then $(x^2 + 2x + 1) + (y^2 + 4y + 4) = 31 + 1 + 4$, so $(x + 1)^2 + (y + 2)^2 = 36$. The center is (-1, -2) and the radius is $\sqrt{36} = 6$.	center (-1, -2), radius $\sqrt{31}$: the group adds 1 and 4 on the left only. The right side stays 31, so $r^2 = 31$.	C
2 M	$x^2 + y^2 - 10x - 2y - 23 = 0$	center (5, 1), radius 7. Move the number to the right, changing its sign. Then $(x^2 - 10x + 25) + (y^2 - 2y + 1) = 23 + 25 + 1$, so $(x - 5)^2 + (y - 1)^2 = 49$. The center is (5, 1) and the radius is $\sqrt{49} = 7$.	center (-5, -1), radius 7: the group copies the signs in the brackets of $(x - 5)^2 + (y - 1)^2 = 49$. The center has the opposite signs.	T
3 T	$x^2 + y^2 + 2x - 10 = 6y$	center (-1, 3), radius $2\sqrt{5}$. Move the y term to the left, changing the sign. Move the number to the right, changing its sign. Then $(x^2 + 2x + 1) + (y^2 - 6y + 9) = 10 + 1 + 9$, so $(x + 1)^2 + (y - 3)^2 = 20$. The center is (-1, 3) and the radius is $\sqrt{20} = 2\sqrt{5}$.	center (-2, 6), radius $5\sqrt{2}$: the group squares the numbers in front of x and y without halving them. It adds 4 and 36 to both sides and writes $(x + 2)^2 + (y - 6)^2 = 50$.	E
4 D	$x^2 - 12y + y^2 + 10x + 45 = 0$	center (-5, 6), radius 4. Move the number to the right, changing its sign. Then $(x^2 + 10x + 25) + (y^2 - 12y + 36) = -45 + 25 + 36$, so $(x + 5)^2 + (y - 6)^2 = 16$. The center is (-5, 6) and the radius is $\sqrt{16} = 4$.	center (-5, 6), radius 16: the group stops at $r^2 = 16$ and writes 16 as the radius.	E
5 L	$6y + x^2 - 12x + y^2 = 36$	center (6, -3), radius 9. Then $(x^2 - 12x + 36) + (y^2 + 6y + 9) = 36 + 36 + 9$, so $(x - 6)^2 + (y + 3)^2 = 81$. The center is (6, -3) and the radius is $\sqrt{81} = 9$.	center (6, -3), radius 6: the group adds 36 and 9 on the left only. The right side stays 36, so $r^2 = 36$.	N
6 E	$x^2 + 12x + y^2 - 2y + 8 = 0$	center (-6, 1), radius $\sqrt{29}$. Move the number to the right, changing its sign. Then $(x^2 + 12x + 36) + (y^2 - 2y + 1) = -8 + 36 + 1$, so $(x + 6)^2 + (y - 1)^2 = 29$. The center is (-6, 1) and the radius is $\sqrt{29}$.	center (6, -1), radius $\sqrt{29}$: the group copies the signs in the brackets of $(x + 6)^2 + (y - 1)^2 = 29$. The center has the opposite signs.	D
7 C	$x^2 - 2x + y^2 - 2y = 16$	center (1, 1), radius $3\sqrt{2}$. Then $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 16 + 1 + 1$, so $(x - 1)^2 + (y - 1)^2 = 18$. The center is (1, 1) and the radius is $\sqrt{18} = 3\sqrt{2}$.	center (2, 2), radius $2\sqrt{6}$: the group squares the numbers in front of x and y without halving them. It adds 4 for each letter to both sides and writes $(x - 2)^2 + (y - 2)^2 = 24$.	E
8 K	$x^2 + y^2 + 12x - 12 = -2y$	center (-6, -1), radius 7. Move the y term to the left, changing the sign. Move the number to the right, changing its sign. Then $(x^2 + 12x + 36) + (y^2 + 2y + 1) = 12 + 36 + 1$, so $(x + 6)^2 + (y + 1)^2 = 49$. The center is (-6, -1) and the radius is $\sqrt{49} = 7$.	center (-6, -1), radius 49: the group stops at $r^2 = 49$ and writes 49 as the radius.	R

Scavenger Hunt — the cards off the ring

For the teacher

A group reaches these eight cards only by a wrong answer. The answer at the top of each one is the wrong answer from one ring card. Its equation is the circle of the ring card just before the one the group got wrong, in another layout. So its right answer is the top of the ring card the group got wrong, and the group goes back and tries it again.

Card	The card's equation	Answer, and why	The common wrong answer on this card, and why	Letter of its top
H	$2x^2 + 2y^2 - 24 = -24x - 4y$	center $(-6, -1)$, radius 7. Move the x and y terms to the left, changing the signs. Move the number to the right, changing its sign. Divide every term by 2. Then $(x^2 + 12x + 36) + (y^2 + 2y + 1) = 12 + 36 + 1$, so $(x + 6)^2 + (y + 1)^2 = 49$. The center is $(-6, -1)$ and the radius is $\sqrt{49} = 7$.	—	E
Q	$x^2 + 4y + y^2 + 2x - 31 = 0$	center $(-1, -2)$, radius 6. Move the number to the right, changing its sign. Then $(x^2 + 2x + 1) + (y^2 + 4y + 4) = 31 + 1 + 4$, so $(x + 1)^2 + (y + 2)^2 = 36$. The center is $(-1, -2)$ and the radius is $\sqrt{36} = 6$.	—	T
B	$x^2 + y^2 - 10x - 2y = 23$	center $(5, 1)$, radius 7. Then $(x^2 - 10x + 25) + (y^2 - 2y + 1) = 23 + 25 + 1$, so $(x - 5)^2 + (y - 1)^2 = 49$. The center is $(5, 1)$ and the radius is $\sqrt{49} = 7$.	—	C
G	$x^2 + 2x + y^2 - 6y = 10$	center $(-1, 3)$, radius $2\sqrt{5}$. Then $(x^2 + 2x + 1) + (y^2 - 6y + 9) = 10 + 1 + 9$, so $(x + 1)^2 + (y - 3)^2 = 20$. The center is $(-1, 3)$ and the radius is $\sqrt{20} = 2\sqrt{5}$.	—	E
N	$x^2 + y^2 + 10x + 45 = 12y$	center $(-5, 6)$, radius 4. Move the y term to the left, changing the sign. Move the number to the right, changing its sign. Then $(x^2 + 10x + 25) + (y^2 - 12y + 36) = -45 + 25 + 36$, so $(x + 5)^2 + (y - 6)^2 = 16$. The center is $(-5, 6)$ and the radius is $\sqrt{16} = 4$.	—	R
P	$x^2 + y^2 - 12x + 6y - 36 = 0$	center $(6, -3)$, radius 9. Move the number to the right, changing its sign. Then $(x^2 - 12x + 36) + (y^2 + 6y + 9) = 36 + 36 + 9$, so $(x - 6)^2 + (y + 3)^2 = 81$. The center is $(6, -3)$ and the radius is $\sqrt{81} = 9$.	—	E
A	$x^2 + y^2 + 8 = -12x + 2y$	center $(-6, 1)$, radius $\sqrt{29}$. Move the x and y terms to the left, changing the signs. Move the number to the right, changing its sign. Then $(x^2 + 12x + 36) + (y^2 - 2y + 1) = -8 + 36 + 1$, so $(x + 6)^2 + (y - 1)^2 = 29$. The center is $(-6, 1)$ and the radius is $\sqrt{29}$.	—	D
J	$x^2 + y^2 - 16 = 2x + 2y$	center $(1, 1)$, radius $3\sqrt{2}$. Move the x and y terms to the left, changing the signs. Move the number to the right, changing its sign. Then $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 16 + 1 + 1$, so $(x - 1)^2 + (y - 1)^2 = 18$. The center is $(1, 1)$ and the radius is $\sqrt{18} = 3\sqrt{2}$.	—	N

If a group makes a mistake on a card off the ring, one of the four mistakes leads to another card off the ring, and that card's line in italics says what to check. On card A, moving the x or y terms across the = sign without changing their signs also gives center $(6, -1)$, radius $\sqrt{29}$, so it leads to card P too.

Other mistakes lead to no card. A group might add only one of the two new numbers to the right side, halve only one number, add the half instead of its square, write the square of a negative half as negative, flip one sign of the center, swap its two coordinates, move a term across the = sign without changing its sign (except on card A, above), or divide only some terms by the common factor. Its answer is then not at the top of any card, and the group knows to check its work. A group whose number on the right comes out 0 or less has made a mistake too: every card here is a circle.

On these sheets the radius is exact, in simplest radical form, such as $3\sqrt{2}$ for $\sqrt{18}$. Some textbooks use other definitions, so tell the class which ones this sheet uses.